MATH 108: Introduction to Combinatorics, Winter 2016
HOMEWORK 2
Due Monday, January 25

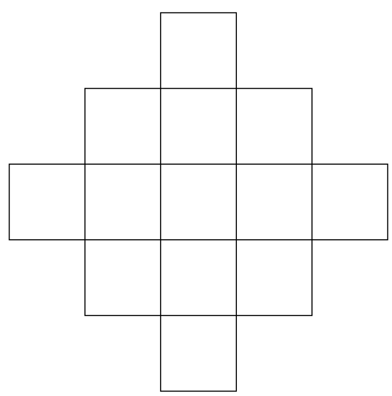
You should solve the homework on your own. Don't use any books or the internet.

Problem 1. Find a construction of orthogonal pairs of Latin squares for $n = 4m$. Start with the following Latin square:

0	1	2	3	4	5	6	7
1	0	3	2	5	4	7	6
2	3	4	5	6	7	0	1
3	2	5	4	7	6	1	0
4	5	6	7	0	1	2	3
5	4	7	6	1	0	3	2
6	7	0	1	2	3	4	5
7	6	1	0	3	2	5	4

First, find a Latin transveral in this square. Then, modify the transversal to obtain 8 disjoint Latin transversals (which solves the case of $n = 8$). Then, figure out how this construction generalizes to every $n = 4m$.

Problem 2. How many dominoes (pieces covering two adjacent squares) can you fit in the following region? Provide a certificate that your solution is optimal.



Problem 3. Prove that every graph with all degrees even is a union of edge-disjoint cycles.

Problem 4. Prove that if there is a labeling of vertices with numbers $r(v)$ such that for each vertex v, at most d neighbors $w \in N(v)$ have $r(w) \leq r(v)$, then the chromatic number is at most $d + 1$.