MATH 108: Introduction to Combinatorics, Winter 2016
HOMEWORK 3
Due Monday, February 1

You should solve the homework on your own. Don't use any books or the internet.

Problem 1. Consider a bipartite graph with vertex parts A, B. Suppose that $S \subseteq A$, $T \subseteq B$, there exists a matching M_1 covering all vertices of S, and a matching M_2 covering all vertices of T. Prove that there is a matching M_3 covering all vertices of $S \cup T$.
Hint: Consider the symmetric difference $M_1 \Delta M_2$.

Problem 2. Consider an instance of stable matching where all the men have the same ranking of the women. (Women might have different rankings of the men.) Prove that there is a unique stable matching and describe what it is.

Problem 3. [Knuth p. 479, exercise 70 (rephrased)]
Find a mapping $f : \{0,1\}^n \to \{0,1\}^n$ such that
(i) $f(f(\sigma)) = \sigma$ (applying the operation twice yields the original string),
(ii) either $\sigma \leq f(\sigma)$ or $\sigma \geq f(\sigma)$ (coordinate-wise),
(iii) $\nu(\sigma) + \nu(f(\sigma)) = n$, where ν denotes the number of 1's.
Hint: Remember the Christmas Tree.

Problem 4. [Knuth p. 479, variant of exercise 71]
What is the largest set $A \subset \{0,1\}^n$ that does not contain any chain of length 3 ($\sigma_1, \sigma_2, \sigma_3$ distinct such that $\sigma_1 \leq \sigma_2 \leq \sigma_3$)?