

MATH 108: Introduction to Combinatorics, Winter 2016
HOMEWORK 5
Due Monday, February 22

You should solve the homework on your own. Don't use any books or the internet.

Problem 1. Every box of cereal contains a coupon of one of 3 types (randomly). Suppose you buy 6 boxes of cereal. What is the probability that you will collect at least 1 coupon of each type? (The probability is the number of configurations containing at least 1 coupon of each type, divided by the total number of configurations which is 3^6 .)

Problem 2. The Fibonacci numbers are defined by $F_1 = F_2 = 1$ and $F_n = F_{n-1} + F_{n-2}$ for $n \geq 3$. Express the following quantities in terms of Fibonacci numbers:

- the number of binary strings $(a_1, a_2, \dots, a_n) \in \{0, 1\}^n$ with no two consecutive ones.
- The number of binary strings $(e_1, e_2, \dots, e_n) \in \{0, 1\}^n$ such that $e_1 \leq e_2 \geq e_3 \leq e_4 \geq \dots e_n$.

Problem 3. Exercises on the cycle index:

- Let $P_n(k)$ be the number of permutations in \mathbb{S}_n with k fixed points. The *mean* number of fixed points is $\sum_{k=0}^n kP_n(k)$. Find a simple formula for this quantity. Check it for $n = 3$.
- Find a formula for the number of permutations in \mathbb{S}_n with no 2-cycles. Check it for $n = 3$.

Problem 4. Read Knuth pages 324-325 and do Problem 21.