WHAT YOU SHOULD KNOW FOR THE MIDTERM

We have covered some sections of the book and some material not in the book. In general, if we didn't cover it in class, we won't ask you about it (there is a lot in the book). For the midterm, you are supposed to know what we covered in class *in the first 4 weeks (January only).* The list of topics is as follows:

- Langford sequences, Latin squares
- graph coloring, edge coloring
- characterization of bipartite graphs (no odd cycles)
- matchings in bipartite graphs (Hall's theorem, König's theorem)
- stable matchings (Gale-Shapley algorithm)
- chains and antichains in the hypercube (Sperner's theorem, the Christmas tree pattern)
- partially ordered sets (Dilworth's theorem)
- Gray codes and de Bruijn cycles (definition and constructions)
- Euler's theorem (walks covering each edge exactly once)

Some of the questions are "Prove this theorem as it was proved in class or in the book". Some are similar to homework problems but generally easier. The idea is to trick you into learning the material. :-)

On the following page you can find a sample midterm for practice.

MATH 108: Introduction to Combinatorics, Winter 2016
Midterm exam - sample questions

Please do all five problems. Show your work (partial credit will be given). All problems count equally. No notes or books allowed except one handwritten 3×5" *card of notes (both sides). Please hand it in with your exam. Good luck!*

Problem 1. A tree is a connected graph with no cycles.

- Prove directly that every tree is 2-colorable.
- Prove this by citing a theorem from class.

Problem 2. The complete bipartite graph $K_{m,n}$ has two disjoint sets of vertices $V_1, V_2, |V_1| = m, |V_2| = n$ with an edge from every vertex in V_1 to every vertex to V_2. For which m, n does $K_{m,n}$ have a Hamiltonian cycle?

Problem 3. An *independent set* in a graph is a set A such that there are no edges between two vertices of A. Let $\alpha(G)$ denote the maximum size of an independent set in G and $\tau(G)$ the minimum size of a vertex cover in G. Prove that $\alpha(G) + \tau(G) = |V(G)|$.

Problem 4. Find an example of a graph which is 3-edge-colorable but not 3-vertex-colorable.

Problem 5. Prove that a graph is bipartite if and only if it does not contain an odd cycle. (Repeat the proof from class or formulate your own.)