MATH 108: Introduction to Combinatorics, Winter 2016
FINAL TERM PROJECTS
Proposals Due Friday, February 12

The final exam in this course is replaced by a final project. This is to be a project based on a section of the book (Don Knuth: The Art of Computer Programming Vol. 4A). We suggest some projects below but you should feel free to propose your own, after consulting with us if needed. The final project takes a small section of the book, or a small set of connected exercises (as suggested below) and weaves them into a paper. Your project should be 'stand alone readable' with discussions, examples and complete proofs. We expect you to 'make the material your own', going beyond what is written is some way. This could be writing some code, it could be showing that some hypothesis is needed by providing a counter-example, or putting in the details where the author says 'by similar arguments'. If you can find mistakes of any sort you will be rewarded (both by us and Knuth).

It's fine to look things up and quote outside papers or web pages but please be careful to do so. We need you to have a proposed project, 3-5 pages, due in class on Friday Feb. 12. We will mark these up and get them back to you to try to help. The book has amazing stuff in it. We are trying to trick you into finding that out and helping us appreciate it too. The final is due in Diaconis math department mail box on or before 5 pm of the scheduled date for the final for this course.

Suggested Topics: (but you are not restricted to these)

- Latin squares, finite geometries and error correcting codes (sec. 7 introduction).
  - See exercises 21-24: solve them and explain the connections in your own words.
- Graph theory (sec. 7 introduction). Here are three potential projects:
  - Look at pblms 133-135 on musical graphs, do these and weave them together into a paper.
  - The same for pblms 137-138 (generalized toruses).
  - Exercises 140-142 treat 'proportional graphs'. Take a look and tell us all about them.
- Fibonacci numbers and tiling of the hyperbolic plane (sec. 7.1.3, p. 167-171).
  - Figure out how this works, and draw a labeled map of a larger piece of the hyperbolic plane. What does Knuth mean by a "hyperbolic cylinder" (at the end)?
- Binary Decision Diagrams (sec. 7.1.4; we will not cover any of this in class).
  - Knuth has told us that "ZDDs [Zero-suppressed Binary Decision Diagrams] are amazing objects". Figure out some of what he has to say about them (p. 249-256) and explain.
- Permutations (sec 7.2.1.2). Three suggestions:
  - John Conway's game of 'top swap' is treated in exercises 107,108. By the way, we recommend Conway's recent (auto)biography "Genius at Play".

  - Derangements: exercises 97-99. Persi just gave two talks in the combinatorics seminar about these (and their variations).
  - Gray codes for permutations: exercises 60-62. We will do some of this in class but there is a lot to say (and they are very useful).

- Combinations (sec 7.2.1.3). Two suggestions:
  - Contingency tables: problems 62-63.
  - Shadows and the Kruskal-Katona theorem (p. 372-379, related exercises 74-81). Read and understand the proof of the generalized theorem; can you explain it in a simpler way for Theorem K?

- Partitions (sec. 7.2.1.4). Classical stuff. Try:
  - Problems 54,55 (on the natural partial orders on partitions).

- Set Partitions (sec. 7.2.1.5). One of our favorite topics (Persi does research in this area).
  - Problems 28,29 (about rook polynomials and permutations with restrictions).

- Trees (sec. 7.2.1.6). This is different as we probably won't get to this section. Start reading, find a little subsection that interests you and 'bring it to life'. It will help if you can tag on a homework problem or two but that's neither necessary nor sufficient for a good paper.