Math 108 Syllabus (Jan Vondrak)

Textbook:
Matousek and Nesetril – Invitation to Discrete Mathematics
(Oxford University Press, 2008)

Some additional material:
Lovasz and Plummer – Matching Theory
Donald Knuth – The Art of Computer Programming, Vol. 4A

Week 1: Introduction
- Latin squares, orthogonal pairs
- Graphs, vertex coloring, edge coloring

Week 2: Eulerian and Hamiltonian graphs
- Eulerian graphs – the 7 bridges of Konigsburg
- directed Eulerian graphs
- Hamiltonian graphs
- Gray codes and de Bruijn sequences

Week 3: Bipartite matching
- Hall's Matching Theorem, König's Theorem
- Edge coloring of bipartite graphs

Week 4: The Boolean cube and posets
- Sperner's theorem (the largest antichain in {0,1}^n)
- Posets; Dilworth's theorem on chains and antichains
- The Christmas tree pattern (*)
- Application to monotone subsequences

Week 5: Double counting
- Sperner's coloring lemma (in the plane)
- Connection to Brouwer's fixed point theorem (*)
- Graphs without 4-cycles

Week 6: Finite projective planes
- definition and construction of finite projective planes
- connection with Latin squares

Week 7: Basics of Enumerative Combinatorics
- Binomial and multinomial coefficients, binomial theorem
- Generating functions, some examples
- Inclusion-exclusion

Week 8: Permutations
- Counting derangements
- Statistics on permutations: cycles, inversions, descents
- Permutations with restricted positions
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Week 9: Trees
- Binary trees and Catalan numbers
- Cayley's formula
- The matrix-tree theorem (*)

Week 10: Partitions
- Partitions of an integer, Young diagrams
- Generating function of partitions
- Euler's pentagonal formula (*)

(*) if time allows