WHAT YOU SHOULD KNOW FOR THE FINAL

We have covered some chapters of the textbook and some material not in the book. For the final, you are supposed to know what we covered in class. If I didn't cover it in class, I won't ask you about it. The list of topics is as follows:

- Latin squares, orthogonal pairs
- graphs, vertex coloring, edge coloring
- characterization of bipartite graphs
- Eulerian graphs, Hamiltonian graphs
- De Bruijn sequences and Gray codes
- matchings in bipartite graphs (Hall's theorem, Konig's theorem)
- chains and antichains in the hypercube (Sperner's theorem)
- chains and antichains in partially ordered sets (Dilworth's theorem)
- finite projective planes, connection with Latin squares
- counting with generating functions
- Fibonnacci numbers, Catalan numbers
- the inclusion-exclusion principle
- permutations without fixed points, cycle structure
- integer partitions, Euler's generating function
- counting trees, the matrix-tree formula

Some of the questions are "Prove this theorem as it was proved in class or in the book". Some are similar to homework problems but generally easier.

On the following page you can find a sample exam for practice.

MATH 108: Introduction to Combinatorics, Winter 2017
Final exam - sample questions (3 hours)

Please try to do all 6 problems. Show your work (partial credit will be given). All problems count equally. No notes or books allowed except one handwritten 3×5" card of notes (both sides). Please hand it in with your exam. Good luck!

Problem 1. Suppose that a tree T contains a vertex of degree k. Prove that T has at least k leaves.

Problem 2. Prove that a graph can be drawn without lifting the pen from the paper (each edge drawn exactly once) if and only if it is connected and has at most 2 vertices of odd degree. (Write down the full proof, without citing theorems from class.)

Problem 3. How many numbers in $\{1, \ldots, 99\}$ are divisible by the square of an integer?

Problem 4. Let A be a set of points in a finite projective plane of order p such that no 3 points lie on the same line. Prove that $|A| \leq p + 2$.

Problem 5. What is the generating function of the sequence $(1, 2, 1, 4, 1, 8, 1, \ldots)$? (in a closed form)

Problem 6. Let a_n be the number of ordered triples $i, j, k \geq 0$ such that $i + 2j + 3k = n$. Find the generating function $A(x) = \sum_{n=0}^{\infty} a_n x^n$ in a closed form.