MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 1
Due Monday, Oct 8

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Decide whether each of the following is a subspace of $\mathbb{R}^3$ (and provide an argument for your answers):

- $A_1 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 0\}$
- $A_2 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 + x_3 = 1\}$
- $A_3 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 = 0 \text{ or } x_2 + x_3 = 0\}$
- $A_4 = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : x_1 + x_2 = 0 \text{ and } x_2 + x_3 = 0\}$

Problem 2. Let $c \in \mathbb{R}$ be a constant. $\mathbb{R}^{[0,1]}$ denotes the vector space of functions $f : [0, 1] \to \mathbb{R}$. Consider

$$W_c = \left\{ f \in \mathbb{R}^{[0,1]} : \int_0^1 f(x)dx = c \right\}.$$

For what values of c is this a subspace of $\mathbb{R}^{[0,1]}$?

Problem 3. Prove that for any subspaces $U_1, U_2 \subset V$, $U_1 \cup U_2$ is a subspace if and only if $U_1 \subseteq U_2$ or $U_2 \subseteq U_1$.

Problem 4. Prove or find a counterexample: For subspaces $U_1, U_2, W \subset V$, if $U_1 + W = U_2 + W$, then $U_1 = U_2$.

Problem 5. Let $V = \{(x, x, y, y) : x, y \in \mathbb{R}\}$. Find a decomposition of V as a direct sum of two non-trivial spaces.

Bonus problem. Prove that for subspaces $U_1, U_2, U_3 \subset V$, the only way that $U = U_1 \cup U_2 \cup U_3$ can be a subspace is that it is equal to U_1 or U_2 or U_3.