MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 2
Due Monday, Oct 15

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Suppose that $v_1, v_2, v_3, v_4 \in V$ are linearly independent. Decide whether

- v_1 and $v_1 + v_2$ are linearly independent?
- $v_1, v_1 - v_2, v_2 - v_3, v_3 - v_4$ are linearly independent?
- $v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4 - v_1$ are linearly independent?

Problem 2. Suppose $v_1, \ldots, v_k$ are linearly independent and $w \notin Span(v_1, \ldots, v_k)$. Prove that then $v_1 + w, v_2 + w, \ldots, v_k + w$ are also linearly independent.

Problem 3. Find a basis of the following vector space:

$$U = \{(z_1, \ldots, z_4) \in \mathbb{C}^4 : 2z_1 = z_2, z_3 + iz_4 = 0\}.$$

Prove that your choice is indeed a basis. Then extend it to a basis of $\mathbb{C}^4$.

Problem 4. Is there a basis of $\mathcal{P}_3(\mathbb{R})$ consisting of polynomials of degree 3 only?

Problem 5. Suppose $u_1, \ldots, u_k$ is a basis of U and $w_1, \ldots, w_\ell$ is a basis of W. Prove that $u_1, \ldots, u_k, w_1, \ldots, w_\ell$ is a basis of $U + W$ if and only if it is a direct sum.

Bonus problem. Prove that the vector space of continuous real-valued functions on $[0, 1]$ is not finite-dimensional.