MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 3
Due Monday, Oct 22

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Let $U = \{p \in \mathcal{P}_3(\mathbb{R}) : p(1) = 0\}$. Find a basis of U, and describe a subspace W such that $\mathcal{P}_3(\mathbb{R}) = U \oplus W$.

Problem 2. Prove that if $v_1, \ldots, v_k$ are linearly independent and w is any vector, then $Span(v_1 + w, \ldots, v_k + w)$ has dimension at least $k - 1$.

Problem 3. Find an example of a function $f : \mathbb{R}^2 \to \mathbb{R}$ such that $f(\alpha v) = \alpha f(v)$ for all $\alpha \in R$ and $v \in \mathbb{R}^2$, but f is not a linear map.

Problem 4. Find a linear map $T : \mathbb{R}^4 \to \mathbb{R}^4$ such that $null(T) = range(T)$.

Problem 5. Let $\mathcal{S} = \{T \in \mathcal{L}(\mathbb{R}^2, \mathbb{R}^2) : T \text{ is surjective}\}$. Is $\mathcal{S}$ a subspace of $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$? Is $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2) \setminus \mathcal{S}$ a subspace of $\mathcal{L}(\mathbb{R}^2, \mathbb{R}^2)$?

Bonus problem. Decide whether it is true that for any 3 subspaces,

$$dim(U_1 + U_2 + U_3) =$$

$$= dim(U_1) + dim(U_2) + dim(U_3) - dim(U_1 \cap U_2) - dim(U_1 \cap U_3) - dim(U_2 \cap U_3) + dim(U_1 \cap U_2 \cap U_3).$$