MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 4
Due Monday, Nov 5

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Suppose $A_1 = u_1 + U_1$ and $A_2 = u_2 + U_2$ are affine subsets of V. Prove that $A_1 \cap A_2$ is either empty or an affine subset.

Problem 2. Let U be a subspace of a finite-dimensional space V. Prove that V is isomorphic to $U \times (V/U)$. (Hint: consider W such that $U \oplus W = V$.)

Problem 3. Suppose that U is a subspace of V such that $\dim(V/U) = 1$. Prove that there is $\phi \in V'$ such that $\operatorname{null}(\phi) = U$.

Problem 4. Verify that $1, x-5, (x-5)^2, \ldots, (x-5)^m$ is a basis of $\mathcal{P}_m(\mathbb{R})$. What is the dual basis of $(\mathcal{P}_m(\mathbb{R}))'$?

Problem 5. Suppose that V is finite-dimensional and U, W are subspaces of V. Prove that $(U \cap W)^0 = U^0 + W^0$.

Bonus problem. Prove that $(\mathcal{P}(\mathbb{R}))'$ is isomorphic to $\mathbb{R}^\infty$.