MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 5
Due Monday, Nov 12

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Prove that every linear functional $\phi : V \to \mathbb{F}$ is either surjective or identically zero.

Problem 2. Recall that $\mathbb{F}^\infty$ is the vector space of countably infinite sequences of elements of $\mathbb{F}$. Let $U = \{x \in \mathbb{F}^\infty : x_i = 0 \text{ for all except finitely many values of } i\}$. Prove that U is a subspace of $\mathbb{F}^\infty$. Is $\mathbb{F}^\infty/U$ finite-dimensional or infinite-dimensional?

Problem 3. Suppose $T \in \mathcal{L}(V, W)$ where W is finite-dimensional. Prove that $T = 0$ if and only if $T' = 0$.

Problem 4. Prove that for any two subspaces U, W of V, $(U + W)^0 = U^0 \cap W^0$.

Problem 5. Suppose that V is finite-dimensional, and $\phi_1, \ldots, \phi_k$ are linearly independent in V'. Prove that

$$\dim(null(\phi_1) \cap \ldots \cap null(\phi_k)) = \dim V - k.$$

Bonus problem. Suppose that $p \in \mathcal{P}(\mathbb{C})$ is a polynomial such that its values at $m + 1$ distinct real points $p(x_0), p(x_1), \ldots, p(x_m)$ are all real. Prove that the coefficients of p are all real.