MATH 113: Linear Algebra, Autumn 2018
HOMEWORK 7
Due Monday, Dec 3

Try solve the homework on your own. If you discuss with others, please list your collaborators. You can use anything that was stated in class, but don't search the internet please.

Problem 1. Let $A \in \mathbb{R}^{n\times n}$ and $c \in \mathbb{R}$. Prove that $\det(cA) = c^n \det(A)$.

Problem 2. Prove or find a counterexample for the following statement:

$$\det(A + B) = \det(A) + \det(B).$$

Problem 3. Suppose that $A \in \mathbb{R}^{n\times n}$ is a matrix which has no real eigenvalues. Prove that $\det(A) > 0$.

Problem 4. Let $A \in \mathbb{C}^{n\times n}$. Recall that the characteristic polynomial of A is $\chi_A(z) = \det(A - zI)$. Find a formula for the coefficients of z^{n-1} and z in $\chi_A(z)$.

Problem 5. Let $b_n = \det(B_n)$ where $B_n \in \mathbb{R}^{n\times n}$ is a matrix where $B_{ij} = 1$ if $|i - j| \le 1$ and $B_{ij} = 0$ otherwise. Find a recursive formula for b_n.

Bonus problem. Given an invertible matrix $A \in \mathbb{R}^{3\times 3}$, find a formula for the volume of the "ellipsoid",

$$E_A = \{x \in \mathbb{R}^3 : x^T A^T A x \le 1\}$$

by relating it by a linear map to a set whose volume you know.