CS364B: Problem Set #3

Due in class on Monday, November 12, 2007.

Instructions:

Collaboration on this homework is actively encouraged. However, your write-up must be your own, and you must list the names of your collaborators on the front page.

Keep an eye on the course web site for a FAQ on this homework.

Problem 1

Assume the following keyword auction setting. There are n bidders with value-per-click $v_1, v_2, v_3 \ldots v_n$. There are k slots with click-through-rates (CTRs) $\Theta_1 \geq \Theta_2 \geq \ldots \Theta_k$. We assume that the CTR of advertiser i's advertisement in slot j is Θ_j. Assume that $k > n$. (This is without loss of generality: if the condition is not satisfied, we can always add $n + 1 - k$ dummy slots with CTR 0.)

Suppose the values of bidders $1, 2, \ldots, n/2$ are drawn i.i.d. from the uniform distribution with support $[0, 4]$ and the values of the remaining bidders are drawn i.i.d. from an exponential distribution with mean 1.

(a) What are the two virtual value functions? Are they monotone in the values?

(b) Describe the allocation rule and the payments of the optimal (revenue-maximizing) keyword auction in this setting. Suppose players $1, \ldots, n/2$ bid 3/2 and the remaining bid 5/4, what is the allocation? Is the auction a rank-by-bid auction?

(c) Suppose all the values were drawn i.i.d. from an exponential distribution with mean 1. Is the optimal auction in this setting a rank-by-bid auction?

Problem 2

Recall the greedy algorithm for online revenue-maximization with budget-constrained bidders: when a keyword k shows up, among all advertisers i whose remaining budget is at least their bid $b(i, k)$, allocate the keyword to the highest bidder i at price $b(i, k)$. Recall that $\epsilon_{\max} = \max_{i,k} b(i, k)/B(i)$, where $B(i)$ is i's initial budget.

(a) Prove that if we don't constrain $\epsilon_{\max}$ to be bounded below 1, then the greedy algorithm is not α-competitive for any $\alpha > 0$.

(b) Prove that the competitive ratio of the greedy algorithm approaches 1/2 as $\epsilon_{\max} \to 0$.

(c) What can you say about the competitive ratio of the following modification of the greedy algorithm?: assign a keyword k to the advertiser with maximum value of $\min\{B'(i), b(i, k)\}$, where $B'(i)$ is the remaining budget of i, and charge the advertiser $\min\{B'(i), b(i, k)\}$.