Homework 4

To be finished individually. Due at the end of class of Thursday, Feb 24, 2011.

1. *(10 points) Use the push-relabel algorithm to find the maximum flow of the following network, and list your intermediate steps.*

$$c(1 \to 3) = 4 \quad c(3 \to 4) = 5 \quad c(1 \to 2) = 2$$
$$c(2 \to 3) = 3 \quad c(2 \to 4) = 1$$

   Here 1 is the source, and 4 is the sink.

2. *(10 points) Suppose in a given network, all edges are undirected (or think of every edge as bi-directional with the same capacity), and the length of the longest simple path from the source s to sink t is at most L. Show that the running time of Edmond-Karp algorithm on such networks can be improved to be $O(L \cdot |E|^2)$. Also show that the running time of push-relabel algorithm can be improved to $O(L^2 \cdot |E|)$, where now you initially set the distance label of the source node to be L. Please only sketch changes needed to the analysis.*

3. *(10 points) We want to show that the edge contraction algorithm does not work well with finding the minimum s-t cut instead of simply the minimum cut. To be concrete, for every size n, construct a network of size n, such that if you run the edge contraction algorithm on this network, the probability that some minimum s-t cut survives at the end can be exponentially small in n.*