Practice Midterm

No need to turn in.

1. (10 points) Recall the vertex cover problem, where you are given a graph $G = (V, E)$, and your task is to choose a minimum number of vertices such that every edge in E is touched by at least one of these vertices.

   Again recall the natural LP relaxation of this problem:

$$\begin{array}{rll} \text{minimize} & \sum_{v \in V} x_v & \\ \text{subject to} & x_u + x_v \geq 1 & \forall (u, v) \in E \\ & x_v \geq 0 & \forall v \in V \end{array}$$

   Now your task is to adapt the 2-approximation algorithm for vertex cover we covered in class to solve this LP optimally.

2. (10 points) Consider the maximum weighted matching problem, where you are given a (not necessarily bipartite) graph $G = (V, E)$ with nonnegative weights on the edges, and your goal is to find a maximum weight set of edges such that no two edges from the set share a vertex, i.e., they form a matching. It's known that this problem can be solved exactly in polynomial time. Your task here however, is to give a linear time 2-approximation algorithm. (Hint: consider visiting the edges in appropriate order, and adding them to the solution greedily when possible.)

3. (10 points) (In real midterm, I'll try to provide pictures for such problems, if any.)

   We have a network as follows: there are 7 nodes labelled 1 to 7, where 1 is the source and 7 is the sink, and the list of arcs and the corresponding capacities are as follows:

$$\begin{array}{ll} c(1 \to 2) = 4 & c(2 \to 4) = 2 \\ c(4 \to 6) = 5 & c(6 \to 7) = 8 \\ c(2 \to 5) = 3 & c(5 \to 6) = 4 \\ c(1 \to 3) = 3 & c(3 \to 4) = 2 \end{array}$$

   Please find the max flow and min cut of this network, and list the intermediate steps.