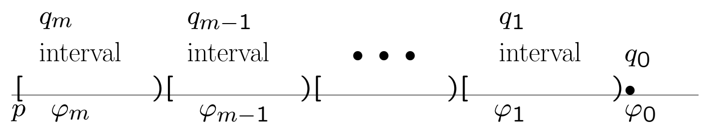
CS256/Spring 2008 – Lecture #10

Zohar Manna

Nested Waiting-for Formulas



Rule nwait (nested waiting-for)

For assertions p, $q_0, q_1, \ldots, q_m$ and $\varphi_0, \varphi_1, \ldots, \varphi_m$

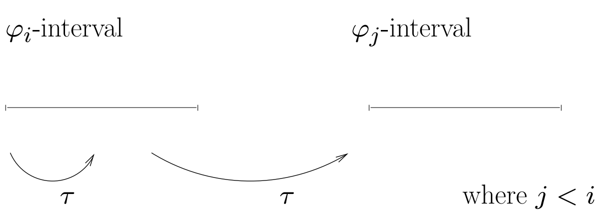
N1. $p \rightarrow \bigvee_{j=0}^{m} \varphi_j$

N2. $\varphi_i \rightarrow q_i$ for $i = 0, 1, \ldots, m$

N3. $\{\varphi_i\} \mathcal{T} \left\{ \bigvee_{j \leq i} \varphi_j \right\}$ for $i = 1, \ldots, m$

$$p \Rightarrow q_m \, \mathcal{W} \, q_{m-1} \cdots q_1 \, \mathcal{W} \, q_0$$

Nested Waiting-for Formulas (Cont'd)



Premise N3 states that for each assertion φ_i, each transition $\tau \in \mathcal{T}$ either preserves φ_i or leads to some φ_j, with $j < i$.

`Example:` Program mux-pet1 (Fig. 3.4)

An example of a nested waiting-for formula is 1-bounded overtaking for MUX-PET1:

$$\underbrace{at\_\ell_3}_{p} \Rightarrow \underbrace{\neg at\_m_4}_{q_3} \; \mathcal{W} \; \underbrace{at\_m_4}_{q_2} \; \mathcal{W} \; \underbrace{\neg at\_m_4}_{q_1} \; \mathcal{W} \; \underbrace{at\_\ell_4}_{q_0}$$

It states that when process P_1 is at ℓ_3, process P_2 can enter its critical section at most once ahead of process P_1.

Example: Program mux-pet1 (Fig. 3.4)

(Peterson's Algorithm for mutual exclusion)

local y_1, y_2: **boolean** **where** $y_1 = \text{F}, y_2 = \text{F}$
s : **integer** **where** $s = 1$

$$P_1 :: \quad \ell_0 : \textbf{loop forever do} \begin{bmatrix} \ell_1: & \textbf{noncritical} \\ \ell_2: & (y_1,\ s) := (\text{T},\ 1) \\ \ell_3: & \textbf{await}\ (\neg y_2) \vee (s \neq 1) \\ \ell_4: & \textbf{critical} \\ \ell_5: & y_1 := \text{F} \end{bmatrix}$$

||

$$P_2 :: \quad m_0 : \textbf{loop forever do} \begin{bmatrix} m_1: & \textbf{noncritical} \\ m_2: & (y_2,\ s) := (\text{T},\ 2) \\ m_3: & \textbf{await}\ (\neg y_1) \vee (s \neq 2) \\ m_4: & \textbf{critical} \\ m_5: & y_2 := \text{F} \end{bmatrix}$$

With the following strengthenings all premises of rule NWAIT become state-valid.

p: $\underline{at\_\ell_3}$

φ_3: $at\_\ell_3 \wedge \underline{\neg at\_m_4} \wedge at\_m_3 \wedge s = 1$
"P_2 has priority over P_1"

φ_2: $at\_\ell_3 \wedge \underline{at\_m_4}$

φ_1: $at\_\ell_3 \wedge \underline{\neg at\_m_4} \wedge (at\_m_3 \rightarrow s = 2)$
"P_1 has priority over P_2"

$\varphi_0 = q_0$: $\underline{at\_\ell_4}$

or equivalently,

p: at_ℓ_3

φ_3: $at\_\ell_3 \ \wedge \ at\_m_3 \ \wedge \ s = 1$

φ_2: $at\_\ell_3 \ \wedge \ at\_m_4$

φ_1: $at\_\ell_3 \ \wedge \ (at\_m_{0..2,5} \ \vee \ (at\_m_3 \ \wedge \ s = 2))$

$\varphi_0 = q_0$: at_ℓ_4

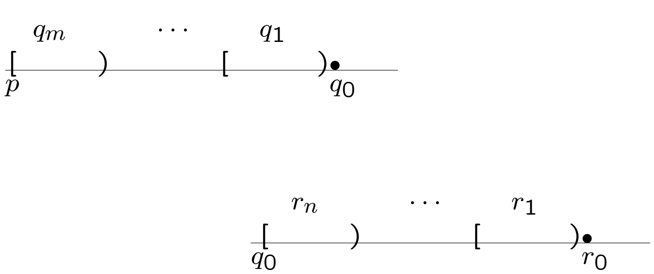
Concatenation of waiting-for formulas

Rule CONC-W

$$p \Rightarrow q_m \, \mathcal{W} \cdots q_1 \, \mathcal{W} \, q_0$$

$$q_0 \Rightarrow r_n \, \mathcal{W} \cdots \mathcal{W} \, r_0$$

$$p \Rightarrow q_m \, \mathcal{W} \cdots \mathcal{W} \, q_1 \, \mathcal{W} \, r_n \, \mathcal{W} \cdots \mathcal{W} \, r_0$$



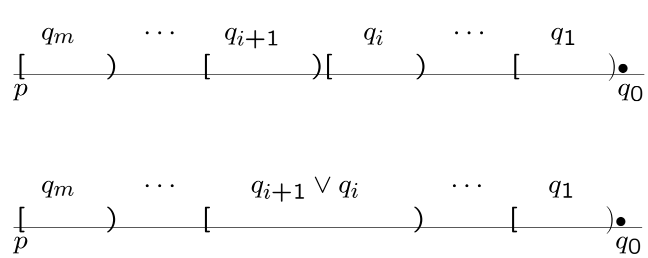
Collapsing of waiting-for formulas

Rule COLL-W

For $i > 0$

$$p \Rightarrow q_m \, \mathcal{W} \cdots \mathcal{W} \, q_{i+1} \, \mathcal{W} \, q_i \, \mathcal{W} \cdots \mathcal{W} \, q_0$$

$$p \Rightarrow q_m \, \mathcal{W} \cdots \mathcal{W} \, (q_{i+1} \vee q_i) \, \mathcal{W} \cdots \mathcal{W} \, q_0$$



Basic Verification Diagrams

A visual summary of verification proofs

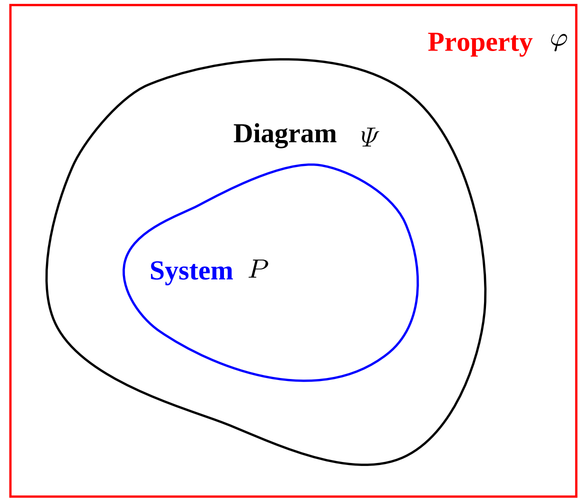
Verification Diagrams (VDs) allow a graphical representation of a proof of a temporal property.

To prove φ is P-valid, find diagram Ψ such that:

$$\mathcal{L}(P) \subseteq \mathcal{L}(\Psi) \subseteq \mathcal{L}(\varphi)$$

i.e., every P-computation σ is a Ψ-sequence and every Ψ-sequence σ is a model of φ (satisfies $\sigma \vDash \varphi$).

Verification Diagrams (VDs)



$\mathcal{L}(P) \subseteq \mathcal{L}(\Psi)$ proved by verification conditions.

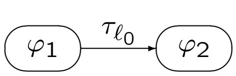
$\mathcal{L}(\Psi) \subseteq \mathcal{L}(\varphi)$ follows from well-formedness of diagram.

Verification Diagram (VD)

Directed labeled graph with

- Nodes – labeled by assertions

- Edges – labeled by names of transitions



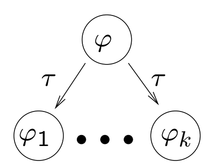
- Terminal Node ("goal") – no edges depart from it

Verification conditions (VCs)

VD provides a concise representation of sets of VCs:

- The verification condition associated with a node labeled by φ and a transition τ is

$$\Rightarrow \ \{\varphi\} \ \tau \ \{\varphi \vee \varphi_1 \vee \ldots \vee \varphi_k\}$$



There is an implicit τ-edge connecting each φ-node to itself.

- Nonterminal node without outgoing edges

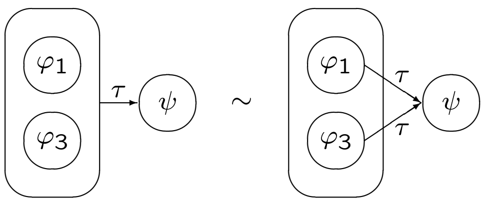
$\Rightarrow \ \{\varphi\} \ \tau \ \{\varphi\}$

Note: No verification conditions for terminal node.

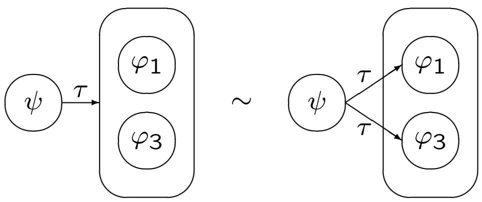
`Definition`: VD is P-valid iff all VCs associated with nodes in the diagram are P-state valid

Compound Nodes: Statecharts Conventions

- Departing edges

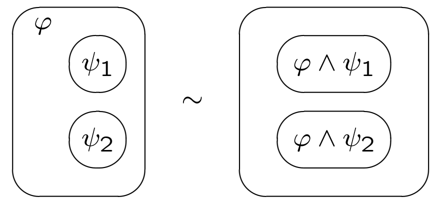


- Arriving edges



Compound Nodes: Statecharts Conventions

- Common factors



Classes of Diagrams

- Proofs of invariance properties

$$\Box\, q$$

are represented by INVARIANCE diagrams

- Proofs of precedence properties

$$p \;\Rightarrow\; q_m \,\mathcal{W}\, q_{m-1} \;\cdots\; q_1 \,\mathcal{W}\, q_0$$

are represented by WAIT diagrams

- Proofs of response properties

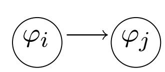
$$p \Rightarrow \Diamond\, q$$

are represented by CHAIN and RANK diagrams (Vol. III)

Wait Diagrams

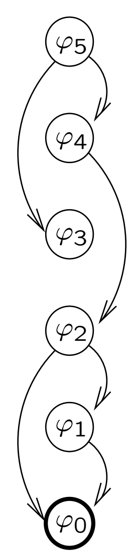
VDs with nodes $\varphi_m, \ldots, \varphi_0$ such that:

- weakly acyclic, i.e.,

  if 

  then $i \geq j$

- φ_0 is a terminal node



`Claim` (wait diagram):

A P-valid WAIT diagram establishes that

$$\bigvee_{j=0}^{m} \varphi_j \;\Rightarrow\; \varphi_m \,\mathcal{W}\, \varphi_{m-1} \cdots \varphi_1 \,\mathcal{W}\, \varphi_0$$

is P-valid.

If, <u>in addition</u>,

(N1) $p \;\rightarrow\; \bigvee_{j=0}^{m} \varphi_j$

(N2) $\varphi_i \;\rightarrow\; q_i$ for $i = 0, 1, \ldots, m$

are P-state valid, then

$$\boxed{p \;\Rightarrow\; q_m \,\mathcal{W}\, q_{m-1} \cdots q_1 \,\mathcal{W}\, q_0}$$

is P-valid.

`Example:` Program MUX-PET1 (Fig 3.4)

1-bounded overtaking from ℓ_3

$$\boxed{\psi:\; \underbrace{at\_\ell_3}_{p} \;\Rightarrow\; \underbrace{(\neg at\_m_4)}_{q_3} \,\mathcal{W}\, \underbrace{at\_m_4}_{q_2} \,\mathcal{W}\, \underbrace{(\neg at\_m_4)}_{q_1} \,\mathcal{W}\, \underbrace{at\_\ell_4}_{q_0}}$$

Proof is summarized in WAIT diagram

(Fig 3.8)

Example: Program mux-pet1 (Fig. 3.4)

(Peterson's Algorithm for mutual exclusion)

local y_1, y_2: **boolean** **where** $y_1 = \text{F}, y_2 = \text{F}$
s : **integer** **where** $s = 1$

$P_1 ::$
ℓ_0 : **loop forever do**
- ℓ_1 : **noncritical**
- ℓ_2 : $(y_1,\ s) := (\text{T},\ 1)$
- ℓ_3 : **await** $(\neg y_2) \lor (s \neq 1)$
- ℓ_4 : **critical**
- ℓ_5 : $y_1 := \text{F}$

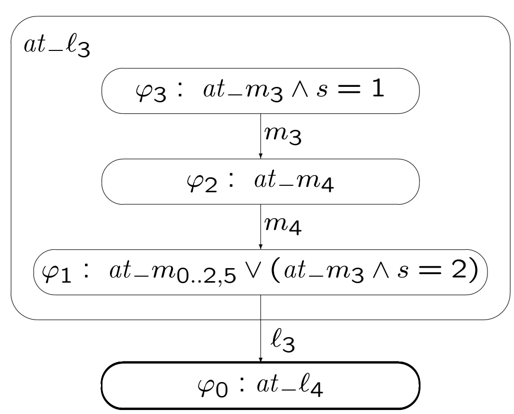
||

$P_2 ::$
m_0 : **loop forever do**
- m_1 : **noncritical**
- m_2 : $(y_2,\ s) := (\text{T},\ 2)$
- m_3 : **await** $(\neg y_1) \lor (s \neq 2)$
- m_4 : **critical**
- m_5 : $y_2 := \text{F}$

`Example:` Program MUX-PET1 (Con't)

WAIT diagram (Fig. 3.8)
(1-bounded overtaking from ℓ_3)

$$\psi:\; \underbrace{at\_\ell_3}_{p} \;\Rightarrow\; \underbrace{(\neg at\_m_4)}_{q_3} \,\mathcal{W}\, \underbrace{at\_m_4}_{q_2} \,\mathcal{W}\, \underbrace{(\neg at\_m_4)}_{q_1} \,\mathcal{W}\, \underbrace{at\_\ell_4}_{q_0}$$



`Example:` Program MUX-PET1 (Con't)

Associated VCs

- From φ_3

$\{\varphi_3\}\ m_3\ \{\varphi_3 \vee \varphi_2\}$

$$\underbrace{\ldots}_{\varphi_3} \wedge \underbrace{\ldots \wedge at'\_m_4}_{\rho_{m_3}} \rightarrow \underbrace{\ldots}_{\varphi_3'} \vee \underbrace{at'\_m_4}_{\varphi_2'}$$

$\{\varphi_3\}\ \overline{m_3}\ \{\varphi_3\}$

for all non-m_3 transitions.
But since we are at_ℓ_3, at_m_3, check only ℓ_3.

$\{\varphi_3\}\ \ell_3\ \{\varphi_3\}$ holds, since

$$\underbrace{at\_m_3 \wedge \ldots \wedge s = 1}_{\varphi_3} \wedge \underbrace{\ldots \wedge ((\neg y_2) \vee (s \neq 1))}_{\rho_{\ell_3}} \rightarrow \underbrace{\ldots}_{\varphi_3'}$$

Recall that by χ_2, $at\_m_3 \rightarrow y_2$.

- From φ_2

$\{\varphi_2\}\ m_4\ \{\varphi_2 \vee \varphi_1\}$

$\{\varphi_2\}\ \overline{m_4}\ \{\varphi_2\}$

- From φ_1

$\{\varphi_1\}\ \ell_3\ \{\varphi_1 \vee \varphi_0\}$

$\{\varphi_1\}\ \overline{\ell_3}\ \{\varphi_1\}$

They are P-state valid
[not state-valid - require invariants $\chi_0, \ldots, \chi_4$]

Therefore,
WAIT diagram is valid over MUX-PET1

`Example:` Program MUX-PET1 (Con't)
Therefore,

$$\bigvee_{i=0}^{3} \varphi_i \Rightarrow \varphi_3\ \mathcal{W}\ \varphi_2\ \mathcal{W}\ \varphi_1\ \mathcal{W}\ \varphi_0$$

is valid over MUX-PET1.

In addition,

$$\underbrace{at\_\ell_3}_{p} \rightarrow \bigvee_{j=0}^{3} \varphi_j$$

$$\varphi_0 \rightarrow \underbrace{at\_\ell_4}_{q_0} \qquad \varphi_1 \rightarrow \underbrace{\neg at\_m_4}_{q_1}$$

$$\varphi_2 \rightarrow \underbrace{at\_m_4}_{q_2} \qquad \varphi_3 \rightarrow \underbrace{\neg at\_m_4}_{q_3}$$

are P-state valid.

Therefore,

$$\psi:\ at\_\ell_3 \Rightarrow (\neg at\_m_4)\ \mathcal{W}\ at\_m_4\ \mathcal{W}\ (\neg at\_m_4)\ \mathcal{W}\ at\_\ell_4$$

is valid over MUX-PET1

Invariance Diagrams

VDs with no terminal nodes (cycles OK)

`Claim (invariance diagram)`:

A P-valid INVARIANCE diagram establishes that

$$\bigvee_{j=1}^{m} \varphi_j \quad \Rightarrow \quad \Box(\bigvee_{j=1}^{m} \varphi_j)$$

is P-valid.

If, in addition,

$$\text{(I1)} \quad \Theta \ \rightarrow \ \bigvee_{j=1}^{m} \varphi_j$$

$$\text{(I2)} \quad \bigvee_{j=1}^{m} \varphi_j \ \rightarrow \ q$$

are P-state valid, then

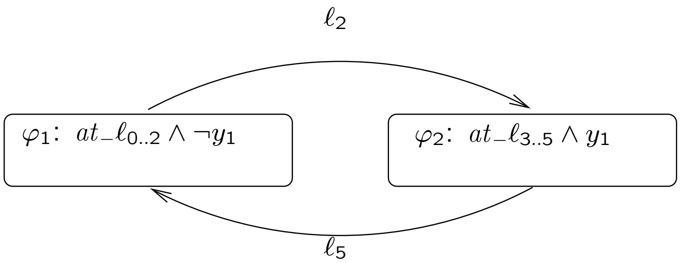
$$\boxed{\Box\, q}$$

is P-valid

Example: Program MUX-PET1 (Fig 3.4)

Establish $\boxed{\Box\,\underbrace{(y_1 \leftrightarrow at\_\ell_{3..5})}_{q}}$

INVARIANCE diagram
valid for program MUX-PET1



because

$\{\varphi_1\}\,\ell_2\,\{\varphi_1 \vee \varphi_2\}$ $\qquad$ $\{\varphi_1\}\,\overline{\ell_2}\,\{\varphi_1\}$

$\{\varphi_2\}\,\ell_5\,\{\varphi_2 \vee \varphi_1\}$ $\qquad$ $\{\varphi_2\}\,\overline{\ell_5}\,\{\varphi_2\}$

Thus

$$\varphi_1 \vee \varphi_2 \Rightarrow \Box(\varphi_1 \vee \varphi_2)$$

Also,

(I1) $\underbrace{at\_\ell_0 \;\wedge\; \neg y_1 \;\wedge\; \cdots}_{\Theta} \;\rightarrow\; \underbrace{at\_\ell_{0..2} \;\wedge\; \neg y_1}_{\varphi_1} \;\vee\; \underbrace{\cdots}_{\varphi_2}$

(I2) $\underbrace{at\_\ell_{0..2} \;\wedge\; \neg y_1}_{\varphi_1} \;\vee\; \underbrace{at\_\ell_{3..5} \;\wedge\; y_1}_{\varphi_2} \;\rightarrow\; \underbrace{y_1 \;\leftrightarrow\; at\_\ell_{3..5}}_{q}$

are state-valid

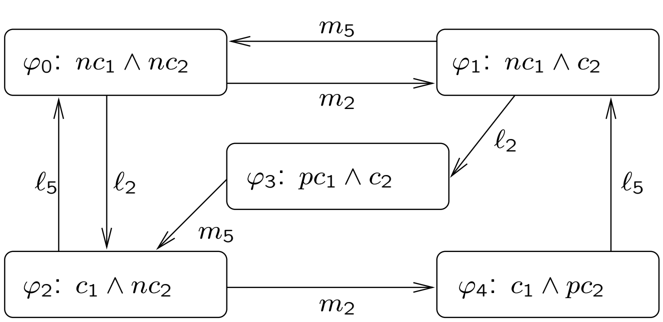
Therefore

$\boxed{\Box\,\underbrace{(y_1 \;\leftrightarrow\; at\_\ell_{3..5})}_{q}}$

is P-valid.

Example: Program MUX-PET1 (Fig. 3.4)

Establish $\boxed{\Box\,\neg(at\_\ell_4 \wedge at\_m_4)}$



non-critical: nc_1: $at\_\ell_{0..2}$
nc_2: $at\_m_{0..2}$

critical: c_1: $at\_\ell_{3..5} \wedge \neg y_2$
c_2: $at\_m_{3..5} \wedge \neg y_1$

pre-critical: pc_1: $at\_\ell_3 \wedge s = 1 \wedge y_2$
pc_2: $at\_m_3 \wedge s = 2 \wedge y_1$