CS256/Spring 2008 — Lecture #13

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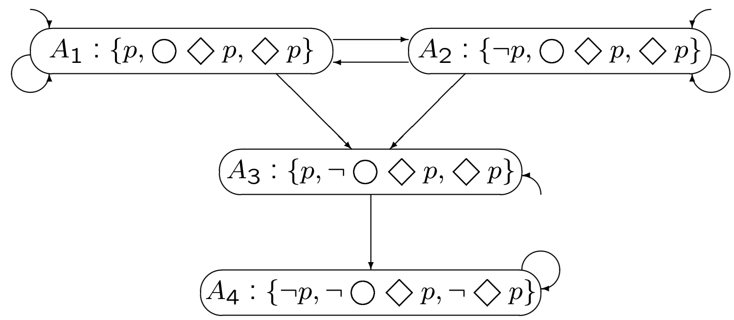
<u>`Example`</u>: $\varphi_0 : \Diamond p$

Tableau T_{φ_0}:



<u>Promising Formula</u>

In $T_{\Diamond p}$, a path can start and stay forever in atom A_2. But A_2 includes $\Diamond p$, i.e., A_2 promises that p will eventually happen, but it is never fulfilled in the path. We want to exclude these paths.

The idea is that if a path contains an atom that includes a <u>promising formula</u>, then the path should fulfill the promise.

A formula $\psi \in \Phi_\varphi$ is said to

<u>promise the formula r</u>

if ψ is one of the forms:

$$\Diamond r \qquad \underbrace{p\, \mathcal{U}\, r}_{\approx \Diamond r \wedge \ldots} \qquad \underbrace{\neg \Box \neg r}_{\approx \Diamond r} \qquad \underbrace{\neg((\neg r)\, \mathcal{W}\, p)}_{\approx \Diamond r \wedge \ldots}$$

<u>`Example:`</u>

$$\boxed{\varphi_1 \colon \Box p \wedge \Diamond \neg p}$$

$$\Phi_{\varphi_1} \colon \left\{ \begin{array}{cccccc} \varphi_1, & \Box p, & \underline{\Diamond \neg p}, & \bigcirc \Box p, & \bigcirc \Diamond p, & p \\ \neg\varphi_1, & \underline{\neg \Box p}, & \neg \Diamond \neg p, & \neg \bigcirc \Box p, & \neg \bigcirc \Diamond p, & \neg p \end{array} \right\}$$

Only 2 promising formulas in Φ_φ

$$\psi_1 : \neg \Box p \quad \text{promises} \quad r_1 : \neg p$$
$$\psi_2 : \Diamond \neg p \quad \text{promises} \quad r_2 : \neg p$$

Promise Fulfillment

Property:

Let σ be an arbitrary model of φ,
and $\psi \in \Phi_\varphi$ a formula that promises r.
If $(\sigma, j) \vDash \psi$ then $(\sigma, k) \vDash r$ for some $k \geq j$

`Proof:` Follows from the semantics of temporal formulas.

`Claim:` (promise fulfillment by models)

Let σ be an arbitrary model of φ,
and $\psi \in \Phi_\varphi$ a formula that promises r.
Then σ contains infinitely many positions $j \geq 0$ such that

$$(\sigma, j) \vDash \neg\psi \quad \text{or} \quad (\sigma, j) \vDash r$$

Proof:

1. Assume σ contains infinitely many ψ-positions. Then σ must contain infinitely many r-positions, since ψ promises r.
2. Assume σ contains finitely many ψ-positions. Then it contains infinitely many $\neg\psi$-positions.

Fulfilling Atoms

Definition: Atom A fulfills $\psi \in \Phi_\varphi$ (which promises r)
if $\neg\psi \in A$ or $r \in A$.

`Example:` In $T_{\diamond p}$,

Only one promising formula:

$$\psi : \ \diamond p \ \text{ promises } \ r : p$$

$A_1^+ :$ $\{p, \ \bigcirc \diamond p, \ \diamond p\}$
fulfills $\diamond p$ since $p \in A_1$

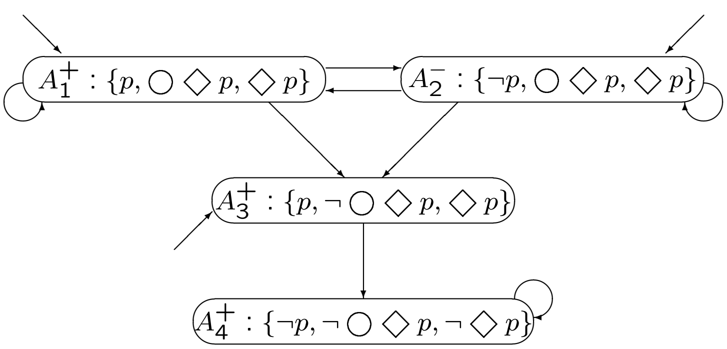
$A_3^+ :$ $\{p, \ \neg \bigcirc \diamond p, \ \diamond p\}$
fulfills $\diamond p$ since $p \in A_3$

$A_4^+ :$ $\{\neg p, \ \neg \bigcirc \diamond p, \ \neg \diamond p\}$
fulfills $\diamond p$ since $\neg \diamond p \in A_4$

But

$A_2^- :$ $\{\neg p, \ \bigcirc \diamond p, \ \diamond p\}$
does not fulfill $\diamond p$ since $\diamond p, \neg p \in A_2$

Tableau $T_{\diamond p}$



Fulfilling Paths

Definition: A path $\pi : A_0, A_1, \ldots$ is fulfilling if for every promising formula $\psi \in \Phi_\varphi$ it contains infinitely many A_j that fulfill ψ.

`Example:` In $T_{\diamond p}$,

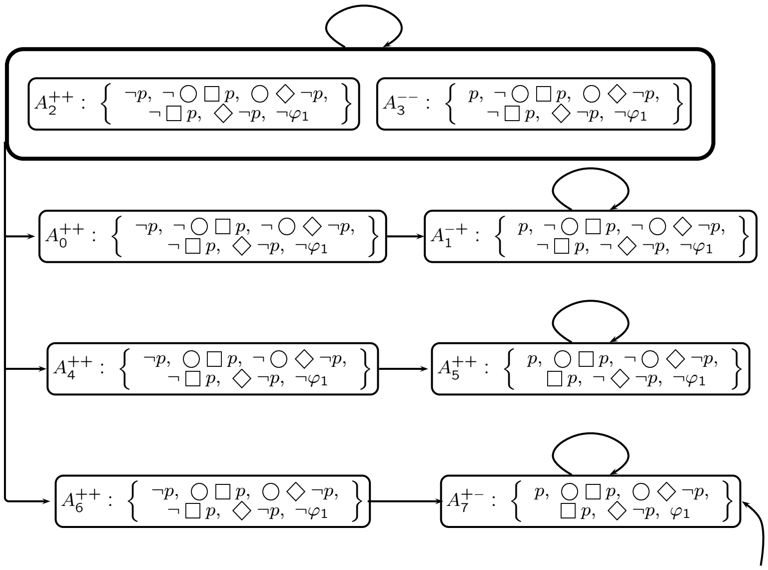
$$A_2^-, A_2^-, A_2^-, A_3^+, A_4^+, A_4^+, \ldots$$
$$A_2^-, A_1^+, A_2^-, A_1^+, A_1^+, A_1^+, \ldots$$

are fulfilling paths, but

$$A_2^-, A_2^-, A_2^-, A_2^-, A_2^-, A_2^-, A_2^-, \ldots$$

is not a fulfilling path.

Fig. 5.3: Tableau T_{φ_1} for formula
φ_1: $\square p \ \wedge \ \diamond \neg p$



Example:

$\varphi_1: \quad \square p \ \wedge \ \diamond \neg p$

T_{φ_1} in Fig 5.3

There are two promising formulas in Φ:

$$\psi_1 : \neg \square p \quad \text{promises} \quad r_1 : \neg p$$
$$\psi_2 : \diamond \neg p \quad \text{promises} \quad r_2 : \neg p$$

$$
\begin{array}{lcllll}
A_0^{++} & : & \{ \ \neg p, & \neg \square p, & \diamond \neg p, & \ldots \ \} \\
A_1^{-+} & : & \{ \ p, & \neg \square p, & \neg \diamond \neg p, & \ldots \ \} \\
A_2^{++} & : & \{ \ \neg p, & \neg \square p, & \diamond \neg p, & \ldots \ \} \\
A_3^{--} & : & \{ \ p, & \neg \square p, & \diamond \neg p, & \ldots \ \} \\
A_4^{++} & : & \{ \ \neg p, & \neg \square p, & \diamond \neg p, & \ldots \ \} \\
A_5^{++} & : & \{ \ p, & \square p, & \neg \diamond \neg p, & \ldots \ \} \\
A_6^{++} & : & \{ \ \neg p, & \neg \square p, & \diamond \neg p, & \ldots \ \} \\
A_7^{+-} & : & \{ \ p, & \square p, & \diamond \neg p, & \ldots \ \}
\end{array}
$$

Example: (Cont'd)

- path $(A_7^{+-})^\omega$ not fulfilling.
- path $(A_2^{++})^\omega$ is fulfilling.
- path $(A_2^{++}, A_3^{--})^\omega$ is fulfilling.
- path $A_4^{++}, (A_5^{++})^\omega$ is fulfilling.
- For arbitrary m, path
  $$\pi: \ (A_2^{++}, A_3^{--})^m, A_4^{++}, (A_5^{++})^\omega$$
  is fulfilling.

Models vs. fulfilling paths

Claim 2 (model → fulfilling path):

If

$$\pi_\sigma : \ A_0, A_1, \ldots$$

is a path induced by a model σ of φ,
then π_σ is fulfilling.

Claim 3 (fulfilling path → model):

If

$$\pi_\sigma : \ A_0, A_1, \ldots$$

is a fulfilling path in T_φ,
then there exists a model σ of φ that induces π_σ.

Proposition 1 (satisfiability by path)

Formula φ is satisfiable

iff

the tableau T_φ contains a fulfilling path $\pi : A_0, A_1, A_2, \ldots$ such that $\varphi \in A_0$

Proof:

($\Leftarrow$) $\pi : A_0, A_1, \ldots$ is a fulfilling path in T_φ with $\varphi \in A_0$
Then, by Claim 3, there exists model σ such that
$\forall j \geq 0, \forall p \in \Phi_\varphi$: $(\sigma, j) \vDash p$ iff $p \in A_j$
Since $\varphi \in A_0$, $(\sigma, 0) \vDash \varphi$ and thus $\sigma \vDash \varphi$.

($\Rightarrow$) $\sigma \vDash \varphi$. Then by Claims 1, 2, there exists a fulfilling path π_σ in T_φ that is induced by σ.
Since $(\sigma, 0) \vDash \varphi$, by the definition of induced, $\varphi \in A_0$.

Examples

In the examples below we use the following optimization: A path starting in A can only visit nodes that are reachable from A in T_φ. So we only need to consider nodes that are reachable from nodes labeled by atoms A such that $\varphi \in A$.

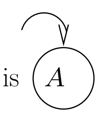
Example: $\boxed{\varphi: \Box p \land \neg \bigcirc p}$

$$\Phi_\varphi = \{ \varphi, \Box p, \bigcirc \Box p, p, \bigcirc p, \neg\varphi, \underline{\neg \Box p}, \neg \bigcirc \Box p, \neg p, \neg \bigcirc p \}$$

Basic formulas: $\{\bigcirc p, \bigcirc \Box p, p\} \rightarrow$ 8 atoms

There is only one atom such that $\varphi \in A$:
$A: \{\neg \bigcirc p, \bigcirc \Box p, p, \Box p, \varphi\}$

Any successor of A requires $\neg p$, $\Box p$, but these cannot coexist in any atom.

So the part of T_φ reachable from A is 

So there is no fulfilling path (no path at all, as A does not have a successor).
Hence, $\boxed{\varphi \text{ is not satisfiable.}}$

Example: $\boxed{\varphi_1: \Box p \land \Diamond \neg p}$

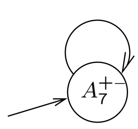
$$\Phi_{\varphi_1} = \{ \varphi_1, \Box p, \Diamond \neg p, p, \bigcirc \Box p, \bigcirc \Diamond \neg p, \neg\varphi_1, \underbrace{\neg \Box p}_{\Diamond \neg p}, \neg \Diamond \neg p, \neg p, \neg \bigcirc \Box p, \neg \bigcirc \Diamond \neg p \}$$

$\neg \Box p$ and $\Diamond \neg p$ promise $\neg p$.

Basic formulas:
$\{p, \bigcirc \Box p, \bigcirc \Diamond \neg p\} \rightarrow$ 8 atoms

There is only one atom s.t. $\varphi_1 \in A$:
$A_7: \{p, \bigcirc \Box p, \bigcirc \Diamond \neg p, \Box p, \Diamond \neg p, \varphi_1\}$

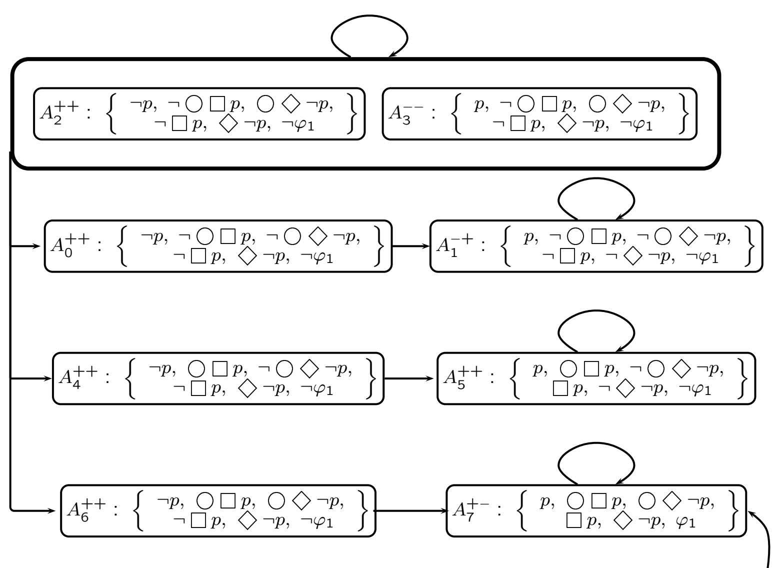
Any successor of A_7 requires $\Box p$, $\Diamond \neg p$, and therefore φ_1.
So the only successor is A_7 itself, and the part of T_{φ_1} reachable from A_7 is



which has the infinite path A_7^ω.

However, A_7^{+-} does not fulfill the promising formula $\Diamond \neg p$, and thus A_7^ω is not a fulfilling path.
Hence, $\boxed{\varphi_1 \text{ is not satisfiable.}}$

Fig. 5.3: Tableau T_{φ_1} for formula $\varphi_1: \Box p \land \Diamond \neg p$



Strongly Connected Subgraphs (SCS's)

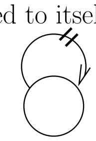
Definitions

- A subgraph $S \subseteq T_\varphi$ is called strongly connected subgraph (SCS) if for every 2 distinct atoms $A, B \in S$,

  there exists a path from A to B
  which only passes through atoms of S

Note: a single-node subgraph is an SCS

- A single-node SCS is called transient ("bad") if it is not connected to itself



- A non-transient ("good") SCS S is fulfilling if every promising formula $\psi \in \Phi_\varphi$ is fulfilled by some atom $A \in S$, i.e.

$$\neg\psi \in A \quad \text{or} \quad r \in A$$

- SCS S is φ-reachable if there exist a path and $k \geq 0$

$$B_0,\ B_1,\ \ldots,\ B_k,\ \ldots$$

  such that $\varphi \in B_0$ and $B_k \in S$.

`Example:` In $T_{\diamond p}$,
$\{A_1^+\},\ \{A_1^+,\ A_2^-\},\ \{A_4^+\}$ are fulfilling
$\{A_2^-\}$ is not fulfilling
All SCSs are $(\diamond p)$-reachable.
A_3 is a transient SCS. All others are good SCSs.

`Example:` In T_{φ_1} (Fig. 5.3),

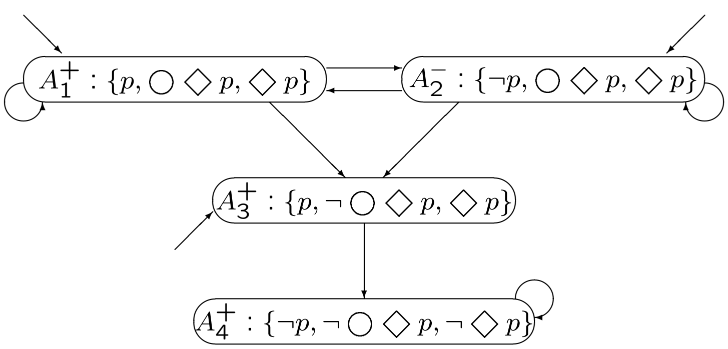
$\{A_4\}$ transient SCS

$\{A_5\}$ good SCS

$\{A_7\}$ is the only φ_1-reachable SCS

$\{A_2^{++}, A_3^{--}\}$ $\{A_5^{++}\}$	fulfilling SCS's
$\{A_1^{-+}\}$ $\{A_7^{+-}\}$	SCS's but not fulfilling

Tableau $T_{\diamond p}$



Why SCS's?

In general a tableau may have infinitely many paths, so we cannot directly determine whether there are any fulfilling paths.

What needs to hold?

- When does a graph have an infinite path?
  → it must have a *non-transient* SCS.

- When is such an infinite path induced by a model of φ?
  → SCS must be *φ-reachable*,
  i.e., reachable from a node labeled by A, s.t. $\varphi \in A$
  → SCS must be *fulfilling*,
  i.e., for every promising formula $\psi \in \Phi_\varphi$ the SCS must have at least one atom that fulfills ψ.

Proposition (satisfiability by SCS)

Formula φ is satisfiable
iff
the tableau T_φ contains a φ-reachable fulfilling SCS

The number of SCS's in a graph is finite, but may be exponential in the size of the graph!

> **Example:** $\varphi_0 : \diamond p$
> In T_{φ_0}, the fulfilling SCS's
>
> $$\{A_1^+\} \;\; \{A_1^+, A_2^-\} \;\; \{A_4^+\}$$
>
> are reachable from an initial node.
>
> Thus, $\varphi_0 : \diamond p$ is satisfiable.
> Satisfying models:
>
> $$p^\omega \quad (p, \neg p)^\omega \quad p, (\neg p)^\omega.$$

Maximal Strongly Connected Subgraphs (MSCS's)

Definition: An SCS is maximal (MSCS) if it is not properly contained in any larger SCS

Example: In T_{φ_1} (Fig. 5.3),

$$\underbrace{\{A_2\}\;\{A_3\}}_{\text{not MSCS}} \quad \underbrace{\{A_2, A_3\}}_{\text{MSCS}}$$

In fact, it is sufficient to determine whether there exists a fulfilling reachable MSCS in T_φ. The number of MSCS in T_φ is bounded by $|T_\varphi|$.

Decomposition into MSCS's

There exists an efficient algorithm [Hopcroft&Tarjan] to decompose T_φ into subgraphs $G_1, \ldots, G_N$ such that

- each G_i is an MSCS (and therefore disjoint)
- $G_1 \cup \ldots \cup G_N = T_\varphi$
- whenever there is an edge from a node in G_i to a node in G_j then $i \leq j$.

Algorithm SAT

(check satisfiability of arbitrary temporal formula φ)

- construct T_φ
- construct T_φ^- by removing all atoms that are not reachable from φ-atom
- decompose T_φ^- into MSCS's $U_1, \ldots, U_k$
- check whether $U_1, \ldots, U_k$ is fulfilling:
  - if some U_i is fulfilling: φ is satisfiable.
    A model is defined by the path leading from a φ-atom to U_i and staying in U_i forever from then on.
  - if no U_i is fulfilling: φ is not satisfiable.

Proposition (satisfiability and MSCS)

Formula φ is satisfiable
iff
The tableau T_φ^- contains a φ-reachable fulfilling MSCS

Check validity of φ

Apply algorithm SAT to $\neg\varphi$

Algorithm reports success:

$\neg\varphi$ is satisfiable $=$ φ is not valid
(the produced σ is a counterexample)

Algorithm reports failure:

$\neg\varphi$ is unsatisfiable $=$ φ is valid

Example: Check satisfiablility of

$$\boxed{\varphi_1\colon\ \Box\, p \ \wedge\ \Diamond\, \neg p}$$

T_{φ_1} (Fig 5.3)
$T_{\varphi_1}^- = \{A_7^{+-}\}$ MSCS of $T_{\varphi_1}^- = \{A_7^{+-}\}$
nonfulfilling $\Longrightarrow$ $\boxed{\varphi_1 \text{ is unsatisfiable}}$

Example:

$$\boxed{\psi_1 = \neg\varphi_1\colon\ \neg(\Box\, p \ \wedge\ \Diamond\, \neg p)}$$

T_{ψ_1} (Fig 5.3)
$T_{\psi_1}^-$: all atoms

MSCS's:

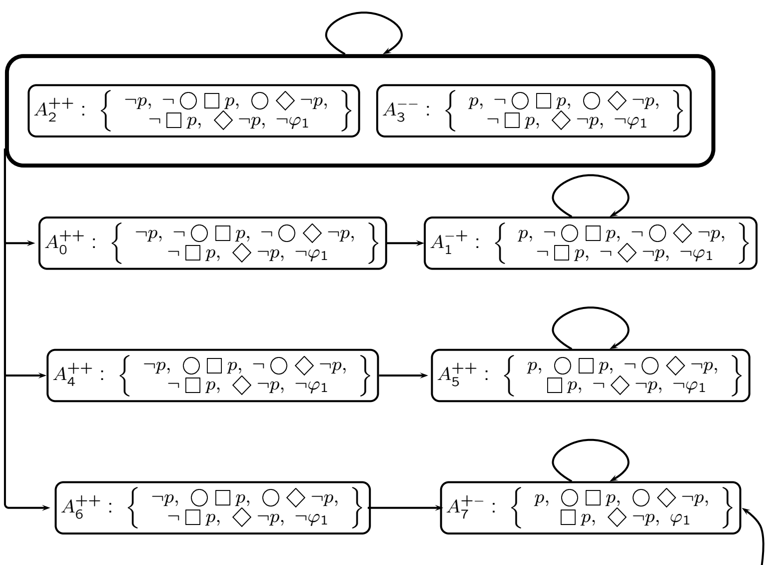
$\{A_0\},\ \{A_4\},\ \{A_6\}$	transient
$\{A_1^{-+}\},\ \{A_7^{+-}\}$	non-fulfilling
$\{A_2^{++}, A_3^{--}\},\ \{A_5^{++}\}$	fulfilling

$\boxed{\psi_1 \text{ satisfiable}}$

For A_5^{++}:	A_5^ω	model $\langle p\colon \mathrm{T}\rangle^\omega$
For $\{A_2^{++}, A_3^{--}\}$:	$(A_2, A_3)^\omega$	model $(\langle p\colon \mathrm{F}\rangle\langle p\colon \mathrm{T}\rangle)^\omega$

each satisfies ψ_1

Fig. 5.3: Tableau T_{φ_1} for formula
$\varphi_1\colon\ \Box\, p \ \wedge\ \Diamond\, \neg p$



Example: Check satisfiability of

$$\boxed{\varphi_2\colon\ \Box(\underbrace{\neg at\_\ell_2 \ \vee\ \Diamond\, at\_\ell_3}_{p_2})}$$

$\Phi_{\varphi_2}^+$: $\{\ \Box\, p_2,\quad \bigcirc\Box\, p_2,\quad p_2,\quad at\_\ell_2,$
$\Diamond\, at\_\ell_3,\quad \bigcirc\Diamond\, at\_\ell_3,\quad at\_\ell_3\ \}$

φ_2-reachable atoms

$\{\ \underbrace{\underbrace{\Box\, p_2}_{\varphi_2},\ \bigcirc\Box\, p_2,\ p_2,}_{\text{fixed}}$

$\underbrace{at\_\ell_2,\ at\_\ell_3,\ \bigcirc\Diamond\, at\_\ell_3,}_{\text{8 possibilities}}\ \underbrace{\Diamond\, at\_\ell_3,\ \neg\Diamond\, at\_\ell_3}_{\text{followed}}\}$

One promising formula in Φ : $\Diamond\, at\_\ell_3$(and $\neg\Box\, p_2$)

A_0^+ :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$\neg at\_\ell_2,$	$\neg at\_\ell_3,$	$\neg\bigcirc\Diamond\, at\_\ell_3,$	$\neg\Diamond\, at\_\ell_3\}$
A_1^- :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$\neg at\_\ell_2,$	$\neg at\_\ell_3,$	$\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$
A_2^+ :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$\neg at\_\ell_2,$	$at\_\ell_3,$	$\neg\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$
A_3^+ :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$\neg at\_\ell_2,$	$at\_\ell_3,$	$\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$
A_4^- :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$at\_\ell_2,$	$\neg at\_\ell_3,$	$\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$
A_5^+ :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$at\_\ell_2,$	$at\_\ell_3,$	$\neg\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$
A_6^+ :	$\{\Box\, p_2,$	$\bigcirc\Box\, p_2,$	$p_2,$	$at\_\ell_2,$	$at\_\ell_3,$	$\bigcirc\Diamond\, at\_\ell_3,$	$\Diamond\, at\_\ell_3\}$

Example: (Cont'd)

Atom #8

$\{\ \underline{\Box p_2,\ \bigcirc\Box p_2,\ p_2}\ ,\ at\_\ell_2,$
$\neg at\_\ell_3,\ \neg\bigcirc\Diamond\, at\_\ell_3,\ \ldots\}$

is not considered since

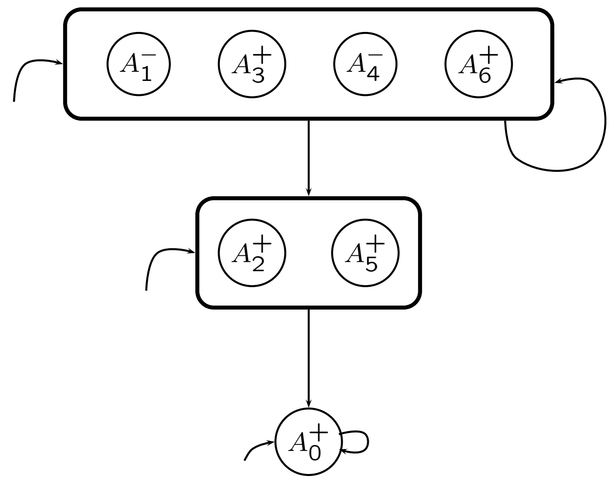
$\underbrace{\neg at\_\ell_2 \vee \Diamond\, at\_\ell_3}_{p_2}$ and $at\_\ell_2 \rightarrow \Diamond\, at\_\ell_3$

$\neg at\_\ell_3$ and $\neg\bigcirc\Diamond\, at\_\ell_3 \rightarrow \neg\Diamond\, at\_\ell_3$

Tableau T_{φ_2} (Fig 5.4) $=$ $T^-_{\varphi_2}$

formula $\Diamond\, at\_\ell_3$ promising at_ℓ_3

Fig. 5.4. Tableau for φ_2: $\Box(\neg at\_\ell_2 \vee \Diamond\, at\_\ell_3)$



Decomposition to MSCS's

$\{A_1^-,\ A_3^+,\ A_4^-,\ A_6^+\}$ $\{A_2^+\}$ $\{A_5^+\}$ $\{A_0^+\}$

fulfilling MSCS's: $\{A_0^+\}$, $\{A_1^-, A_3^+, A_4^-, A_6^+\}$
($\{A_2\}$ and $\{A_5\}$ are transient)

$\boxed{\varphi_2 \text{ is satisfiable}}$

model (by A_0^ω)

$\langle at\_\ell_2 \colon f,\ at\_\ell_3 \colon f\rangle^\omega$

Pruning the tableau

Definition: MSCS S is terminal if

there are no edges leading from
atoms of S to atoms outside S

Example: Consider $\psi_1 = \neg\varphi_1 : \neg(\Box p \wedge \Diamond\neg p)$
In T_{ψ_1} (same as T_{φ_1}, Fig 5.3, except for initial nodes)

$\{A_1\}$ $\{A_5\}$ $\{A_7\}$ are terminal MSCS's

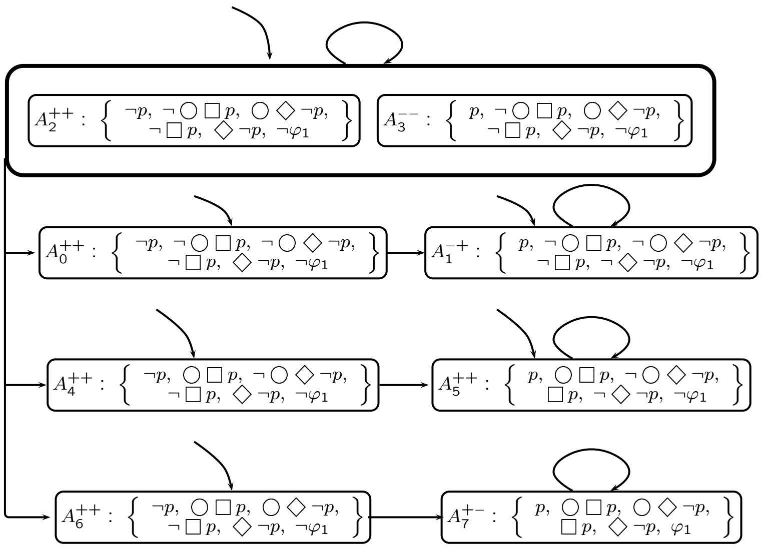
$\{A_6\}$ $\{A_2, A_3\}$ are not

After constructing T_φ, remove useless atoms:

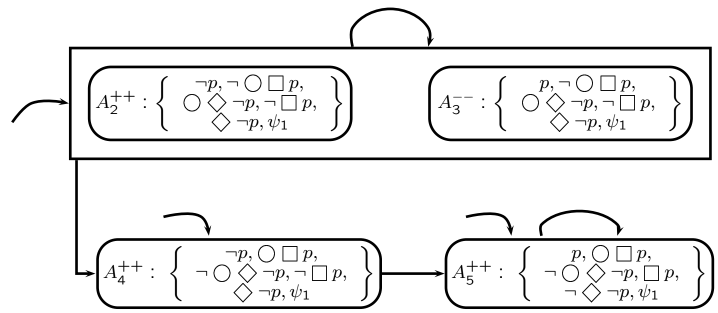
- Remove an MSCS that is not φ-reachable.
- Remove a terminal MSCS that is not fulfilling.

Iterate until no further atoms can be removed.

Fig. 5.3: Tableau T_{ψ_1} for formula
ψ_1: $\neg(\Box\, p \;\wedge\; \Diamond\, \neg p)$.



Pruned Tableau $T^-_{\psi_1}$ for
$\psi_1 : \neg(\Box\, p \wedge \Diamond\, \neg p)$



Fulfilling MSC's: $\{A_2^{++}, A_3^{-+}\}$, $\{A_5^{++}\}$
ψ_1 : $\neg(\Box\, p \wedge \Diamond\, \neg p)$ is satisfiable.

Example:

φ_3: $\Box\,\Diamond(x = 3)$

$\Phi^+_{\varphi_3}$: $\{\ \varphi_3,\ \Diamond(x = 3),\ x = 3,\ \bigcirc\Diamond(x = 3),\ \bigcirc\varphi_3\ \}$

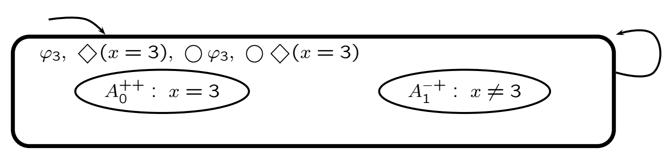
8 atoms $A_0, \ldots, A_7$ (see list)

$\{\underbrace{x = 3,\ \bigcirc\Diamond(x = 3),\ \bigcirc\varphi_3}_{\text{8 possibilities}},\ \ldots\}$

Promising formulas: $\Diamond(x = 3)$ and $\neg\underbrace{\Box\,\Diamond(x = 3)}_{\varphi_3}$

A_0^{++} :	$\{x = 3,$	$\bigcirc\Diamond(x = 3),$	$\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\varphi_3\}$
A_1^{-+} :	$\{x \neq 3,$	$\bigcirc\Diamond(x = 3),$	$\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\varphi_3\}$
A_2^{++} :	$\{x = 3,$	$\neg\bigcirc\Diamond(x = 3),$	$\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\varphi_3\}$
A_3^{++} :	$\{x \neq 3,$	$\neg\bigcirc\Diamond(x = 3),$	$\bigcirc\varphi_3,$	$\neg\Diamond(x = 3),$	$\neg\varphi_3\}$
A_4^{+-} :	$\{x = 3,$	$\bigcirc\Diamond(x = 3),$	$\neg\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\neg\varphi_3\}$
A_5^{--} :	$\{x \neq 3,$	$\bigcirc\Diamond(x = 3),$	$\neg\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\neg\varphi_3\}$
A_6^{+-} :	$\{x = 3,$	$\neg\bigcirc\Diamond(x = 3),$	$\neg\bigcirc\varphi_3,$	$\Diamond(x = 3),$	$\neg\varphi_3\}$
A_7^{++} :	$\{x \neq 3,$	$\neg\bigcirc\Diamond(x = 3),$	$\neg\bigcirc\varphi_3,$	$\neg\Diamond(x = 3),$	$\neg\varphi_3\}$

Fig. 5.6. Pruned tableau $T^-_{\varphi_3}$



The φ_3-reachable MSCS's: $\{A_0^{++}, A_1^{-+}\}$

$\{A_0^{++}, A_1^{-+}\}$ is fulfilling.
Therefore, φ_3 is satisfiable.

Model (by $(A_0, A_1)^\omega$): $(\langle x{:}\,3\rangle, \langle x{:}\,0\rangle)^\omega$
↑
arbitrary $x \neq 3$