Static Analysis Techniques for Predicting the Behavior of Database Production Rules

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Abstract

Methods are given for statically analyzing sets of database production rules to determine if the rules are (1) guaranteed to terminate, (2) guaranteed to produce a unique final database state, and (3) guaranteed to produce a unique stream of observable actions. If the analysis determines that one of these properties is not guaranteed, it isolates the rules responsible for the problem and determines criteria that, if satisfied, guarantee the property. The analysis methods are presented in the context of the Starburst Rule System.

1 Introduction

Production rules in database systems allow specification of data manipulation operations that are executed automatically whenever certain events occur or conditions are met, making the database system *active* [DW92]. Database production rules provide a general and powerful mechanism for many database features, including integrity constraint enforcement, derived data maintenance, triggers, alerters, authorization checking, and versioning. In addition, active database systems provide a convenient platform for large and efficient knowledge-bases and expert systems. A significant drawback of active database systems, however, lies in the development of correct rule applications: it can be very difficult in general to predict how a set of database production rules will behave. Rule processing occurs as a result of arbitrary database changes; certain rules are triggered initially, and their execution can trigger additional rules or trigger the same rules additional times. The unstructured, unpredictable, and often nondeterministic behavior of rule processing can become a nightmare for the database rule programmer.

A significant step in aiding the database rule programmer is to provide a facility that statically analyzes sets of rules, providing information about the following three properties of rule behavior:

- *Termination*: Is rule processing guaranteed to terminate after any set of changes to the database in any state?

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- *Confluence*: Can the execution order of non-prioritized rules make any difference in the final database state? That is, if multiple rules are triggered at the same time during rule processing, can the final database state at termination of rule processing depend on which is considered first? If not, the rule set is *confluent.*
- *Observable Determinism*: If a rule action is visible to the environment (e.g., if it performs data retrieval or a rollback statement), then we say it is *observable.* Can the execution order of non-prioritized rules make any difference in the order or appearance of observable actions? If not, the rule set is *observably deterministic.*

These properties can be very difficult or impossible to decide in the general case. However, we have developed conservative static analysis algorithms that:

- guarantee that a set of rules will terminate or say that it may not terminate;
- guarantee that a set of rules is confluent or say that it may not be confluent;
- guarantee that a set of rules is observably deterministic or say that it may not be observably deterministic.

Furthermore, when the answer is "may not" for any of these properties, the analysis algorithms isolate the rules responsible for the problem and determine criteria that, if satisfied, guarantee the property. Hence the analysis can form the basis of an interactive environment where the rule programmer invokes the analyzer to obtain information about rule behavior. If termination, confluence, or observable determinism is desired but not guaranteed, then the user may verify that the necessary criteria are satisfied or may modify the rule set and try again.

Our analysis methods have been developed and are presented in the context of the *Starburst Rule System* [WCL91], a production rules facility integrated into the Starburst extensible relational DBMS prototype at the IBM Almaden Research Center [H^+90]. Although some aspects of the analysis are dependent on Starburst rules, we have tried to remain as general as possible, and our methods certainly can be adapted to other database rule languages.

1.1 Related Work

Most previous work in static analysis of production rules [HH91,Ras90,ZH90] differs from ours in two ways. First, it considers simplified versions of the *OPS5* production rule language [BFKM85]. OPS5 has a quite different model of rule processing than most active database systems, including Starburst. Second, the goal of previous work is to impose restrictions and/or orderings on OPS5 rule sets such that unique fixed points are guaranteed. Our goal, on the other hand, is to permit arbitrary rule sets and provide useful information about their behavior in the database setting. In Section 11 we make some additional, more technical, comparisons, and we explain how our analysis techniques subsume results in [HH91,Ras90,ZH90].

In [KU91], the issue of rule set termination is discussed, along with the issue of *conflicting updates*—determining when one rule may undo changes made by a previous rule. Although models and a problem-solving architecture for rule analysis are proposed, no algorithms are given. In [CW90] we presented preliminary methods for analyzing termination in the context of deriving production rules for integrity constraint maintenance; these methods form the basis of our approach to termination in this paper. An initial presentation of our analysis methods appears in [AWH92]; in this paper we provide additional intuition, refine and extend the methods in [AWH92], include numerous examples, and provide proofs for all lemmas and theorems.

1.2 Outline of Paper

As an introduction to active databases and to establish a basis for our analysis techniques, in Section 2 we give a syntax and semantics for the Starburst production rule language; Section 3 then motivates the termination, confluence, and observable determinism properties in the context of this language. In Section 4 we provide examples of Starburst production rules, illustrating rule sets that do and do not satisfy termination, confluence, and observable determinism. In Section 5 we introduce notation and definitions needed for rule analysis and we define some straightforward preliminary analysis techniques. In Section 6 we present a model of rule processing to be used as the formal basis for our analysis algorithms. Termination analysis is covered in Section 7 and confluence in Section 8. In Section 9 we give methods for analyzing *partial confluence*, which specifies that a rule set is confluent with respect to a portion of the database. Observable determinism is covered in Section 10. Finally, in Section 11 we draw conclusions and discuss future work.

2 The Starburst Rule System

We provide a brief overview of the Starburst database production rule language. Examples are given in Section 4; additional examples and further details appear in [WCL91,WF90].

Starburst production rules are based on the notion of *transitions*. A transition is a database state change resulting from execution of a sequence of data manipulation operations. Rules consider only the *net effect* of transitions, meaning that: (1) if a tuple is updated several times, only the composite update is considered; (2) if a tuple is updated then deleted, only the deletion is considered; (3) if a tuple is inserted then updated, this is considered as inserting the updated tuple; (4) if a tuple is inserted then deleted, this is not considered at all. A formal theory of transitions and their net effects appears in [WF90].

The syntax for defining a rule is:

> **create rule** *name* **on** *table*
> **when** *triggering-operations*
> [**if** *condition*]
> **then** *action*
> [**precedes** *rule-list*]
> [**follows** *rule-list*]

The *triggering-operations* are one or more of **inserted**, **deleted**, and **updated**($c_1, \ldots, c_n$), where $c_1, \ldots, c_n$ are columns of the rule's *table*. The rule is triggered by a given transition if at least one of the specified operations occurred in the net effect of the transition. The optional *condition* specifies an SQL predicate.[1] The *action* specifies an arbitrary sequence of database operations to be executed when the rule is triggered and its condition is true; these operations may be SQL data modification operations (**insert**, **delete**, **update**), SQL data retrieval operations (**select**), and transaction abort (**rollback**). The optional **precedes** and **follows** clauses are used to induce a partial ordering on the set of defined rules. If a rule r_1 specifies a rule r_2 in its **precedes** list, or if r_2 specifies r_1 in its **follows** list, then r_1 is higher than r_2 in the ordering. (We also say that r_1 has *precedence* or *priority* over r_2.) When no direct or transitive ordering is specified between two rules, their order is arbitrary.[2]

A rule's condition and action may refer to the current state of the database through top-level and nested SQL **select** operations. In addition, rule conditions and actions may refer to *transition tables*, which are logical tables reflecting the changes to the rule's table that have occurred during the triggering transition. At the end of a given transition, transition table **inserted** in a rule refers to those tuples of the rule's table that were inserted by the transition, transition table **deleted** refers to those tuples that were deleted, and transition tables **new-updated** and **old-updated** refer to the new and old values (respectively) of the updated tuples. A rule may refer only to transition tables corresponding to its triggering operations.

Rules are activated at *rule processing points*. There is an automatic rule processing point at the end of each transaction, and there may be additional user-specified rule processing points within a transaction. We describe the semantics of rule processing at an arbitrary point. The state change resulting from the user-generated data modification operations executed since the last rule processing point (or start of the transaction) creates the first relevant transition, and some rules are triggered by this transition. As rule actions are executed, additional transitions are created which may trigger additional rules or trigger the same rules additional times. Rule processing follows an iterative algorithm in which:

1. A triggered rule r is selected for *consideration* such that no other triggered rule has precedence over r.
2. r's condition is evaluated.
3. If r's condition is true then r's action is executed.

For step 1 in this algorithm, a rule is triggered if one or more of its triggering operations occurred in the composite transition since the last time the rule was considered, or since the start of the transaction if the rule has not yet been considered. (The effect of this semantics is that each rule

[1]In the current implementation rule conditions are SQL **select** statements, where the condition is true iff the **select** statement produces one or more tuples. In the context of rule conditions and for the purposes of rule analysis, predicates and **select** statements are equivalent [WCL91].

[2]The system actually orders non-prioritized rules using an algorithm based on rule creation time [ACL91], but this is an implementation feature separate from the semantics of the rule language.

sees each modification exactly once.) Rule processing terminates when a **rollback** operation is executed or when there are no more triggered rules.

User-invoked rule processing may specify that only a subset of the defined rules should be considered for execution, rather than all rules [Wid92]. Hence, the rule programmer may want to predict the behavior of a subset of the rules. The semantics of rule processing for rule subsets is identical to rule processing in the general case, so our analysis methods can be applied directly to arbitrary rule subsets.

The analysis techniques we present are based on the Starburst rule language and rule processing semantics, but with modifications the methods also could apply to other similar languages; see Section 11.

3 Termination, Confluence, and Observable Determinism

Suppose a set of Starburst rules has been defined using the language described in Section 2. Further suppose that arbitrary data modification operations have been performed, creating an initial triggering transition, and then rule processing is invoked. From the algorithm given in Section 2 it is clear that rule processing may not terminate—rules could trigger each other forever. A set of Starburst rules has the *termination* property if, for any initial triggering transition, rule processing is guaranteed to terminate, i.e. eventually **rollback** is executed or there are no more triggered rules.

In step 1 of the rule processing algorithm, a rule r is selected such that no other triggered rule has precedence over r. Since the precedence of rules may be only a partial order (indeed, no ordering is required), many rules may be eligible for selection in this step. A set of Starburst rules has the *confluence* property if, for any initial triggering transition, there is some database state D such that if rule processing terminates it terminates with the database in state D, regardless of which eligible rule is selected each time step 1 is performed.

Finally, when rule actions are executed (step 3 in the rule processing algorithm), these actions may be visible to the environment in which the transaction is executing. For example, actions may perform data retrieval, or they may abort the transaction. A set of Starburst rules has the *observable determinism* property if, for any initial triggering transition, the sequence of **select** and **rollback** operations executed during rule processing (including the values returned by **select** operations) is identical, regardless of which eligible rule is selected each time step 1 is performed.

4 Examples

For examples we consider a simple database schema with three tables:

emp(id, rank, salary)
bonus(emp-id, amount)
sales(emp-id, month, number)

Table **emp** records each employee's rank and salary, table **bonus** records a bonus amount to be awarded to each employee, and table **sales** records each employee's number of sales on a monthly basis. Because our example rules are quite simple, none includes an **if** clause; this does not detract from illustrating the salient points of termination, confluence, and observable determinism.

Example 4.1 (nonterminating) Nontermination is illustrated with two (admittedly contrived) rules. The first rule, **bonus-rank**, states that whenever an employee's bonus is increased by more than 100, that employee's rank is increased by 1:

```
create rule bonus-rank on bonus
when updated(amount)
then update emp
     set rank = rank + 1
     where id in (select emp-id from new-updated, old-updated
                  where new-updated.emp-id = old-updated.emp-id
                  and new-updated.amount - old-updated.amount > 100)
```

The second rule, **rank-bonus**, states that whenever an employee's rank is modified, that employee's bonus is increased by 10 times the new rank:

```
create rule rank-bonus on emp
when updated(rank)
then update bonus
     set amount = amount + 10 * (select rank from new-updated
                                 where new-updated.id = bonus.emp-id)
     where emp-id in (select id from new-updated)
```

With these two rules, whenever an employee's rank is modified to greater than 10, or whenever an employee with rank at least 10 receives a bonus greater than 100, rule processing does not terminate—the rules trigger each other forever. □

Example 4.2 (terminating, non-confluent) This is the most subtle and interesting of our examples. We specify three rules that are guaranteed to terminate. We suggest an ordering for the rules that appears to guarantee confluence, then we explain how this ordering is in fact insufficient for confluence. The first rule, **good-sales**, increases an employee's salary by 10 whenever that employee posts sales greater than 50 for a month:

```
create rule good-sales on sales
when inserted
then update emp
     set salary = salary + 10
     where id in (select emp-id from inserted where number > 50)
```

The second rule, **great-sales**, increases an employee's rank by 1 whenever that employee posts sales greater than 100 for a month:

```
create rule great-sales on sales
when inserted
then update emp
     set rank = rank + 1
     where id in (select emp-id from inserted where number > 100)
```

Although rules **good-sales** and **great-sales** may be triggered at the same time, their actions cannot affect each other, so no relative ordering between the rules is needed for confluence. The third rule, **rank-raise**, increases an employee's salary by 10% whenever that employee's rank reaches 15 (we assume that ranks do not decrease):

```
create rule rank-raise on emp
when updated(rank)
then update emp
     set salary = 1.1 * salary
     where id in (select id from new-updated where rank = 15)
```

For confluence it is clear that a relative ordering is required between rule **rank-raise** and rule **good-sales** since, if both rules are triggered at the same time, the order in which their actions execute influences the final salary. We specify that **rank-raise** has priority over **good-sales**, so the 10% increase given by **rank-raise** does not reflect the increase of 10 given by **good-sales**. It also is clear that a relative ordering is required between rule **rank-raise** and rule **great-sales** since, if both rules are triggered at the same time, executing **great-sales** first could increase an employee's rank to 16 before **rank-raise** awards the 10% increase. Hence, we add the following clause to rule **rank-raise**:

```
precedes good-sales, great-sales
```

It turns out that these orderings still are insufficient for confluence. Suppose rules **good-sales** and **great-sales** are triggered at the same time, and there is an employee whose newly posted sales exceeds 100, whose salary is 60, and whose rank is 14. Suppose rule **good-sales** is executed first. Then **good-sales** increases the employee's salary to 70, **great-sales** increases the employee's rank to 15, and finally **rank-raise** is triggered and increases the employee's salary to 77. Now suppose instead that rule **great-sales** is executed first. Then **great-sales** increases the employee's rank to 15, **rank-raise** is triggered and increases the employee's salary to 66, and finally **good-sales** increases the employee's salary to 76. Hence there are two possible final values for the employee's salary, and the rules are not confluent.

The important property to observe here is that there are two rules (**good-sales** and **great-sales**) that appear to be unrelated and therefore appear to need no relative ordering. However, the existence of a third rule (**rank-raise**) that is related to both of the first two rules means that the first two rules must be ordered to achieve confluence. □

Example 4.3 (terminating, confluent, not observably deterministic) Consider rules **good-sales** and **rank-raise** from Example 4.2, ordered so that **rank-raise** precedes **good-sales**. Now add a third rule, **new-rank**, that displays an employee's ID, rank, and salary, along with a "new-rank" message, whenever that employee's rank is modified:

```
create rule new-rank on emp
when updated(rank)
then select id,rank,salary,"new-rank" from new-updated
```

These three rules are guaranteed to terminate and they are confluent—the final database state is guaranteed to be unique. However, the rules are not observably deterministic. Suppose all three rules are triggered at the same time. Then the salaries displayed by rule **new-rank** may take on three different values, depending on whether **new-rank** is executed before **rank-raise**, after **rank-raise** but before **good-sales**, or after **good-sales**. □

Example 4.4 (terminating, confluent, observably deterministic) Consider rule **bonus-rank** from Example 4.1, rule **good-sales** from Example 4.2, and rule **new-rank** from Example 4.3, with **new-rank** specified to follow **bonus-rank** and **good-sales**. This set of rules exhibits all three properties: termination, confluence, and observable determinism. □

5 Definitions and Preliminary Analysis

Let $R = \{r_1, r_2, \ldots, r_n\}$ denote an arbitrary set of Starburst production rules to be analyzed. Analysis is performed on a fixed set of rules—when the rule set is changed, analysis must be repeated. (Incremental methods are certainly possible; see Section 11.) Let P denote the set of user-defined priority orderings on rules in R (as specified by their **precedes** and **follows** clauses), including those implied by transitivity. $P = \{r_i > r_j,\ r_k > r_l,\ \ldots\}$, where $r_i > r_j$ denotes that rule r_i has precedence over r_j. Let $T = \{t_1, t_2, \ldots, t_m\}$ denote the tables in the database schema, and let $C = \{t_i.c_j,\ t_k.c_l,\ \ldots\}$ denote the columns of tables in T. Finally, let O denote the set of database modification operations:

$$O \;=\; \{\langle \mathbf{I}, t\rangle \mid t \in T\} \;\cup\; \{\langle \mathbf{D}, t\rangle \mid t \in T\} \;\cup\; \{\langle \mathbf{U}, t.c\rangle \mid t.c \in C\}$$

$\langle \mathbf{I}, t\rangle$ denotes insertions into table t, $\langle \mathbf{D}, t\rangle$ denotes deletions from table t, and $\langle \mathbf{U}, t.c\rangle$ denotes updates to column c of table t.

The following definitions are computed using straightforward preliminary analysis of the rules in R:

- *Triggered-By* takes a rule r and produces the set of operations in O that trigger r. *Triggered-By* is trivial to compute based on rule syntax.

- *Performs* takes a rule r and produces the set of operations in O that may be performed by r's action. *Performs* is trivial to compute based on rule syntax.

- *Triggers* takes a rule r and produces all rules r' that can become triggered as a result of r's action (possibly including r itself). $\mathit{Triggers}(r) = \{r' \in R \mid \mathit{Performs}(r) \cap \mathit{Triggered\text{-}By}(r') \neq \emptyset\}$.

- *Uses* takes a rule r and produces all columns in C that may be referenced when evaluating r's condition or executing a data modification operation in r's action. $\mathit{Uses}(r)$ contains every $t.c$ referenced in r's condition, every $t.c$ referenced in the **where** clause of a **delete** or **update** operation in r's action, and every $t.c$ referenced in a nested **select** expression in an **insert**, **delete**, or **update** operation in r's action. In addition, for every $\langle \mathit{trans}\rangle.c$ referenced in this same way, where $\langle \textit{trans} \rangle$ is one of **inserted**, **deleted**, **new-updated**, or **old-updated**, $t.c$ is in $\textit{Uses}(r)$ for r's triggering table t. (Recall from Section 2 that **inserted**, **deleted**, **new-updated**, and **old-updated** are transition tables based on changes to t.)

- *Can-Untrigger* takes a set of operations $O' \subseteq O$ and produces all rules that can be "untriggered" as a result of operations in O'. A rule is untriggered if it is triggered at some point during rule processing but not chosen for consideration, then subsequently no longer triggered because all triggering changes were undone by other rules.[3] $\textit{Can-Untrigger}(O') = \{r \in R \mid \langle \mathbf{D}, t \rangle \in O' \text{ and } \langle \mathbf{I}, t \rangle \text{ or } \langle \mathbf{U}, t.c \rangle \in \textit{Triggered-By}(r) \text{ for some } t \in T,\ t.c \in C\}$.

- *Choose* takes a set of triggered rules $R' \subseteq R$ and produces a subset of R' indicating those rules eligible for consideration (based on priorities). $\textit{Choose}(R') = \{r_i \mid r_i \in R' \text{ and there is no } r_j \in R' \text{ such that } r_j > r_i \in P\}$.

- *Rollback* takes a rule r and indicates whether executing r's action is guaranteed to abort the transaction. In Starburst, $\textit{Rollback}(r)$ is true iff one of the operations comprising r's action is **rollback**.

- *Observable* takes a rule r and indicates whether r's action may be observable. In Starburst, a rule's action may be observable iff it includes **rollback** or a top-level **select** statement.

6 Execution Model

We now define a formal model of execution-time rule processing. The model is based on *execution graphs* and accurately captures the semantics of rule processing described in Section 2. Note that execution graphs are used to discuss and to prove the correctness of our analysis techniques, but these graphs are never actually constructed and they are not part of the analysis itself.

A directed execution graph has a distinguished initial state representing the start of rule processing (at any processing point) and zero or more final states representing termination of rule processing. The paths in the graph represent all possible execution sequences during rule processing; branches in the graph result from choosing different rules to consider when more than one is eligible. (Hence any graph for a totally ordered rule set has no branches.) The graph may have infinitely long paths (possibly but not necessarily due to cycles in the graph); these paths represent nontermination of rule processing.

More formally, a *state* (node) S in an execution graph has two components: (1) a database state D; (2) a set TR containing each triggered rule and its associated transition tables. We denote this state as $S = (D, TR)$. The initial state I is created by an initial transition, which results from a sequence of user-generated database operations. Hence, $I = (D_I, TR_I)$ where D_I is a database state and there is some set of operations $O' \subseteq O$ such that:

[3]As an example, a rule r_1 might be triggered by insertions, but another rule r_2 might delete all inserted tuples before r_1 is chosen for consideration. Untriggering is rare in practice.

$$TR_I = \{r \in R \mid O' \cap \textit{Triggered-By}(r) \neq \emptyset\}$$

O' is the set of operations producing the initial transition, and TR_I contains the rules triggered by those operations. A final state F is some $(D_F, \emptyset)$, since no rules are triggered when rule processing terminates. Note that a final state F may correspond to normal termination or it may correspond to termination due to a **rollback** statement; in the latter case $F = (D_T, \emptyset)$, where D_T is the database state at the start of the transaction.

Each directed *edge* in an execution graph is labeled with a rule r and represents the consideration of r during rule processing. (This includes determining whether r's condition is true and, if so, executing r's action.) Using definitions from Section 5, the following lemma states certain properties that hold for all execution graphs. The lemma is stated without proof—it follows directly from the semantics of rule processing described in Section 2.

Lemma 6.1 (Properties of Execution Graphs) Consider any execution graph edge from a state (D_1, TR_1) to a state (D_2, TR_2) labeled with a rule r. Then:

- $r \in \textit{Choose}(TR_1)$
- Either:
  (1) $\textit{Rollback}(r)$, $D_2 = D_T$, and $TR_2 = \emptyset$; or
  (2) There is some (possibly empty) set of operations $O' \subseteq \textit{Performs}(r)$ such that the triggered rules in TR_2 can be derived from the triggered rules in TR_1 by:
    (a) removing rule r
    (b) removing some subset of the rules in $\textit{Can-Untrigger}(O')$
    (c) adding all rules $r' \in R$ such that $O' \cap \textit{Triggered-By}(r') \neq \emptyset$ $\square$

Case (1) corresponds to the situation in which r's condition is true and its action includes **rollback**. Case (2) corresponds to the situation in which r's condition is false, or r's condition is true and its action does not include **rollback**. In case (2), the operations in O' are those executed by r's action, where O' is empty if r's condition is false. If r's condition is true then O' still may be a proper subset of *Performs*(r) since, by the semantics of SQL, for most operations there are certain database states on which they have no effect. Finally, note that although rule r is removed in step (a), r may be added again in step (c) if $O' \cap \textit{Triggered-By}(r) \neq \emptyset$.

The properties in Lemma 6.1 are guaranteed for all execution graphs. By performing more complex analysis on rule conditions and actions, by incorporating properties of database states, and by considering a variety of special cases, we probably can identify additional properties of execution graphs. Since our analysis techniques are based on execution graph properties, more accurate properties may result in more accurate rule analysis. We believe that the properties used here, although somewhat conservative, are sufficiently accurate to yield strong analysis techniques.

7 Termination

We want to determine whether the rules in R are guaranteed to terminate. That is, we want to determine if for all user-generated operations and initial database states, rule processing always reaches a point at which there are no triggered rules to consider. We take as an assumption that individual rule actions terminate. Hence, in terms of execution graphs, the rules in R are guaranteed to terminate iff all paths in every execution graph for R are finite.

As suggested in [CW90], termination is analyzed by constructing a directed *triggering graph* for the rules in R, denoted TG_R. The nodes in TG_R represent all rules $r \in R$ such that *Rollback*(r) is false; the edges in TG_R represent the *Triggers* relationship. That is, there is an edge from r_i to r_j in TG_R iff $r_j \in$ *Triggers*(r_i). We exclude a rule r_i from the graph if *Rollback*(r_i), since if r_i contains **rollback** then it will not trigger any other rule r_j, even if technically $r_j \in$ *Triggers*(r_i).[4]

Theorem 7.1 (Termination) If there are no cycles in TG_R then the rules in R are guaranteed to terminate.

Proof: We must prove that all paths in every execution graph for R are finite. Suppose, for the sake of a contradiction, that there are no cycles in graph TG_R but there is an infinite path p in some execution graph for R. Then, since there are only finitely many rules, some rule r must appear on infinitely many edges on path p. By the properties of execution graphs in Lemma 6.1, r must be added to set TR of triggered rules (by step 3) infinitely many times. Hence there must be some operation $o \in$ *Triggered-By*(r) that is performed infinitely many times. Since o is in *Performs*(r_1) for only a finite number of rules r_1, there must be some rule r_1 such that $o \in$ *Performs*(r_1) and r_1 appears on infinitely many edges on path p. Note that, by definition, $r \in$ *Triggers*(r_1), so there is an edge from r_1 to r in TG_R. Since r_1 appears on infinitely many edges, r_1 must be added to set TR infinitely many times. By the same reasoning as above, there is some rule r_2 such that $r_1 \in$ *Triggers*(r_2) (so there is an edge from r_2 to r_1 in TG_R) and r_2 appears on infinitely many edges on path p. This reasoning continues, generating rules r_3, r_4, etc. Since, by assumption, there are no cycles in graph TG_R, this reasoning generates infinitely many rules, a contradiction. $\square$

Hence, to determine whether the rules in R are guaranteed to terminate, triggering graph TG_R is constructed and checked for cycles. As an example, consider the triggering graph TG_R for rules R = {**bonus-rank**, **rank-bonus**} of Example 4.1. There are two nodes in TG_R, one representing rule **bonus-rank** (call the node *br*) and one representing rule **rank-bonus** (call the node *rb*). Since **rank-bonus** $\in$ *Triggers*(**bonus-rank**), there is an edge in TG_R from node *br* to node *rb*. Similarly, since **bonus-rank** $\in$ *Triggers*(**rank-bonus**), there is an edge in TG_R from node *rb* to node *br*. Therefore, there is a cycle in TG_R, and our analysis determines that this set of rules may not terminate. In Examples 4.2, 4.3, and 4.4, the triggering graphs are acyclic, so our analysis determines that the rules are guaranteed to terminate.

[4]If *Rollback*(r_i) and $r_j \in$ *Triggers*(r_i), then r_i must include both a data modification operation and a **rollback** operation in its action. This is allowed syntactically, but semantically it makes little sense—the effect of the data modification operation always will be undone when the **rollback** operation is executed.

Although our approach to termination may appear to be very conservative, by considering only the known properties of our execution graph model (Lemma 6.1), we see that whenever there is a cycle in the triggering graph, our analysis cannot rule out the possibility that there is an execution graph with an infinite path. Clearly, however, there are a number of special cases in which there is a cycle in the triggering graph but other properties (not captured in Lemma 6.1) guarantee termination. Examples are:

- The action of some rule r on the cycle only deletes from a table t, and no other rules on the cycle insert into t. Eventually r's action has no effect.
- The action of some rule r on the cycle only performs a "monotonic" update (e.g. increments values), guaranteeing that the condition of some rule r' on the cycle eventually becomes false (e.g. some value is less than 10).

Although some such cases may be detected automatically, for now we assume that they are discovered by the user through the interactive analysis process: Once the analyzer has built the triggering graph for the rules in R, the user is notified of all cycles (or strong components). If the user is able to verify that, on each cycle, there is some rule r such that repeated consideration of the rules on the cycle guarantee that r's condition eventually becomes false or r's action eventually has no effect, then the rules in R are guaranteed to terminate.

As part of a case study, we used this approach to establish termination for a set of rules in a power network design application [CW90].

8 Confluence

Next we want to determine whether the rules in R are confluent. That is, we want to determine if the final database state at termination of rule processing can depend on which rule is chosen for consideration when multiple non-prioritized rules are triggered. In terms of execution graphs, the rules in R are confluent if every execution graph for R has at most one final state. (Recall that all final states in an execution graph have an empty set of triggered rules, so two different final states cannot represent the same database state.)

Confluence for production rules is a particularly difficult problem because, in addition to the standard problems associated with confluence [Hue80], we must take into account the interactions between rule triggering and rule priorities. For example, it is not sufficient to simply consider the combined effects of two rule actions; it also is necessary to consider all rules that can become triggered, directly or indirectly, by those actions, and the relative ordering of these triggered rules (recall Example 4.2). These issues are discussed further as we develop our requirements for confluence in Section 8.3. As preliminaries, we first introduce the notion of *rule commutativity*, and we make a useful observation about execution graphs.

$$\begin{tikzcd}
 & S \arrow[dl, "r_i"'] \arrow[dr, "r_j"] & \\
S_i \arrow[dr, "r_j"'] & & S_j \arrow[dl, "r_i"] \\
 & S' &
\end{tikzcd}$$

Figure 1: Commutative rules

8.1 Rule Commutativity

We say that two rules r_i and r_j are *commutative* (or r_i and r_j *commute*) if, given any state S in any execution graph, considering rule r_i and then rule r_j from state S produces the same execution graph state S' as considering rule r_j and then rule r_i; this is depicted in Figure 1. If this equivalence does not always hold, then r_i and r_j are *noncommutative* (or r_i and r_j *do not commute*).

Each rule clearly commutes with itself. Based on the definitions of Section 5, we give a set of conditions for analyzing whether pairs of distinct rules commute.

Lemma 8.1 For distinct rules r_i and r_j, if any of the following conditions hold then r_i and r_j may be noncommutative; otherwise they are commutative:

1. $r_j \in \mathit{Triggers}(r_i)$, i.e. r_i can cause r_j to become triggered
2. $r_j \in \mathit{Can\text{-}Untrigger}(\mathit{Performs}(r_i))$, i.e. r_i can untrigger r_j
3. $\langle \mathbf{I}, t\rangle$, $\langle \mathbf{D}, t\rangle$, or $\langle \mathbf{U}, t.c\rangle$ is in $\mathit{Performs}(r_i)$ and $t.c$ is in $\mathit{Uses}(r_j)$ for some $t.c \in C$, i.e. r_i's operations can affect what r_j uses
4. $\langle \mathbf{I}, t\rangle$ is in $\mathit{Performs}(r_i)$ and $\langle \mathbf{D}, t\rangle$ or $\langle \mathbf{U}, t.c\rangle$ is in $\mathit{Performs}(r_j)$ for some $t \in T$ or $t.c \in C$, i.e. r_i's insertions can affect what r_j updates or deletes[5]
5. $\langle \mathbf{U}, t.c\rangle$ is in both $\mathit{Performs}(r_i)$ and $\mathit{Performs}(r_j)$, i.e. r_i's updates can affect r_j's updates
6. any of 1–5 with r_i and r_j reversed $\square$

It is straightforward to verify that if a pair of rules does not satisfy any of 1–6 then the rules are guaranteed to commute.

As illustration, consider the examples from Section 4:

- *Example 4.1* – Rules **bonus-rank** and **rank-bonus** are noncommutative according to condition 1 of Lemma 8.1, because **rank-bonus** $\in$ *Triggers*(**bonus-rank**).

[5]In SQL it is possible to delete from or update a table without referencing columns of the table, which is why cases 4 and 5 are distinct from case 3.

- *Example 4.2* – Rules **good-sales** and **great-sales** are commutative. Rules **good-sales** and **rank-raise** are noncommutative according to condition 5, because $\langle$**U**, **emp.salary**$\rangle$ is in both *Performs*(**good-sales**) and *Performs*(**rank-raise**). Rules **great-sales** and **rank-raise** are noncommutative according to condition 3, because $\langle$**U**, **emp.rank**$\rangle$ is in *Performs*(**great-sales**) and **emp.rank** is in *Uses*(**rank-raise**). (Recall from Section 5 that **emp.rank** is in *Uses*(**rank-raise**) because **new-updated.rank** is referenced and **emp** is the triggering table.) Rules **great-sales** and **rank-raise** are noncommutative also according to condition 1, because **rank-raise** $\in$ *Triggers*(**great-sales**).

- *Example 4.3* – Rules **good-sales** and **rank-raise** are noncommutative as in Example 4.2. Rules **good-sales** and **new-rank** are commutative; rules **rank-raise** and **new-rank** also are commutative. (Rule **new-rank** commutes with rules **good-sales** and **rank-raise** because, although **new-rank** does access column **emp.salary**, this column is not in *Uses*(**new-rank**)—it is not used in the rule's condition or in a data modification action.)

- *Example 4.4* – Rules **bonus-rank** and **good-sales** are commutative. Rules **bonus-rank** and **new-rank** are noncommutative according to condition 1, because **new-rank** $\in$ *Triggers*(**bonus-rank**). Rules **good-sales** and **new-rank** are commutative.

The conditions in Lemma 8.1 are somewhat conservative and probably could be refined by performing more complex analysis on rule conditions and actions and by considering a variety of special cases. As two examples of this, consider rules r_i and r_j such that

1. r_i inserts into a table t and r_j deletes from t, but the tuples inserted by r_i never satisfy the delete condition of r_j

2. r_i and r_j update the same table and column but never the same tuples

In the first example, r_i and r_j are noncommutative according to condition 4 of Lemma 8.1, but they do actually commute. In the second example, r_i and r_j are noncommutative according to condition 5 but do commute. Although some such cases may be detected automatically, for now we assume that they are specified by the user during the interactive analysis process: We allow the user to declare that pairs of rules that appear noncommutative according to Lemma 8.1 actually do commute. The analysis algorithms then treat these rules as commutative.

8.2 Observation

We say that two rules r_i and r_j are *unordered* if neither $r_i > r_j$ nor $r_j > r_i$ is in P. (Similarly, we say two rules r_i and r_j are *ordered* if $r_i > r_j$ or $r_j > r_i$ is in P.) Based on our execution graph model, we make the following observation about possible states, which is used in the next section to develop our criteria for confluence.

Observation 8.2 Consider any two unordered rules r_i and r_j in R. It is very likely that there is an execution graph with a state that has (at least) two outgoing edges, one labeled r_i and one labeled r_j. (Informally, there is very likely a scenario in which both r_i and r_j are triggered and eligible for consideration. Recall that a triggered rule r is eligible for consideration iff there is no other triggered rule with precedence over r.)

Justification: Let $O' = \mathit{Triggered\text{-}By}(r_i) \cup \mathit{Triggered\text{-}By}(r_j)$. Consider an execution graph for which the operations in O' are the initial user-generated operations, so that r_i and r_j are both triggered in the initial state. Consider any path of length 0 or more from the initial state to a state $S = (D, TR)$ in which there are no rules $r \in TR$ such that $r > r_i$ or $r > r_j$ is in P, i.e. there are no triggered rules with precedence over r_i or r_j.[6] State S has at least two outgoing edges, one labeled r_i and one labeled r_j. □

8.3 Analyzing Confluence

We now return to the question of confluence. We want to determine if every execution graph for R is guaranteed to have at most one final state. For two execution graph states S_i and S_j, let $S_i \rightarrow S_j$ denote that there is an edge in the execution graph from state S_i to state S_j and let $S_i \xrightarrow{*} S_j$ denote that there is a path of length 0 or more from S_i to S_j. ($\xrightarrow{*}$ is the reflexive-transitive closure of $\rightarrow$.) Our first Lemma establishes conditions for confluence based on $\xrightarrow{*}$:

Lemma 8.3 (Path Confluence) Consider an arbitrary execution graph EG and suppose that for any three states S, S_i, and S_j in EG such that $S \xrightarrow{*} S_i$ and $S \xrightarrow{*} S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$ (Figure 2a). Then EG has at most one final state.[7]

Proof: Suppose, for the sake of a contradiction, that EG has two distinct final states, F_1 and F_2. Let I be the initial state, so $I \xrightarrow{*} F_1$ and $I \xrightarrow{*} F_2$. Then, by assumption, there must be a fourth state S such that $F_1 \xrightarrow{*} S$ and $F_2 \xrightarrow{*} S$. Since F_1 and F_2 are both final states, $S = F_1$ and $S = F_2$, contradicting $F_1 \neq F_2$. □

It is quite difficult in general to determine when the supposition of Lemma 8.3 holds, since it is based entirely on arbitrarily long paths. The following Lemma gives a somewhat weaker condition that is easier to verify and implies the supposition of Lemma 8.3; it does, however, add the requirement that rule processing is guaranteed to terminate:

Lemma 8.4 (Edge Confluence) Consider an arbitrary execution graph EG with no infinite paths. Suppose that for any three states S, S_i, and S_j in EG such that $S \rightarrow S_i$ and $S \rightarrow S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$ (Figure 2b). Then for any three states S, S_i, and S_j in EG such that $S \xrightarrow{*} S_i$ and $S \xrightarrow{*} S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$.

Proof: Classic result; see e.g. [Hue80].

[6]Such a path does not exist if r_i or r_j is untriggered along all potential paths, or if rules with precedence over r_i or r_j are considered indefinitely along all potential paths. These are highly unlikely (and probably undesirable) circumstances, but are why this is an observation rather than a theorem.

[7]Sometimes the term confluence is used to denote the supposition of this Lemma [Hue80], which then implies confluence in the sense that we've defined it.

$$\begin{tikzcd}
& S \arrow[dl, "*"'] \arrow[dr, "*"] & \\
S_i \arrow[dr, "*"'] & & S_j \arrow[dl, "*"] \\
& S' &
\end{tikzcd}$$

(a) Based on paths

$$\begin{tikzcd}
& S \arrow[dl, "r_i"'] \arrow[dr, "r_j"] & \\
S_i \arrow[dr, "*"'] & & S_j \arrow[dl, "*"] \\
& S' &
\end{tikzcd}$$

(b) Based on edges

Figure 2: Conditions for confluence

We use Lemma 8.4 as the basis for our analysis techniques. Based on this Lemma (along with Lemma 8.3), we can guarantee confluence for the rules in R if we know

1. there are no infinite paths in any execution graph for R (i.e., the rules in R are guaranteed to terminate), and

2. in any execution graph for R, for any three states S, S_i, and S_j such that $S \rightarrow S_i$ and $S \rightarrow S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$.

We assume that the first condition has been established through the analysis techniques of Section 7; we focus our attention on analysis techniques for establishing the second condition.

Consider any execution graph for R and any three states S, S_i, and S_j such that $S \rightarrow S_i$ and $S \rightarrow S_j$. This configuration is produced by every state S that has at least two unordered triggered rules that are eligible for consideration. Let r_i be the rule labeling edge $S \rightarrow S_i$ and r_j be the rule labeling edge $S \rightarrow S_j$, as in Figure 2b. We want to prove that there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$. It is tempting to assume that if r_i and r_j are commutative, then r_j can be considered from state S_i and r_i from S_j, producing a common state S' as in Figure 1. Unfortunately, this is not always possible: If r_i causes a rule r with precedence over r_j to become triggered, then r_j is not eligible for consideration in state S_i (similarly for r_i in state S_j). Since the new triggered rule r must be considered before rule r_j, r must commute with r_j. Furthermore, r may cause additional rules with precedence over r_j to become triggered.

With this in mind, we motivate the requirements for the existence of a common state S' that is reachable from both S_i and S_j. We do this by attempting to "build" valid paths from S_i and S_j towards S'; call these paths p_1 and p_2, respectively. From state S_i, triggered rules with precedence over r_j are considered until r_j is eligible; call these rules R_1. Similarly, from S_j triggered rules with precedence over r_i are considered until r_i is eligible; call these rules R_2. After this, r_j can be considered on path p_1 and r_i can be considered on path p_2. Paths p_1 and p_2 up to this point are depicted in Figure 3.

Now suppose that from state S_i' we can continue path p_1 by considering the rules in R_2 (in the same order), i.e. suppose the rules in R_2 are appropriately triggered and eligible. Similarly, suppose

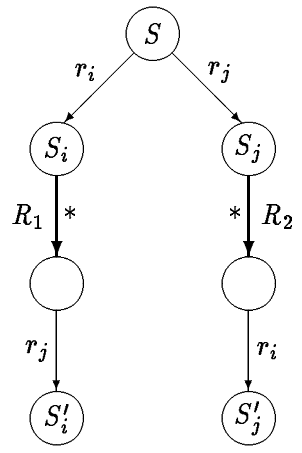

Figure 3: Paths towards common state S'

that from S_j' we can consider the rules in R_1. Then the same rules are considered along both paths. Consequently, if each rule in $\{r_i\} \cup R_1$ commutes with each rule in $\{r_j\} \cup R_2$, then the two paths are equivalent and reach a common state S'; this is depicted in Figure 4.

Unfortunately, even this scenario is not necessarily valid: There is no guarantee that the rules in R_2 are triggered and eligible from state S_i'; similarly for R_1 and S_j'. (For example, a rule in R_2 may not be eligible from state S_i' because r_j triggered a rule with higher priority.) We can guarantee this, however, if we extend the rules originally considered in R_1 to include all eligible rules with precedence over rules in R_2, and extend the rules in R_2 similarly. Using this mutually recursive definition of R_1 and R_2, the pairwise commutativity of rules in $\{r_i\} \cup R_1$ with rules in $\{r_j\} \cup R_2$ guarantees the existence of state S', and consequently guarantees confluence.

To establish confluence for the rules in R, then, we must consider in this fashion every pair of rules r_i and r_j such that some state in some execution graph for R may have two outgoing edges, one labeled with r_i and one with r_j. Recall Observation 8.2: For any two unordered rules r_i and r_j, it is very likely that there is an execution graph with a state that has two outgoing edges, one labeled r_i and one labeled r_j. Consequently, we consider every pair of unordered rules, and our analysis requirement for confluence is stated as follows.

Definition 8.5 (Confluence Requirement) Consider any pair of unordered rules r_i and r_j in R. Let $R_1 \subseteq R$ and $R_2 \subseteq R$ be constructed by the following algorithm:

```
$R_1 \leftarrow \{r_i\}$
$R_2 \leftarrow \{r_j\}$
repeat until unchanged:
    $R_1 \leftarrow R_1 \cup \{r \in R \mid r \in \mathit{Triggers}(r_1)$ for some $r_1 \in R_1$
                           and $r > r_2 \in P$ for some $r_2 \in R_2$ and $r \neq r_j\}$
    $R_2 \leftarrow R_2 \cup \{r \in R \mid r \in \mathit{Triggers}(r_2)$ for some $r_2 \in R_2$
                           and $r > r_1 \in P$ for some $r_1 \in R_1$ and $r \neq r_i\}$
```

For every pair of rules $r_1 \in R_1$ and $r_2 \in R_2$, r_1 and r_2 must commute. □

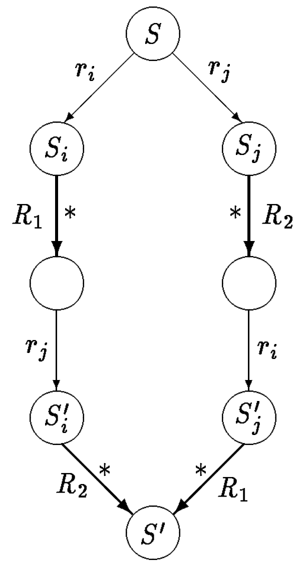

Figure 4: Paths reaching common state S'

The following lemma and theorem formally prove that the requirement of Definition 8.5 indeed guarantees confluence.

Lemma 8.6 (Confluence Lemma) Suppose the Confluence Requirement (Definition 8.5) holds for R. Then in any execution graph EG for R, for any three states S, S_i, and S_j in EG such that $S \rightarrow S_i$ and $S \rightarrow S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$.

Proof: See Appendix A.1. (The formal proof parallels the motivation shown in Figure 4, although the full construction is slightly more complex.)

Theorem 8.7 (Confluence Theorem) Suppose the Confluence Requirement holds for R and there are no infinite paths in any execution graph for R. Then any execution graph for R has exactly one final state, i.e. the rules in R are confluent.

Proof: Let EG be any execution graph for R. By Confluence Lemma 8.6, for any three states S, S_i, and S_j in EG such that $S \rightarrow S_i$ and $S \rightarrow S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$. Therefore, by Edge Confluence Lemma 8.4, for any three states S, S_i, and S_j in EG such that $S \xrightarrow{*} S_i$ and $S \xrightarrow{*} S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$. By Path Confluence Lemma 8.3, EG has at most one final state, hence (since there are no infinite paths) EG has exactly one final state. □

Thus, analyzing whether the rules in R are confluent requires considering each pair of unordered rules r_i and r_j in R: Sets R_1 and R_2 are built from r_i and r_j according to Definition 8.5, and the rules in R_1 and R_2 are checked pairwise for commutativity.

8.4 Examples

Consider Example 4.2 in which there are three rules: **good-sales**, **great-sales**, and **rank-raise**. The only pair of unordered rules is **good-sales** and **great-sales**. Letting r_i = **good-sales** and

r_j = **great-sales**, we construct sets R_1 and R_2 of Definition 8.5. R_1 = {**good-sales**}, since there are no rules in *Triggers*(**good-sales**). R_2 = {**great-sales**, **rank-raise**}, since **rank-raise** $\in$ *Triggers*(**great-sales**) and **rank-raise** $>$ **good-sales**. Now, since **good-sales** $\in R_1$ and **rank-raise** $\in R_2$ do not commute (recall Section 8.1), then according to Definition 8.5 we cannot conclude that this set of rules is confluent. To see this in terms of execution graphs, consider a state S in which **good-sales** and **great-sales** are both triggered. If **good-sales** is considered first, then path p_1 from S to a final state considers rules **good-sales**, **great-sales**, and **rank-raise**, in that order. If **great-sales** is considered first, then path p_2 from S to a final state considers rules **great-sales**, **rank-raise**, and **good-sales**, in that order. Although rules **good-sales** and **great-sales** do commute, rules **good-sales** and **rank-raise** do not. Hence paths p_1 and p_2 may lead to different final states.

Now consider Example 4.3 in which there are three rules: **good-sales**, **rank-raise**, and **new-rank**. There are two pairs of unordered rules: **good-sales**/**new-rank** and **rank-raise**/**new-rank**. For both pairs, sets R_1 and R_2 of Definition 8.5 contain only the rules themselves, and the rules commute. Hence by Theorem 8.7 the rules are confluent.

Finally consider Example 4.4 in which there are three rules: **bonus-rank**, **good-sales**, and **new-rank**. The only pair of unordered rules is **bonus-rank** and **good-sales**. Letting r_i = **bonus-rank** and r_j = **good-sales**, we construct sets R_1 and R_2 of Definition 8.5; consider R_2 first. R_2 = {**good-sales**}, since there are no rules in *Triggers*(**good-sales**). R_1 = {**bonus-rank**}, since although **new-rank** $\in$ *Triggers*(**bonus-rank**), there is no rule $r_2 \in R_2$ such that **new-rank** $> r_2$. Since **good-sales** and **bonus-rank** commute, the rules are confluent.

8.5 Using Confluence Analysis

If our analysis determines that the rules in a set R are not confluent, it can be attributed to pairs of unordered rules r_i and r_j that generate sets R_1 and R_2 such that rules $r_1 \in R_1$ and $r_2 \in R_2$ do not commute. (In the most common case, r_1 and r_2 are r_i and r_j themselves; see Corollary 8.8 below.) With this information, it appears that the user has three possible courses of action towards confluence (short of modifying the rules themselves):

1. Certify that rules r_1 and r_2 actually do commute
2. Specify a user-defined priority between rules r_i and r_j so they no longer must satisfy the Confluence Requirement
3. Remove user-defined priorities so r_1 or r_2 is no longer part of R_1 or R_2

Approach 1 is clearly the best when it is valid. Approach 3 is non-intuitive and in fact useless: removing orderings to eliminate r_1 or r_2 from R_1 or R_2 simply produces a corresponding violation to the Confluence Requirement elsewhere. Hence, if Approach 1 is not applicable (i.e. rules r_1 and r_2 do not commute) then Approach 2 should be used. Note, however, that adding an ordering between rules r_i and r_j does not immediately guarantee confluence—sets R_1 or R_2 may increase for other pairs of rules and indicate that the rule set is still not confluent.[8]

As initial guidelines for developing confluent rule sets, the following Corollaries indicate simple properties that must be satisfied by a set of rules R if the rules in R are found to be confluent using our methods.

Corollary 8.8 If R is found to be confluent and r_i and r_j are unordered rules in R, then r_i and r_j commute.

Proof: Unordered rules r_i and r_j generate sets R_1 and R_2 such that $r_i \in R_1$ and $r_j \in R_2$. Hence, by the Confluence Requirement, r_i and r_j must commute. $\Box$

Corollary 8.9 If R is found to be confluent and $P = \emptyset$ (i.e. there are no user-defined priorities between any rules in R), then every pair of rules in R commutes.

Proof: Follows directly from Corollary 8.8. $\Box$

Corollary 8.10 If R is found to be confluent and r_i and r_j in R are such that r_i may trigger r_j (or vice-versa), then r_i and r_j are ordered.

Proof: Since $r_j \in \mathit{Triggers}(r_i)$, by our conditions for noncommutativity (Lemma 8.1), r_i and r_j do not commute. Suppose, for the sake of a contradiction, that r_i and r_j are unordered. Then by Corollary 8.8 they must commute. $\Box$

Additional similar Corollaries certainly exist and provide useful initial tools for the rule programmer.

We used our approach to analyze confluence for several medium-sized rule applications. In most cases the rule sets were initially found to be non-confluent. However, for those rule sets that actually were confluent, user specification of rule commutativity eventually allowed confluence to be verified. Furthermore, for some rule sets the analysis uncovered previously undetected sources of non-confluence, i.e. confluence analysis successfully revealed errors in rule programming.

9 Partial Confluence

Confluence may be too strong a requirement for some applications. It sometimes is useful to allow rule set R to be non-confluent for certain "unimportant" (e.g. scratch) tables in the database, but to ensure that R is confluent for other "important" (e.g. data) tables. We call this *partial confluence*, or *confluence with respect to* T', where T' is a subset of the set of tables T in the database schema. In terms of execution graphs, the rules in R are confluent with respect to T' if, given any execution graph EG for R and any two final states $F_1 = (D_1, \emptyset)$ and $F_2 = (D_2, \emptyset)$ in EG, the tables in T'

[8]Intuitively, a source of non-confluence can appear to "move around", requiring an iterative process of adding orderings (or certifying commutativity) until the rule set is made confluent. This happens because our analysis techniques simply detect that confluence requires two rules to be ordered—the user chooses an ordering, and this choice affects which additional rules must be ordered.

are identical in database states D_1 and D_2. (Partial confluence obviously is implied by confluence, since confluence guarantees at most one final state.)

Partial confluence is analyzed by analyzing confluence for a subset of the rules in R: those rules that can directly or indirectly affect the final value of tables in T'.

Definition 9.1 (Significant Rules) Let $T' \subseteq T$ be a set of tables. The set of rules that are ***significant with respect to*** T', denoted $\mathit{Sig}(T')$, is computed by the following algorithm:

```
$\mathit{Sig}(T') \leftarrow \{r \in R \mid \langle \mathbf{I}, t\rangle, \langle \mathbf{D}, t\rangle$, or $\langle \mathbf{U}, t.c\rangle$ is in $\mathit{Performs}(r)$ for some $t \in T'\}$
repeat until unchanged:
    $\mathit{Sig}(T') \leftarrow \mathit{Sig}(T') \cup \{\, r \in R \mid$ there is an $r' \in \mathit{Sig}(T')$ such that
                                        $r'$ and $r$ do not commute $\}$          $\square$
```

That is, $\mathit{Sig}(T')$ contains all rules that modify any table in T', along with (recursively) all rules that do not commute with rules in $\mathit{Sig}(T')$. This algorithm determines whether rules commute using our conservative conditions for noncommutativity from Lemma 8.1. Hence, the user can influence the computation of $\mathit{Sig}(T')$ by specifying that pairs of rules that appear noncommutative according to Lemma 8.1 actually do commute.

As in Confluence Theorem 8.7, partial confluence requires that rules are guaranteed to terminate. In this case, however, the rule set under consideration is $\mathit{Sig}(T')$. Thus, before analyzing partial confluence, termination of the rules in $\mathit{Sig}(T')$ must be established using the techniques of Section 7.[9]

Theorem 9.2 (Partial Confluence) Let $T' \subseteq T$ be a set of tables. Suppose the Confluence Requirement (Definition 8.5) holds for the rules in $\mathit{Sig}(T')$ and there are no infinite paths in any execution graph for $\mathit{Sig}(T')$. Then given any two final states F_1 and F_2 in any execution graph for R, the tables in T' are identical in F_1 and F_2, i.e. the rules in R are confluent with respect to T'.

Proof: See Appendix A.2.

Hence, analyzing whether the rules in R are confluent with respect to T' requires first computing $\mathit{Sig}(T')$, then considering each pair of unordered rules r_i and r_j in $\mathit{Sig}(T')$: Sets R_1 and R_2 are built according to Definition 8.5 and checked pairwise for commutativity. If the analysis determines that the rules in R are not partially confluent, then the same interactive approach as that described in Section 8.5 for confluence can be used here to establish partial confluence. Examples of partial confluence analysis are given in Section 10.1 below.

10 Observable Determinism

In some database production rule languages, such as Starburst, the final database state may not be the only effect of rule processing—some rule actions may be visible to the environment (***observable***)

[9]That is, even though the rules in $\mathit{Sig}(T')$ are never processed on their own, it must be established that if they were processed on their own they would terminate. As in Section 8.3, this is necessary for Definition 8.5 to guarantee confluence.

while rules are being processed. When this is the case, the user may want to determine whether a rule set is ***observably deterministic***, i.e. whether the order and appearance of observable rule actions is the same regardless of which rule is chosen for consideration when multiple non-prioritized rules are triggered. Note that observable determinism and confluence are orthogonal properties: a rule set may be confluent but not observably deterministic or vice-versa. (E.g., the rules in Example 4.2 are observably deterministic but non-confluent, while the rules in Example 4.3 are confluent but not observably deterministic.)

We analyze observable determinism using our techniques for partial confluence. Intuitively, we add a fictional table *Obs* to the database, and we pretend that those rules with observable actions also "timestamp and log" their observable actions in table *Obs*. We analyze the resulting rule set for confluence with respect to table *Obs*; if partial confluence holds, then the rule set is observably deterministic.

More formally, recall the definitions of Section 5. Let $T_{obs} = T \cup \{Obs\}$ be an extended set of tables, let $C_{obs} = C \cup \{Obs.c\}$ be an extended set of columns, and let O_{obs} be the corresponding extended set of operations. Let $Uses_{obs}$ and $Performs_{obs}$ extend the definitions of *Uses* and *Performs* as follows. For every $r \in R$ such that *Observable*(r), add *Obs.c* to *Uses*(r) and $\langle \mathbf{I}, Obs \rangle$ to *Performs*(r). In addition, for every $r \in R$ such that *Observable*(r), add to *Uses*(r) every $t.c$ referenced in a top-level **select** operation in r's action and, for every $\langle trans \rangle.c$ referenced in such a **select** operation, where $\langle trans \rangle$ is one of **inserted**, **deleted**, **new-updated**, or **old-updated**, add $t.c$ to *Uses*(r) for r's triggering table t. Hereafter, for convenience we say that a rule r *is observable* if *Observable*(r).

Theorem 10.1 (Observable Determinism) Suppose, using extended definitions T_{obs}, C_{obs}, O_{obs}, $Uses_{obs}$, and $Performs_{obs}$, that our analysis methods for partial confluence determine that rule set R is confluent with respect to *Obs*. That is, suppose (from Theorem 9.2) that the Confluence Requirement of Definition 8.5 holds for the rules in *Sig*($\{Obs\}$) and there are no infinite paths in any execution graph for *Sig*($\{Obs\}$). Then the rules in R are observably deterministic.

Proof: By supposition, any hypothetical behavior of the rules in R that is consistent with the definitions of $Uses_{obs}$ and $Performs_{obs}$ is confluent with respect to *Obs*. Consider the following such behavior. Suppose each observable rule r, in addition to its existing actions, inserts a new tuple into *Obs* that contains the current number of tuples in *Obs* (the "timestamp") and a complete description of r's observable actions (the "log"). Since there is a unique final value for *Obs*, the hypothetical tuples written to *Obs* must be identical on all execution paths. Consequently, there is only one possible order and appearance of observable actions, and the rules in R are observably deterministic. □

If, using the analysis methods indicated by this theorem, the rules in R are not found to be observably deterministic, then the same interactive approach as that described in Section 8.5 can be used to establish confluence with respect to *Obs*, and consequently observable determinism. Although this requires the user to be aware of fictional table *Obs*, the use of *Obs* in the analysis techniques is quite intuitive and may actually guide the user in establishing observable determinism.

The following Corollary gives a simple property that is satisfied by the observable rules in R if they are found to be deterministic using our methods. Additional useful Corollaries certainly exist.

Corollary 10.2 If R is found to be observably deterministic and r_i and r_j are distinct observable rules in R, then r_i and r_j are ordered.[10]

Proof: Since r_i is observable, $\mathit{Obs.c} \in \mathit{Uses}(r_i)$ and $\langle \mathbf{I}, \mathit{Obs} \rangle \in \mathit{Performs}(r_i)$; similarly for r_j. Therefore, by Definition 9.1, r_i and r_j are both in $\mathit{Sig}(\{\mathit{Obs}\})$. In addition, by Lemma 8.1, r_i and r_j satisfy our conditions for noncommutativity. Suppose, for the sake of a contradiction, that r_i and r_j are unordered. r_i and r_j generate sets R_1 and R_2 (from Definition 8.5) such that $r_i \in R_1$ and $r_j \in R_2$. Hence, by the Confluence Requirement, r_i and r_j must commute, a contradiction. $\Box$

10.1 Examples

In Examples 4.1 and 4.2 there are no observable rules. Hence, in both cases $\mathit{Sig}(\{\mathit{Obs}\}) = \emptyset$, the rules are partially confluent with respect *Obs* (vacuously by Theorem 9.2), and the rules are observably deterministic (by Theorem 10.1).

Now consider Example 4.3 in which there are three rules: **good-sales**, **rank-raise**, and **new-rank**. Since **new-rank** is observable, Uses_{obs} and $\mathit{Performs}_{obs}$ are derived from *Uses* and *Performs* as follows: *Uses*(**new-rank**) is extended to include *Obs.c*, **emp.id**, **emp.rank**, and **emp.salary**; *Performs*(**new-rank**) is extended to include $\langle \mathbf{I}, \mathit{Obs} \rangle$. With these extensions, rule **new-rank** no longer commutes with rule **good-sales** (by condition 3 of Lemma 8.1) or with rule **rank-raise** (also by condition 3). By Definition 9.1 of significant rules, $\mathit{Sig}(\{\mathit{Obs}\})$ = {**new-rank**, **good-sales**, **rank-raise**}. Hence, to analyze observable determinism, we analyze confluence for all three rules. When we considered confluence for these rules in Section 8.4, without the extended definitions of *Uses* and *Performs*, we found them to be confluent. However, with the extended definitions this is not the case: unordered rules **good-sales** and **new-rank** do not commute, so by Corollary 8.8 we cannot guarantee confluence; similarly for unordered rules **rank-raise** and **new-rank**. Therefore we determine that the rules are not observably deterministic.

Finally consider Example 4.4 in which there are three rules: **bonus-rank**, **good-sales**, and **new-rank**. Uses_{obs} and $\mathit{Performs}_{obs}$ are extended for observable rule **new-rank** as in the previous example. With these extensions, rules **good-sales** and **new-rank** no longer commute. By Definition 9.1, $\mathit{Sig}(\{\mathit{Obs}\})$ = {**new-rank**, **bonus-rank**, **good-sales**}. Hence, to analyze observable determinism, we analyze confluence for all three rules. The only pair of unordered rules in the set is **bonus-rank** and **good-sales**. As in Section 8.4, R_1 and R_2 are constructed according to Definition 8.5, then Theorem 8.7 is applied to conclude that the rules are confluent. Therefore the rules are observably deterministic.

[10]Note that this is not an if and only if condition: orderings between all pairs of observable rules does not necessarily guarantee observable determinism.

11 Conclusions and Future Work

We have given static analysis methods that determine whether arbitrary sets of database production rules are guaranteed to terminate, are confluent, are partially confluent with respect to a set of tables, or are observably deterministic. Our algorithms are conservative—they may not always detect when a rule set satisfies these properties. However, they isolate the responsible rules when a property is not satisfied, and they determine simple criteria that, if satisfied, guarantee the property. Furthermore, for the cases when these criteria are not satisfied, our methods often can suggest modifications to the rule set that are likely to make the property hold. Consequently, our methods can form the basis of a powerful interactive development environment for database rule programmers.

Although our methods have been designed for the Starburst Rule System, we expect that they can be adapted to accommodate the syntax and semantics of other database rule languages. In particular, the fundamental definitions of Section 5 (*Triggers*, *Performs*, *Choose*, etc.) can simply be redefined for an alternative rule language. Alternative rule processing semantics will probably require that the execution graph model is modified, which consequently will cause algorithms (and proofs) to be modified. However, our fundamental "building blocks" of rule analysis techniques can remain the same: the triggering graph for analyzing termination, the Edge and Path Lemmas for analyzing confluence, the notion of partial confluence, and the use of partial confluence in analyzing observable determinism.

Some technical comparisons can be drawn between this work and the results in [HH91,Ras90, ZH90]. In [HH91], a version of the OPS5 production rule language is considered, and a class of rule sets is identified that (conservatively) guarantees the *unique fixed point property*, which essentially corresponds to our notion of confluence. By defining a (reasonable) mapping between our language and the language in [HH91], we have shown that our confluence requirements properly subsume their fixed point requirements: if a rule set has the unique fixed point property according to [HH91], then our methods determine that the corresponding rule set is confluent, but not always vice-versa. The methods in [HH91] have previously been shown to subsume those in [Ras90,ZH90], hence our approach, although still conservative, appears quite accurate when compared with previous work.

Finally, we plan a number of improvements and extensions to this work:

- **Incremental methods**: In our current approach, complete analysis is performed after any change to the rule set. In many cases it is clear that most results of previous analysis are still valid and only incremental additional analysis needs to be performed. We plan to modify our methods to incorporate incremental analysis. At the coarsest level, most rule applications can be partitioned into groups of rules such that, across partitions, rules reference different sets of tables and have no priority ordering. Although rules from different partitions are processed at the same time and their execution may be interleaved, they have no effect on each other. Hence, analysis can be applied separately to each partition, and it needs to be repeated for a partition only when rules in that partition change.

- **Less conservative methods**: As discussed throughout the paper, many of our assumptions, definitions, and algorithms are conservative, and there is room for refinement. This may include more complex analysis of SQL, more accurate properties of our execution model, and a suite of special cases.

- **Restricted user operations**: Our analysis assumes that the user-generated operations that initiate rule processing are arbitrary. However, in some cases it may be known that these will be of a particular type, i.e. the user will only perform certain operations on certain tables. This may reduce possible execution paths during rule processing, and consequently may guarantee properties that otherwise do not hold. We plan to extend our methods so that termination, confluence, and observable determinism can be analyzed in the context of limited user-generated operations.

- **Implementation and experimentation**: We plan to implement our algorithms as part of an interactive development environment for the Starburst Rule System. Although we have verified by hand that our methods are indeed useful, implementation will allow practical experimentation with large and realistic rule applications.

A Appendix

Here we provide the two (lengthy) proofs omitted from the body of the paper.

A.1 Confluence

To prove Lemma 8.6, we first introduce some additional notation and definitions. Let S and S' be execution graph states and let r be a rule in R. We write $S \overset{r}{\leadsto} S'$ if S' follows from S by consideration of rule r, whether or not r is triggered and eligible in S. If r is not triggered in S, then the consideration of r in S is an identity transition, i.e. $S \overset{r}{\leadsto} S$. (This corresponds with the intuition that a non-triggered rule can have no effect.) With this notation we give a precise definition of commutativity.

Definition A.1 (Commutativity) Two rules r_i and r_j *commute* if for all execution graph states S, $S \overset{r_i}{\leadsto} S_i \overset{r_j}{\leadsto} S'$ if and only if $S \overset{r_j}{\leadsto} S_j \overset{r_i}{\leadsto} S'$.

If rule r is triggered and eligible in state S, then we write $S \xrightarrow{r} S'$, which also denotes an execution graph edge (as defined in Section 8.3). Recall that $\xrightarrow{*}$ is the reflexive-transitive closure of $\rightarrow$.

Let a *path* be a sequence of rule considerations $S_0 \overset{r_1}{\leadsto} S_1 \overset{r_2}{\leadsto} S_2 \ldots S_{n-1} \overset{r_n}{\leadsto} S_n$.[11] In the proofs below, the start state S_0 always is fixed and obvious from context, so we generally abbreviate a path as just the sequence of considered rules $\langle r_1, \ldots, r_n \rangle$. Two paths P and Q are *equivalent*, denoted $P \equiv Q$, if they begin in the same state S_0 and end in the same state S_n. A path is *valid* if all

[11] Note that a path as defined here is not necessarily a path in an execution graph, since there are no edges in execution graphs that correspond to consideration of ineligible rules.

rules are eligible when they are considered; i.e. for all i, $1 \leq i \leq n$, $S_{i-1} \xrightarrow{r_i} S_i$. Hence, a valid path is a path in an execution graph. Finally, given two sequences of rules A and B, $A; B$ is the path consisting of consideration of the rules in A followed by consideration of the rules in B.

The proof of Lemma 8.6 uses the following simple lemma about paths. For a path P, let $\overline{P}$ denote P with all considerations of non-triggered rules deleted.

Lemma A.2 Let P be a path and assume that for each rule in P, either the rule is not triggered when it is considered, or the rule is eligible when it is considered. Then $\overline{P} \equiv P$ and $\overline{P}$ is a valid path.

Proof: Obvious.

The following two lemmas are used several times in the proof of Lemma 8.6.

Lemma A.3 Let r_i, r_j, and r be rules such that r_i and r_j are unordered, and let R_1 and R_2 be the sets of rules constructed from r_i and r_j in Definition 8.5. If $r \in (R_1 \cup R_2) - \{r_i, r_j\}$, then either $r > r_i$ or $r > r_j$.

Proof: Recall the algorithm in Definition 8.5 for constructing R_1 and R_2. The proof is by induction on the number of loop iterations required by this algorithm to add r to either R_1 or R_2. As the base case, let the number of iterations be 0. Then $R_1 = \{r_i\}$ and $R_2 = \{r_j\}$, so the result holds vacuously. Assume the result holds for n iterations. If r is added in iteration $n+1$, then by definition there is some r' added previously such that $r > r'$. If $r' = r_i$ or $r' = r_j$ then $r > r_i$ or $r > r_j$. Otherwise, by the induction hypothesis $r' > r_i$ or $r' > r_j$, so by transitivity $r > r_i$ or $r > r_j$. $\square$

Lemma A.4 Let r_i and r_j be unordered rules, let R_1 and R_2 be the sets of rules constructed from r_i and r_j in Definition 8.5, and let r and r' be rules such that $r > r'$ and $r' \in R_1 \cup R_2$. Then $r \neq r_i$ and $r \neq r_j$.

Proof: For the sake of a contradiction, suppose $r = r_i$. Since $r > r'$, $r' \neq r_i$ (since a rule cannot have higher priority than itself) and $r' \neq r_j$ (since r_i and r_j are unordered). Hence, by Lemma A.3, either $r' > r_i$ or $r' > r_j$. By transitivity, then, either $r > r_i$ (contradicting the fact that rules cannot have higher priority than themselves) or $r > r_j$ (contradicting the assumption that r_i and r_j are unordered). Thus $r \neq r_i$. A parallel argument shows $r \neq r_j$. $\square$

Lemma 8.6 (Confluence Lemma) Suppose the Confluence Requirement (Definition 8.5) holds for R. Then in any execution graph EG for R, for any three states S, S_i, and S_j in EG such that $S \rightarrow S_i$ and $S \rightarrow S_j$, there is a fourth state S' such that $S_i \xrightarrow{*} S'$ and $S_j \xrightarrow{*} S'$.

Proof: Let r_i and r_j label edges $S \rightarrow S_i$ and $S \rightarrow S_j$ respectively, i.e. $S \xrightarrow{r_i} S_i$ and $S \xrightarrow{r_j} S_j$, where r_i and r_j are distinct unordered rules.[12] We must prove that there is a state S' such that $S_i \xrightarrow{*} S'$

[12]Other than that, rules r_i and r_j are chosen arbitrarily, since by Observation 8.2 any pair is very likely to be possible.

and $S_j \xrightarrow{*} S'$. We do this by exhibiting valid paths $\overline{P_1} = S \xrightarrow{r_i} S_i \xrightarrow{*} S'$ and $\overline{P_2} = S \xrightarrow{r_j} S_j \xrightarrow{*} S'$. The proof involves three steps. First, we give an algorithm to construct initial paths P_1 and P_2. Second, we extend P_1 and P_2 with some additional rules so that every rule in P_1 and P_2 is either eligible or not triggered when it is considered; by Lemma A.2, at this point $\overline{P_1}$ and $\overline{P_2}$ are valid, $P_1 \equiv \overline{P_1}$, and $P_2 \equiv \overline{P_2}$. Third, we show that $\overline{P_1}$ and $\overline{P_2}$ have the same last state, i.e. $\overline{P_1} \equiv \overline{P_2}$, which proves the lemma.

Let R_1 and R_2 be the sets of rules constructed from r_i and r_j in Definition 8.5, let $R_1' = R_1 - \{r_i\}$, and let $R_2' = R_2 - \{r_j\}$. The following algorithm constructs initial paths P_1 and P_2.

```
$A \leftarrow \langle\rangle$
**while** there is a rule $r \in R_1'$ eligible in the last state of path $\langle r_i\rangle; A$ **do** $A \leftarrow A; \langle r\rangle$
$B \leftarrow \langle\rangle$
**while** there is a rule $r \in R_2'$ eligible in the last state of path $\langle r_j\rangle; B$ **do** $B \leftarrow B; \langle r\rangle$
$P_1 \leftarrow \langle r_i\rangle; A; \langle r_j\rangle; B$
$P_2 \leftarrow \langle r_j\rangle; B; \langle r_i\rangle; A$
```

Before proceeding, we establish an important property of paths P_1 and P_2. Consider path P_1 and let S_0 be the last state of path $\langle r_i\rangle; A$. We show that there is no rule r such that $r \in R_1'$ and r is triggered in S_0. (A symmetric argument shows that on path P_2 there is no rule r such that $r \in R_2'$ and r is triggered in the last state of $\langle r_j\rangle; B$.) Clearly, there is no rule r such that $r \in R_1'$ and r is eligible in S_0, because this would violate the termination condition of the first **while** loop in the algorithm. Suppose $r \in R_1'$ and r is triggered but not eligible in S_0. Then there is some r' such that $r' > r$ and r' is eligible in S_0. We show $r' \in R_1'$, a contradiction. By $R_1' = R_1 - \{r_i\}$ and Definition 8.5 of R_1, $r' \in R_1'$ if:

(1) there is a rule $r_1 \in R_1$ such that $r' \in \mathit{Triggers}(r_1)$,

(2) there is a rule $r_2 \in R_2$ such that $r' > r_2$, and

(3) $r' \neq r_i$ and $r' \neq r_j$.

For (1), note that by Lemma A.3, $r' > r_i$ or $r' > r_j$. Thus, r' could not be triggered in the initial state S, or else one of r_i or r_j would not be eligible in S. So r' must be triggered by some rule in $\langle r_i\rangle; A$, which implies $r' \in \mathit{Triggers}(r_1)$ for some $r_1 \in R_1$. For (2), note that $r \in R_1'$ implies that there is some $r_2 \in R_2$ such that $r > r_2$; by transitivity $r' > r_2$. For (3), $r' > r$ and $r \in R_1'$ together imply $r \neq r_i$ and $r \neq r_j$ by Lemma A.4 . Therefore $r' \in R_1'$ and r' is eligible in S_0. But, as noted above, this is a contradiction, since the **while** loop adds all eligible rules in R_1'. We conclude that there are no rules in R_1' triggered in S_0. Similarly, there are no rules in R_2' triggered in the last state of $\langle r_j\rangle; B$.

As the second step of the proof, we show that $\overline{P_1}$ and $\overline{P_2}$ are valid. To do this, we modify P_1 and P_2 so that every rule is either eligible or not triggered when it is considered—that is, there should be no rules that are triggered but not eligible when they are considered. We show how to modify path P_1; the construction for P_2 is symmetric. In path P_1, every rule in the portion $\langle r_i\rangle; A$ is eligible, because r_i is eligible in the initial state S and every rule added by **while** loop is eligible.

Thus, we consider latter portion $\langle r_j \rangle; B$. As above, let S_0 be the last state of $\langle r_i \rangle; A$. Assume, for the sake of a contradiction, that r_j is triggered in S_0 but is not eligible. Then there must be a rule r such that $r > r_j$ and r is eligible in S_0. We show $r \in R_1'$, a contradiction. Consider (1), (2), and (3) above. For (1), note that r cannot be triggered in the initial state S since r_j is eligible in S and $r > r_j$. So r is triggered by a rule in $\langle r_i \rangle; A$, which implies $r \in \mathit{Triggers}(r_1)$ for some $r_1 \in R_1$. For (2), note that $r > r_j$ and $r_j \in R_2$. For (3), $r \neq r_i$ and $r \neq r_j$ because $r > r_j$. Thus, $r \in R_1'$. But r cannot be in R_1', because there are no eligible rules in R_1' at S_0. We conclude that if r_j is triggered in S_0, then r_j is eligible.

Now we must guarantee that rules in B are either eligible or not triggered when they are considered on path P_1. For this property, it may be necessary to add more rules from R_2' to B. We simultaneously consider paths $P_1 = \langle r_i \rangle; A; \langle r_j \rangle; B$ and $P_2 = \langle r_j \rangle; B; \langle r_i \rangle; A$. Let r be a rule in B, let S_1 be the state where r is considered in P_1, and let S_2 be the state where r is considered in P_2. By construction, r is eligible in S_2, because the second **while** loop adds only eligible rules to B. Suppose r is triggered but not eligible in S_1. Then there is some rule r' such that $r' > r$ and r' is eligible in S_1. Clearly r' is not triggered in S_2, or else r could not be eligible in S_2. So we can insert r' before r in B with two effects: an eligible rule is added to P_1 and a non-triggered rule is added to P_2.

This observation motivates the following modification to B: Repeatedly select the first rule r from B that is triggered but not eligible in the state S_1 where r is considered in P_1. Insert a rule r' before r in B, where $r' > r$ and r' is eligible in S_1. By repeating this procedure for all rules with precedence over r, eventually r becomes an eligible rule in P_1. Thus, eventually all rules in P_1 are either triggered and eligible or not triggered when they are considered. This procedure terminates because at each step P_1 is extended by an eligible rule; since (by assumption) rule processing always terminates, only a finite number of rules can be added. Note that when this procedure terminates, $\overline{P_1}$ is valid, and all rules added to B are not triggered when they are considered in P_2.

Before proceeding to the third step, we show that for every r' added to B by the procedure above, $r' \in R_2'$. In P_1, r' can be triggered in three ways: it is triggered in the initial state S, it is triggered by a rule in $\langle r_i \rangle; A$, or it is triggered by a rule in $\langle r_j \rangle; B$. We consider each of these separately:

- Suppose r' is triggered in the initial state S. We know $r' > r$ and $r \in R_2'$, so by Lemma A.3 and transitivity, $r' > r_i$ or $r' > r_j$. But r_i and r_j are eligible in S, so r' cannot be triggered in S.

- Suppose r' is triggered by a rule in $\langle r_i \rangle; A$. To derive a contradiction, we first show $r' \in R_1'$. For (1), every rule in $\langle r_i \rangle; A$ is in R_1, implying that $r' \in \mathit{Triggers}(r_1)$ for some $r_1 \in R_1$. For (2), $r' > r$ and $r \in R_2$. For (3), $r' > r$ and $r \in R_2$ together imply $r' \neq r_i$ and $r' \neq r_j$ by Lemma A.4 . Thus, $r' \in R_1'$. But there are no eligible rules of R_1' in the last state of $\langle r_i \rangle; A$. Hence r' is not triggered by a rule in $\langle r_i \rangle; A$.

- Then r' must be triggered by a rule in $\langle r_j \rangle; B$. This implies $r' \in R_2'$: For (1), every rule in $\langle r_j \rangle; B$ is in R_2, implying $r' \in \mathit{Triggers}(r_2)$ for some $r_2 \in R_2$. For (2), because $r \in R_2'$ there is a rule r_1 such that $r_1 \in R_1$ and $r > r_1$; by transitivity $r' > r_1$. For (3), $r' > r$ and $r \in R_2$ together imply $r' \neq r_i$ and $r' \neq r_j$ by Lemma A.4.

For a path P, let the set of rules appearing on P be denoted $\mathit{Rules}(P)$. We have shown that rules can be added to B such that $\overline{P_1}$ is valid, $\mathit{Rules}(A) \subseteq R_1$, $\mathit{Rules}(B) \subseteq R_2$, and the modifications to B add only non-triggered rules to P_2. A symmetric argument adds rules to A such that $\overline{P_2}$ is valid, $\mathit{Rules}(A) \subseteq R_1$, $\mathit{Rules}(B) \subseteq R_2$, and the modifications to A add only non-triggered rules to P_1. Thus, we can construct P_1 and P_2 such that $P_1 = \langle r_i \rangle; A; \langle r_j \rangle; B$, $P_2 = \langle r_j \rangle; B; \langle r_i \rangle; A$, $\overline{P_1}$ and $\overline{P_2}$ are valid, $\mathit{Rules}(A) \subseteq R_1$, and $\mathit{Rules}(B) \subseteq R_2$.

To complete the proof, we show $\overline{P_1} \equiv \overline{P_2}$, i.e. $\overline{P_1}$ and $\overline{P_2}$ have the same last state. By Lemma A.2, $P_1 \equiv \overline{P_1}$ and $P_2 \equiv \overline{P_2}$, so it suffices to show $P_1 \equiv P_2$. Consider any path with two consecutive rules, one from R_1 and the other from R_2. Since R_1 and R_2 are pairwise commutative, we can interchange the two rules without changing the last state. Path $\langle r_i \rangle; A$ consists entirely of rules from R_1, while path $\langle r_j \rangle; B$ consists entirely of rules from R_2. Hence, by repeatedly interchanging consecutive rules from R_1 and R_2, we can prove $\langle r_i \rangle; A; \langle r_j \rangle; B \equiv \langle r_j \rangle; B; \langle r_i \rangle; A$, and consequently $P_1 \equiv P_2$. $\square$

A.2 Partial Confluence

To prove Theorem 9.2, we use some notation introduced in Appendix A.1 and give some additional notation and definitions. Our method for establishing partial confluence uses the confluence of a set of rules $\mathit{Sig}(T') \subseteq R$ to prove partial confluence for the rules in R. Confluence is a property of execution graphs, but $\mathit{Sig}(T')$ and R may not have the same execution graphs. Thus, to prove the correctness of our method, we need to translate between execution graphs of $\mathit{Sig}(T')$ and execution graphs of R. As a first step, we subscript $\rightsquigarrow$ and $\rightarrow$ with sets of rules. Let $R' \subseteq R$. We write $S \overset{r}{\rightsquigarrow}_{R'} S'$ if S' follows from S by consideration of rule r when the set of rules is R'. We write $S \xrightarrow{r}_{R'} S'$ if, additionally, r is triggered and eligible in state S. Since R denotes the entire set of rules, for consistency with previous sections we write $S \rightsquigarrow S'$ and $S \rightarrow S'$ for $S \rightsquigarrow_R S'$ and $S \rightarrow_R S'$, respectively.

Let $P = S_0 \overset{r_1}{\rightsquigarrow} \dots \overset{r_n}{\rightsquigarrow} S_n$ be any path. P is a *path with respect to* R' if each r_i is in R' and for each state $S_i = (D_i, TR_i)$, the set of triggered rules in TR_i is a subset of R'. If $S_{i-1} \xrightarrow{r_i}_{R'} S_i$, $1 \leq i \leq n$, then P is a *valid* path with respect to R', and P is a path in an execution graph for R' (recall Section A.1). Let S be any state. A rule r is *eligible with respect to* R' *in* S if $r \in R'$, r is triggered in S, and there is no rule r' such that $r' \in R'$, $r' > r$, and r' is triggered in S. The following lemma provides a way to convert certain paths with respect to R into valid paths with respect to R'.

Lemma A.5 Let $R' \subseteq R$, let $(D_0, TR_0) \overset{r_1}{\rightsquigarrow} \dots \overset{r_n}{\rightsquigarrow} (D_n, TR_n)$ be a path (with respect to R), and suppose r_i is eligible with respect to R' in state (D_{i-1}, TR_{i-1}), $1 \leq i \leq n$. Then $(D_0, TR_0 \cap R') \xrightarrow{r_1}_{R'} \dots \xrightarrow{r_n}_{R'} (D_n, TR_n \cap R')$, i.e. $(D_0, TR_0 \cap R') \overset{r_1}{\rightsquigarrow} \dots \overset{r_n}{\rightsquigarrow} (D_n, TR_n \cap R')$ is a valid path with respect to R'.

Proof: Let $TR'_i = TR_i \cap R'$. Rule r_i is eligible in (D_{i-1}, TR'_{i-1}), $1 \le i \le n$, since r_i is eligible with respect to R' in (D_{i-1}, TR_{i-1}). Therefore, $(D_{i-1}, TR'_{i-1}) \xrightarrow{r_i}_{R'} (D_i, TR'_i)$, $1 \le i \le n$. $\square$

The following lemma provides the key construction for proving Theorem 9.2. This lemma shows that any path in an execution graph for R can be transformed by commuting rules into an equivalent path $RS; RN$ such that RS contains only rules from $Sig(T')$, RN contains only rules from $R - Sig(T')$, and all rules in RS are eligible with respect to $Sig(T')$.

Lemma A.6 Let $T' \subseteq T$ be a set of tables and assume there are no infinite paths in any execution graph for $Sig(T')$. If P is a path in an execution graph for R ending in a final state then there exists a path P' such that

(1) $P' = RS; RN$, where $Rules(RS) \subseteq Sig(T')$ and $Rules(RN) \subseteq R - Sig(T')$,

(2) $P' \equiv P$,

(3) in P', every rule in $Sig(T')$ is eligible with respect to $Sig(T')$ when it is considered, and

(4) $TR \cap Sig(T') = \emptyset$, where (D, TR) is the last state of RS.

Proof: Let path P' be constructed by the following algorithm:

```
$P' \leftarrow P$
**while** $P' \ne RS; RN$ where $Rules(RS) \subseteq Sig(T')$ and $Rules(RN) \subseteq R - Sig(T')$ **do**
  **let** $P' = RS; RN; \langle r_i \rangle; A$
    where $Rules(RS) \subseteq Sig(T')$, $Rules(RN) \subseteq R - Sig(T')$, $|RN| > 0$, and $r_i \in Sig(T')$
  **if** $r_i$ is eligible with respect to $Sig(T')$ in last state of $RS$ **then**
    [a] $P' \leftarrow RS; \langle r_i \rangle; RN; A$
  **else if** $r_i$ is not triggered in the last state of $RS$ **then**
    [b] $P' \leftarrow RS; RN; A$
  **else**
    **let** $r_j > r_i$ be eligible with respect to $Sig(T')$ in the last state of $RS$
    [c] $P' \leftarrow RS; \langle r_j \rangle; RN; \langle r_i \rangle; A$
**while** $P' = RS; RN$ and $r \in Sig(T')$ is eligible with respect to $Sig(T')$ in the last state of $RS$ **do**
  $P' \leftarrow RS; \langle r \rangle; RN$
```

We must show that (1)–(4) hold and that the algorithm for constructing P' always terminates. We first show that (1)–(3) hold after the first **while** loop and that the loop terminates. We then show that the second **while** loop preserves (1)–(3), establishes (4), and terminates.

Consider the first **while** loop. Note that if the loop condition is true, then P' must have the form described by the first **let** clause; i.e. P' must begin with a (possibly empty) set of rules RS from $Sig(T')$, followed by a non-empty set of rules RN from $R - Sig(T')$, followed by at least one rule r_i from $Sig(T')$. From the termination condition of this loop, it is clear that (1) holds when the loop terminates. We prove by induction on the number of iterations that (2) and (3) also hold when the loop terminates. For the base case, (2) holds after 0 iterations since $P' = P$. Clearly (3) holds as well, since P is a valid path for R. For the induction step, let $P'_n = RS_n; RN_n; \langle r_i \rangle; A_n$ be

P' after n iterations. As the induction hypothesis, assume $P'_n \equiv P$ and for every rule r in P'_n, if r is in $Sig(T')$ then r is eligible with respect to $Sig(T')$ when it is considered. In iteration $n+1$ of the loop either [a], [b], or [c] is executed; we consider each separately.

Let P'_{n+1} be the result of interchanging r_i and RN_n (branch [a]). Recall from Definition 9.1 of $Sig(T')$ that rules in $R - Sig(T')$ commute with rules in $Sig(T')$; therefore r_i commutes with all rules in RN_n. Hence:

$$\begin{aligned} P'_n &= RS_n; RN_n; \langle r_i \rangle; A_n \\ &\equiv RS_n; \langle r_i \rangle; RN_n; A_n && \text{since } r_i \text{ commutes with all rules in } RN_n \\ &= P'_{n+1} \end{aligned}$$

This shows that (2) holds. For (3), we must show that every rule in $Sig(T')$ is eligible with respect to $Sig(T')$ when it is considered in P'_{n+1}. A rule in RS_n is considered in the same state in P'_{n+1} as in P'_n, so rules in RS_n are eligible with respect to $Sig(T')$ when considered in P'_{n+1}. By the condition for the [a] branch, r_i is eligible with respect to $Sig(T')$ in the last state of RS_n. By definition, no rules in RN_n are in $Sig(T')$. Finally, because r_i commutes with rules in RN_n, the first state of A_n is the same in P'_n and P'_{n+1}, so each rule in A_n is considered in the same state in P'_n and P'_{n+1}. Therefore, rules in A_n that are also in $Sig(T')$ are eligible with respect to $Sig(T')$ in P'_{n+1}.

Now suppose $P'_{n+1} = RS_n; RN_n; A_n$ (branch [b]). Let S be the last state of RS_n. By the condition for the [b] branch, we know that r_i is not triggered in S, i.e. $S \overset{r_i}{\rightsquigarrow} S$. Hence:

$$\begin{aligned} P'_n &= RS_n; RN_n; \langle r_i \rangle; A_n \\ &\equiv RS_n; \langle r_i \rangle; RN_n; A_n && \text{since } r_i \text{ commutes with all rules in } RN_n \\ &\equiv RS_n; RN_n; A_n && \text{since } r_i \text{ is not triggered in } S \\ &= P'_{n+1} \end{aligned}$$

This shows that (2) holds. For (3), we must show that every rule in $Sig(T')$ is eligible with respect to $Sig(T')$ when it is considered in P'_{n+1}. A rule in RS_n is considered in the same state in P'_{n+1} as in P'_n, so rules in RS_n are eligible with respect to $Sig(T')$ when considered in P'_{n+1}. By definition, no rules in RN_n are in $Sig(T')$. Finally, by the equivalences above, the first state of A_n is the same in P'_n and P'_{n+1}, so each rule in A_n is considered in the same state in P'_n and P'_{n+1}. Therefore, rules in A_n that are also in $Sig(T')$ are eligible with respect to $Sig(T')$ in P'_{n+1}.

For the last case, suppose $P'_{n+1} = RS_n; \langle r_j \rangle; RN_n; \langle r_i \rangle; A_n$ (branch [c]). Let S be the last state of RS_n and let S' be the last state of RN_n. The new rule r_j must exist, since if r_i is triggered but not eligible with respect to $Sig(T')$ in S, then there is a rule r_j such that $r_j > r_i$ and r_j is eligible with respect to $Sig(T')$ in S. Now, r_j cannot be triggered in S', or else r_i is not eligible with respect to $Sig(T')$ in S'. Note also that r_j commutes with rules in RN_n, since $r_j \in Sig(T')$ and $Rules(RN_n) \subseteq R - Sig(T')$. Hence:

$$\begin{aligned} P'_n &= RS_n; RN_n; \langle r_i \rangle; A_n \\ &\equiv RS_n; RN_n; \langle r_j, r_i \rangle; A_n && \text{since } r_j \text{ is not triggered in } S' \\ &\equiv RS_n; \langle r_j \rangle; RN_n; \langle r_i \rangle; A_n && \text{since } r_j \text{ commutes with all rules in } RN_n \\ &= P'_{n+1} \end{aligned}$$

This shows that (2) holds. For (3), we must show that every rule in $Sig(T')$ is eligible with respect to $Sig(T')$ when it is considered in P'_{n+1}. A rule in RS_n is considered in the same state in P'_{n+1} as in P'_n, so rules in RS_n are eligible with respect to $Sig(T')$ when considered in P'_{n+1}. By the choice of r_j in the **let** clause preceding [c], r_j is eligible with respect to $Sig(T')$ in the last state of RS_n. By definition, no rules in RN_n are in $Sig(T')$. Finally, by the equivalences above, the first state of $\langle r_i \rangle; A_n$ is the same in P'_n and P'_{n+1}, so each rule in $\langle r_i \rangle; A_n$ is considered in the same state in P'_n and P'_{n+1}. Therefore, rules in $\langle r_i \rangle; A_n$ that are also in $Sig(T')$ are eligible with respect to $Sig(T')$ in P'_{n+1}.

We have shown that (1)–(3) hold after the first **while** loop. Next we show that the first **while** loop always terminates. In iteration n, either RS_n is extended by one rule, or a rule not in RS_n is deleted from the path. Hence, to prove termination it suffices to show that RS_n cannot be infinite. By (3) and the fact that $Rules(RS_n) \subseteq Sig(T')$, we know that every rule in RS_n is eligible with respect to $Sig(T')$. Therefore, by Lemma A.5, there is a path RS'_n that is valid with respect $Sig(T')$, and $|RS'_n| = |RS_n|$. By assumption, there are no infinite paths in any execution graph for $Sig(T')$; hence RS'_n must be finite, implying that RS_n is finite as well. We conclude that the first **while** loop terminates.

Now consider the second **while** loop. Condition (1) obviously holds throughout loop execution. We prove by induction on the number of loop iterations that (2) and (3) also hold. The result is trivial for 0 iterations. Let $P'_n = RS; RN$ be P' after n iterations of the second **while** loop. Assume that there is a rule $r \in Sig(T')$ eligible with respect to $Sig(T')$ in the last state of RS. Recall that no rules are triggered in the last state of P because P ends in a final state. Since $P'_n \equiv P$, no rules are triggered in the last state of P'_n. Therefore:

$$\begin{aligned} P'_n &= RS; RN \\ &\equiv RS; RN; \langle r \rangle && \text{since } r \text{ is not triggered after } RN \\ &\equiv RS; \langle r \rangle; RN && \text{by commutativity of } r \text{ with rules in } RN \\ &= P'_{n+1} \end{aligned}$$

This shows that (2) holds. For (3), note that all rules in RS are considered in the same state in both P'_n and P'_{n+1}, by definition r is eligible with respect to $Sig(T')$ in the last state of RS, and no rules in RN are in $Sig(T')$. Finally, note that the second **while** loop does not terminate until (4) holds. To complete the proof, we must show that the second **while** loop always terminates. This directly parallels the proof of termination for the first **while** loop. $\square$

Theorem 9.2 (Partial Confluence) Let $T' \subseteq T$ be a set of tables. Suppose the Confluence Requirement (Definition 8.5) holds for the rules in $Sig(T')$ and there are no infinite paths in any execution graph for $Sig(T')$. Then given any two final states F_1 and F_2 in any execution graph for R, the tables in T' are identical in F_1 and F_2, i.e. the rules in R are confluent with respect to T'.

Proof: Let P_1 and P_2 be any two execution graph paths leading to final states F_1 and F_2, respectively. Let $P'_1 = \langle RS_1; RN_1 \rangle$ and $P'_2 = \langle RS_2; RN_2 \rangle$ be the paths of Lemma A.6 for P_1 and P_2, respectively. By part (2) of Lemma A.6, $P'_1 \equiv P_1$ and $P'_2 \equiv P_2$. We show that the tables in T' are identical in the last states of RS_1 and RS_2. Because none of the rules in RN_1 or RN_2 modify the tables in T' (by Definition 9.1 of $Sig(T')$), it then follows that the tables in T' are identical in the last states of P'_1 and P'_2. Consequently, the tables in T' are identical in the last states of P_1 and P_2, which are F_1 and F_2.

Consider RS_1. By Lemma A.5 and part (3) of Lemma A.6, a path RS'_1 that is valid with respect to $Sig(T')$ can be constructed from RS_1. Let (D_1, TR_1) be the last state of RS_1 and let (D_1, TR'_1) be the last state of RS'_1. By Lemma A.5 and part (4) of Lemma A.6, $TR'_1 = TR_1 \cap Sig(T') = \emptyset$, so RS'_1 ends in final state $(D_1, \emptyset)$. A symmetric argument shows that a path RS'_2 that is valid with respect to $Sig(T')$ can be constructed from RS_2, where RS_2 ends in a state (D_2, TR_2) and RS'_2 ends in final state $(D_2, \emptyset)$. Paths RS'_1 and RS'_2 have the same start state, which is produced by the same initial operations producing the start state of paths P_1 and P_2. Hence, since paths RS'_1 and RS'_2 both end in a final state, RS'_1 and RS'_2 are two paths in an execution graph for $Sig(T')$. By assumption, Definition 8.5 holds for the rules in $Sig(T')$ and there are no infinite paths in any execution graph for $Sig(T')$. Hence, by Confluence Theorem 8.7, every execution graph for $Sig(T')$ has exactly one final state. Therefore $D_1 = D_2$, which shows that the tables in T' are identical in the last states of RS_1 and RS_2. □

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