Separation Logic and the Mashup Isolation Problem

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Phd Qualifier Exam Talk

Outline

1. Background
   - Hoare Logic
   - Intuition behind Separation Logic
2. The Mashup Isolation problem ?
   - Formal Definition of Mashups
   - Isolation Property
   - How does Separation Logic help ?
3. Basic Separation Logic
   - Assertion language and Inference rules
   - Solving the Isolation Problem
4. Separation Logic with Permissions
   - Assertion language and Inference rules
   - Solving the Isolation Problem
5. Ongoing and Future Work

While Programs

$$
\begin{array}{lcl}
\textit{Expressions } E & : & x \mid n \mid E + E \mid E * E \\
\textit{Boolean Expressions } B & : & \textit{true} \mid \textit{false} \mid E = E \mid B \implies B \\
\textit{Commands } C, D & : & x := E \mid \textit{if } B \textit{ then } C \textit{ else } C \mid \\
 & & \textit{while } B \textit{ then } C \mid C; C
\end{array}
$$

Store

$Vars$: Set of Variables $\quad$ $StoreValues$: Nat

$Stores\ A, B : Vars \rightharpoonup StoreValues$

Each program C evaluates with respect to store: $A_1, C_1 \rightarrow A_2, C_2$

Hoare Logic

- *Axiomatic method for proving properties of while-programs, invented by Hoare in 1969.*
- **Central Idea:** Assign meaning to a program C using a *Hoare triple* $\{P\}C\{Q\}$
  - P : Assertion on variables *before* C begins execution.
  - Q: Assertion on variables *after* C finishes execution
- Example:
  - $\{x = 10\}y = x + 1\{y = 11\}$.
  - $\{x = y\}$*while* $x = 10$ *then* $x = x + 1; y = y + 1\{x = y\}$.

Validity of triples

A Hoare triple $\{P\}C\{Q\}$ is valid IFF:
IF P holds initially and C terminates THEN Q holds finally.

The Inference System

The assertions P, Q in the triple $\{P\}C\{Q\}$ are predicates on the store.

$$\{P[E/x]\}x := E\{P\} \qquad [\text{S-ASSIGNMENT}]$$

$$\frac{\{P\}C_1\{P'\} \quad \{P'\}C_2\{Q\}}{\{P\}C_1; C_2\{Q\}} \qquad [\text{S-SEQ}]$$

$$\frac{\{P \wedge B\}C_1\{Q\} \quad \{P \wedge \neg B\}C_2\{Q\}}{\{P\}\mathit{if}\ B\ \mathit{then}\ C_1\ \mathit{else}\ C_2\{Q\}} \qquad [\text{S-IF}]$$

$$\frac{\models P \implies P' \quad \{P'\}C\{Q'\} \quad \models Q' \implies Q}{\{P\}C\{Q\}} \qquad [\text{S-CONSEQ}]$$

- Notice the simplicity of the assignment axiom.
- Assignment axiom also provides Weakest-pre-condition.

$\mathcal{L}$: While-programs + references and records

Introduce Heaps: Mappings from locations to records

- Variables can be locations or numbers.
- Two new boolean expressions: $isNat?(x), isLoc?(x)$.
- Three new commands:
  - $x.p := E$ Update property p of record at location x.
  - $x_1 := x_2.p$ Lookup property p of records at location x_2.
  - $x := \{p_i : E_i\}_{i \in \{1,\ldots,n\}}$ Record Creation.

Heaps and Stores

Loc : Set of locations $\qquad$ $\mathbb{P}$: Set of Property names

$StoreValues : Loc \cup Nat$

Stores $A, B : Vars \rightharpoonup StoreValues$

Heaps $H, C : Loc \rightharpoonup \mathbb{P} \rightharpoonup StoreValues$

Each program C evaluates with respect to a heap and a store.

Hoare logic for $\mathcal{L}$

- We need assertions on heap-stores now: $cont\ x.p = E$
  - Meaning: Property p of record at location x has value of expression E.
  - Think of $cont$ as the function $Loc \rightharpoonup \mathbb{P} \rightharpoonup StoreValues$.
- Is $\{cont\ y\ p = 10\}x.p = 11\{cont\ y\ p = 10\}$ valid ?
  - No, x and y may contain same location.
  - Rule of Constancy does not hold.
- Correct Triple: $\{cont_{x.p:=11}\, y.p = 10\}x.p = 11\{cont\ y\ p = 10\}$
  - $cont_{x.p:=11} = \lambda l, q : \mathit{If}\ (l = x)\ \wedge\ (p = q)\ \mathit{then}\ 10\ \mathit{else}\ cont\ l\ q$
  - This is too complex, imagine multiple assignments to $x.p$.
- Intuitively, $cont\ y\ p = E$ is preserved during execution of C if y is disjoint from location-properties touched by C.
- Can I prove that $cont\ y\ p = E$ is preserved without threading it through the entire analysis of C ?

This is where Separation Logic comes in !

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Separation Logic

History:

1. *Burstall, 1972*: *separate program texts which work on separated sections of the store can be reasoned about independently*.
2. *Reynolds, MPC 2000*: An Intuitionistic logic based on Burstall's observation. Introduced $*$.
3. *Ishtiaq and O'Hearn, POPL 2001*: A classical version of Reynold's logic. Introduced $-\!\!*$
4. *Reynolds, LICS 2002*: Generalized the above logic to arbitrary pointer arithmatic.
5. Several variants for specific domains have been discovered:
   - *O'Hearn et al, POPL 2004*: Separation and Information hiding.
   - *Bornat et al, POPL 2005*: Separation logic with permissions.
   - Many more ...

Basic Separation Logic

Change of Notation: $x.p \mapsto E$ instead of *cont* x p E.

- **Separating conjunction** $*$: $(x.p \mapsto 10) * (y.p \mapsto 10)$ means
  - Field p of locations x and y, contains 10.
  - x and y are different locations.
  - Therefore $\{x.p \mapsto 10 * y.p \mapsto 10\}x.p := 11\{x.p \mapsto 11 * y.p \mapsto 10\}$ is valid.
- **Local Reasoning:** A specification should *only* reason about the heap locations and variables accessed.
  - Assignment axiom for $x.p := E$ is simply $\{x.p \mapsto \_\}x.p := E\{x.p \mapsto E\}$
  - How do we derive that $y.p \mapsto 10$ is preserved under the assignment $x.p := 11$ if $x \neq y$?
- **Frame Rule:** $\frac{\{P\}C\{Q\}}{\{P*R\}C\{Q*R\}}$
  - Helps in going from local specifications to global specifications.
  - Soundness of frame rule $\implies$ *A specified program never looks beyond what is present in its pre-condition !*

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Run-time Safety

- *Robin Milner*: *Well specified programs never go wrong*.
- Any specification $\{P\}C\{Q\}$ is such that for any heap-store H, A which satisfies P, executing C on H, A will never lead to a run-time error. This means:
  - All locations and variables accessed are mentioned by P so that there are no memory errors (critical for frame rule).
  - P includes sufficient conditions on the values of variables so that there are no type errors.
- Therefore a specification $\{x.p \mapsto 5\}C\{x. \mapsto 10\}$ means that C only accesses property p of location contained in x.
- This was the main motivation behind using separation logic for proving mashup isolation.

Separation Logic with Permissions

- Invented by Bornat et al in 2005 by incorporating fractional permissions in Separation logic.
- Main motivation was to track permissions in threads.
- We consider it in a sequential setting with 3 permissions - $\{r\}, \{w\}, \{r, w\}$.

Key Ideas:

- Add more information in the specification to reason about what is read-only.
- Read $x.p \mapsto v$ as "program has permission to read/write property p of record at x".
- Three new assertions: $x.p \overset{\{r\}}{\mapsto} E$, $x.p \overset{\{w\}}{\mapsto} E$, $x.p \overset{\{r,w\}}{\mapsto} E$.
- Modified $*$: $x.p \overset{r}{\mapsto} 10 * y.p \overset{r}{\mapsto} 10$ can hold even if $x = y$.
- Frame rule stays the same !

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Web 2.0

All about mixing and merging content (data and code) from multiple content providers in the users browser, to provide high-value applications known as mashups

- Notation:
  - Individual contents being mixed - *Components*.
  - Content Providers - *Principals*.
  - Publisher of the mashup- *Host*.
- Examples:
  - Basic Mashup: Any web page with advertisements, Facebook page with applications
  - More complex mashups: Yelp, Yahoo Newsglobe ...

Security Issues in Mashups

- Principals participating in a mashup are usually mutually untrusting.
- Each component must be protected from malicious behavior of other components.
  - Each Facebook application wants to make sure that its variables are not over-written by other applications.
  - Current *FBJS* mechanism not sufficient for ensuring this.

This Work:

- Focus on non-interacting basic mashups.
- Verify complete inter-component isolation.

Our Model

Basic Mashups (defined first in our Oakland 2010 paper):

- Components are programs in some sequential prog. language
- Mashup is a sequential composition of the components after variable renaming: $Rn(C_1); \ldots ; Rn(C_n)$.
- Reasonable model for a web page with multiple advertisements.

Isolation Property: *Behavior of each component as part of the mashup should be similar to the behavior obtained by executing it independently.*

- Isolation property in Oakland paper is a special case of the above.
- This work focusses on verifying the property whereas the Oakland paper focusses on enforcing it.

Prog. Language: Simple Imperative language with references and records. Far from *JavaScript*, but good starting point for testing out new theoretical techniques.

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Formal Definition of Mashups

- Principals: $id_1, \ldots, id_n$.
- Components $(C_1, id_1), \ldots, (C_n, id_n)$: Programs from $\mathcal{L}$.
- Initial execution environment: Heap-store H, A
- $Rn(C, a)$: Command obtained by replacing all $x \in C$ with $a.x$.
- $Rn(A, a)$: Store obtained by replacing all x in A with $a.x$.

Variable-separated mashup

A *variable-separated mashup* $\mathcal{M}(H, A, (C_1, id_1), \ldots, (C_n, id_n))$ is defined as the state $H_{mash}, A_{mash}, D_1; \ldots; D_n$ where

- $H_{mash} := H$.
- $A_{mash} := Rn(A, id_1). \ldots .Rn(A, id_n)$.
- $D_i := Rn(C_i, id_i)$.

Variable renaming is done so that components cannot influence each other via the store.

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Operational Semantics of $\mathcal{L}$

- Expressions $[\![E]\!]_{Exp} : Stores \rightarrow StoreValues \cup \{error\}$.
- Boolean Expr $[\![B]\!]_{Bexp} : Stores \rightarrow \{true, false, error\}$.
- Program states S, T are formalized as triples (H, A, C)
- Commands: $\dfrac{\langle premise \rangle}{H_1, A_1, C_1 \rightarrow H_2, A_2, C_2}$(small step)

Example rules:

$$\frac{H, A, C_1 \longrightarrow K, B, C_1'}{H, A, C_1; C_2 \longrightarrow K, B, C_1'; C_2} \quad [\text{C-SequenceContinue}]$$

$$\frac{l \in dom(H) \;\; AND \;\; x \in dom(A)}{H, A, x := l.p \rightarrow H, A[x \rightarrow H(l).p], normal} \quad [\text{C-LookupNormal}]$$

$$\frac{l \notin dom(H) \;\; OR \;\; x \notin dom(A)}{H, A, x := l.p \rightarrow H, A, abort} \quad [\text{C-LookupAbort}]$$

Notations and Definitions

$dom(H)$: $\{(l, p) \mid H(l)(p) \text{ is defined}\}$.
$dom(A)$: $\{x \mid A(x) \text{ is defined}\}$.
Given states S, T:

- $\mathcal{H}(S), \mathcal{S}(S), \mathcal{C}(S)$ denote the heap store and term part of the trace.
- $S \rightsquigarrow T$: S goes to T in zero or more steps.
- $Traces(S)$: Set of reduction traces of S.
- $S\uparrow \quad \stackrel{def}{=} \neg\exists T : S \rightsquigarrow T \not\rightarrow$.
- $Safe(S) \quad \stackrel{def}{=} \forall T : S \rightsquigarrow T \implies \mathcal{C}(T) \neq abort$.

The Simulation Relation

Heap Actions: (l, p, a, v) where $a \in \{r, w\}$ and $v \in StoreValues$
Store Actions: (x, a, v) where $a \in \{r, w\}$ and $v \in StoreValues$

- Given a trace τ, $Acc(\tau)$ is the action sequence corresponding to the trace

State simulation $S \sim T$

There exists a variable renaming $ren : Vars \rightarrow Vars$:

1. *Safe Monotonicity.* $Safe(S) \implies Safe(T)$
2. *Termination Monotonicity.* $\neg S \uparrow \wedge Safe(S) \implies \neg T \uparrow \wedge Safe(S)$
3. *Access Similarity* If $Safe(S)$ holds then for all $\tau_T \in Traces(T)$, there exists $\tau_S \in Traces(S)$ such that $ren(Acc(\tau_S)) = Acc(\tau_T)$

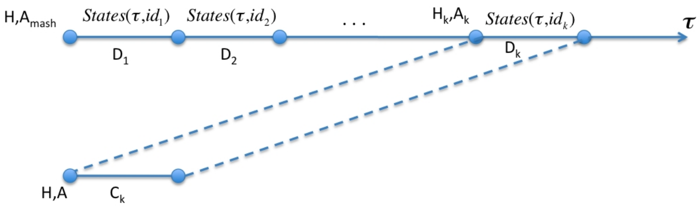
Isolation Property

Let $\mathcal{M}(H, A, (C_1, id_1), \ldots, (C_n, id_n)) = H, A_{mash}, D_1; \ldots ; D_n$, where D_i is a renamed version of C_i.

Pick any $\tau \in Traces(H, A_{mash}, D_1; \ldots ; D_n)$.

- By semantics of sequential compositions, starting heap-store for D_{i+1} is the final heap-store for D_i.
- Define $States(\tau, id_i))$ as the sub-trace of τ corresponding to execution of D_i.
- $States(\tau, id_i)) = \emptyset$ if for some $j < i$, D_j doesn't terminate normally.

Isolation Property, formally



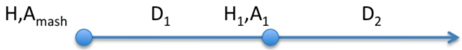
Isolation Property

$\mathcal{M}(H, A, (C_1, id_1), \ldots, (C_n, id_n))$ is isolated IFF:
for all traces $\tau \in Traces(\mathcal{M}((C_1, id_1), \ldots, (C_n, id_n)))$,

$$\forall i : States(\tau, id_i) \neq \emptyset \implies (H, A, C_i) \sim (H_i, A_i, D_i)$$

where $H_i, A_i = \mathcal{HS}(States(\tau, id_i))$

How does Separation Logic help ?



Consider heap store H, A and two components C_1, C_2.
$\mathcal{M}(H, A, (C_1, id_1), (C_2, id_2))) = H, A_{mash}, D_1; D_2$:
Let H_1, A_1 be such that $H, A_{mash}, D_1 \rightsquigarrow H_1, A_1, normal$ (assuming termination).

Result

$Isolation(\mathcal{M}(H, A, (C_1, id_1), (C_2, id_2))))$ IFF:

$$\forall l, p : l, p \in Read(H, A_{mash}, D_2) \cap dom(H) \implies H(l).p = H_1(l).p$$

How does separation logic help

$\forall l, p : l, p \in Read(H, A_{mash}, D_2) \cap dom(H) \implies H(l).p = H_1(l).p$

IFF: one of the following holds:

Any location-property pair that is read during the reduction of D_2 on H, A_{mash} is:

A. Not accessed during the reduction of D_1 on H, A
  - Basic Separation logic can tell me what is accessed and whether it is disjoint from a certain portion of the heap.

B. At most read during the reduction of D_1 on H, A
  - Definition of isolation in Oakland paper.
  - Separation logic with permissions - modified separating conjunction allows overlap on read-only portion.

C. At most written and restored during the reduction of D_1 on H, A
  - Tricky, I have a naive solution.
  - Separation logic and Information hiding.

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Basic Separation Logic SL_1

Assertion language $\mathcal{A}_1$

$$P := B \mid emp \mid x.p \mapsto E \mid P * P \mid P \mathrel{-\!\!*} P \mid P \implies P \mid \exists x.P$$

Satisfaction of Assertions: $H, A \models_{\mathcal{A}_1} P$

- Boolean Exp: $H, A \models_{\mathcal{A}_1} B \quad iff \quad [\![B]\!]_{Bexp} A = true$
- Points-to: $H, A \models_{\mathcal{A}_1} E_1.p \mapsto E_2$ iff
  1. $dom(H) = \{([\![E_1]\!]_{Exp} A, p)\}$
  2. $H([\![E_1]\!]_{Exp} A).p = [\![E_2]\!]_{Exp} A$
- Notice that the points-to relation is exact.

Satisfaction of Assertions

Separating Conjunction: $H, A \models_{\mathcal{A}_2} P_1 * P_2$ iff
$\exists H_1, H_2$:

- $dom(H_1) \cap dom(H_2) = \emptyset$
- $H_1.H_2 = H$
- $H_1, A \models_{\mathcal{A}_1} P_1 \wedge H_2, A \models_{\mathcal{A}_1} P_2$

Remarks:

- No store separation, we can write $(l_1.p \mapsto x) * (l_2.p \mapsto x)$.
- Semantics is exact.

Empty Heap: $H, A \models_{\mathcal{A}_1} emp \quad iff \quad dom(H) = \emptyset$
Implication: $H, A \models_{\mathcal{A}_1} P_1 \implies P_2 \quad iff$
$H, A \models_{\mathcal{A}_1} P_1 \implies H, A \models_{\mathcal{A}_1} P_2$

Satisfaction of Assertions

We have local specifications and frame rule, but how do we express pre-conditions for an arbitrary assertion P ?

$\{??\}x.p = 10\{P\}$

Separating Implication ($-\!\!*$):

- $x.p \mapsto 10 -\!\!* P$ holds for heaps to which if a heap satisfying $x.p \mapsto 10$ is concatenated then assertion P holds.
- Therefore, $\{x.p \mapsto \_ * (x.p \mapsto 10 -\!\!* P)\}x.p = 10\{P\}$ is valid.

Formally, $H, A \models_{\mathcal{A}_1} P_1 -\!\!* P_2 \quad iff$

$\forall H_1, H_2$:

- $dom(H_1) \cap dom(H) = \emptyset$
- $H.H_1 = H_2$
- $H, A \models_{\mathcal{A}_1} P_1 \wedge H_2, A \models_{\mathcal{A}_1} P_2$

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$\forall H_1, H_2$:

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- $H.H_1 = H_2$
- $H, A \models_{\mathcal{A}_1} P_1 \wedge H_2, A \models_{\mathcal{A}_1} P_2$

Satisfaction of Assertions

We have local specifications and frame rule, but how do we express pre-conditions for an arbitrary assertion P ?

$\{??\}x.p = 10\{P\}$

Separating Implication ($-\!\!*$):

- $x.p \mapsto 10 -\!\!* P$ holds for heaps to which if a heap satisfying $x.p \mapsto 10$ is concatenated then assertion P holds.
- Therefore, $\{x.p \mapsto \_ * (x.p \mapsto 10 -\!\!* P)\}x.p = 10\{P\}$ is valid.

Formally, $H, A \models_{\mathcal{A}_1} P_1 -\!\!* P_2 \quad iff$

$\forall H_1, H_2$:

- $dom(H_1) \cap dom(H) = \emptyset$
- $H.H_1 = H_2$
- $H, A \models_{\mathcal{A}_1} P_1 \wedge H_2, A \models_{\mathcal{A}_1} P_2$

Axioms and Inference rules (SL_1)

Hoare logic rules apply. In addition:

$$\{E_1.p \mapsto \_ \wedge Wt(E_2)\} E_1.p = E_2 \{E_1.p \mapsto E_2 \wedge Wt(E_2)\} \qquad []$$

$$\frac{y \text{ not free in } E}{\{present(x) \wedge \exists y : E.p \mapsto y\} x = E.p \{\exists y : E[y/x].p \mapsto x\}} \qquad []$$

$$\frac{y \text{ not free in any } E_i}{\{present(x) \wedge Wt(\tilde{E_i}) \wedge emp\} x := \{\tilde{p_i} : \tilde{E_i}\}\{\exists y : (x : \{p_i \mapsto E_i[y/x]\})\}} \qquad []$$

- "Backwords" rules for arbitrary post-conditions can be written for each of the above commands.
- Ishtiaq and O'Hearn (POPL 2001) proved that the backwords axioms express weakest-pre-conditions.

Frame rule

$$\frac{\{P\}C\{Q\}}{\{P * R\}C\{Q * R\}}[\mathit{modifies}(C) \cap \mathit{free}(R) = \emptyset]$$

$\mathit{modifies}(C)$ can be syntactically derived from C.

Proposition

For H, A and K, B such that $H \subseteq K$ and $A \subseteq B$, for all commands C

1. *Safe Monotonicity.* $Safe(H, A, C) \implies Safe(K, B, C)$
2. *Frame Property.* If $Safe(H, A, C)$ holds then:
   For all K', B', C' such that $K, B, C \rightsquigarrow K', B', C'$, exists H', A'':

$$H, A, C \rightsquigarrow H', A', C' \bigwedge K' - H' = K - H \bigwedge B - A = B' - A'$$

Soundness of frame rule follows from above proposition.

Frame rule

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Soundness of frame rule follows from above proposition.

Soundness

$$[\![P]\!]_{\mathcal{A}_1} \stackrel{def}{=} \{H, A \mid H, A \models_{\mathcal{A}_1} P\}$$

Validity $\models_{SL_1}$

$\{P\}C\{Q\}$ is SL_1-*valid* IFF: for all heap-stores $H, A \in [\![P]\!]_{\mathcal{A}_1}$,

1. $Safe(H, A, C)$
2. For all K, B: $H, A, C \rightsquigarrow K, B, normal \implies K, B \in [\![Q]\!]_{\mathcal{A}_1}$

Provability $\vdash_{SL_1}$

$\{P\}C\{Q\}$ is SL_1-*provable* IFF: it can be derived using the inference rules of Separation logic and $\mathcal{A}_1$-valid assertions.

Soundness

$\vdash_{SL_1} \{P\}C\{Q\} \implies \models_{SL_1} \{P\}C\{Q\}$

Solving the Isolation Problem

$\mathcal{M}(H, A, (C_1, id_1), \dots, (C_n, id_n)) = H, A_{mash}, D_1; \dots; D_n$

Case A: Any location-property pair that is read during the reduction of D_i on H, A_{mash} is not accessed during the reduction of D_j on H, A_{mash}

Procedure 1 (sufficient for case A)

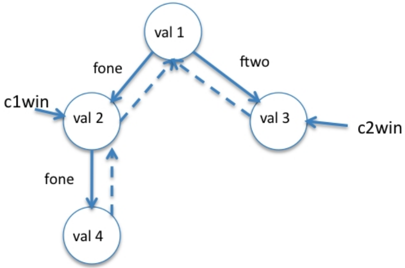
1. Deduce specifications $\{P_1\}C_1\{Q_1\}, \dots, \{P_n\}C_n\{Q_n\}$ in SL_1.
2. Show that $H, A \in [\![P_1 * \dots * P_n * true]\!]_{\mathcal{A}_1}$ holds.

Theorem

Procedure 1 is sound.

- Spent most of my time proving the above theorem.

Example: Tree access



$$H \quad := \quad \left\{ \begin{array}{lcl} l & : & \{val : 1, fone : l_1, ftwo : l_2\} \\ l_1 & : & \{val : 2, fone : l_3, par : l\} \\ l_2 & : & \{val : 3, fone : l_4, par : l\} \\ l_3 & : & \{val : 4, par : l_1\} \end{array} \right\}$$

Example: Tree access

$$\begin{array}{lcl} A_{mash} & := & \{c1win : l_1, c1tmp1 : 0, c1tmp2 : 0, c2win : l_2, c2tmp1 : 0, c2tmp2 : 0\} \\ C_1 & := & c1tmp1 = c1win.par; c1tmp2 = c1tmp1.val; c1win.val = c1tmp2+; \\ C_2 & := & c2tmp1 = c2win.val; c2win.val = c2tmp1 + 1; \end{array}$$

Prove Isolation.

Solution

1. $\{\exists x.c1win.val \mapsto 2 * c1win.par \mapsto x * x.val \mapsto 1\}C_1\{true\}$ $\{c2win.val \mapsto 3\}C_2\{true\}$
2. $H, A_{mash} \in [\![(\exists x.c1win.val \mapsto 2 * c1win.par \mapsto x * x.val \mapsto 1) * c2win.val \mapsto 3 * true]\!]_{\mathcal{A}_1}$

Issues

- Not in Algorithmic form
- Validity of $\mathcal{A}_1$ assertions is not decidable in general.
  - Yang and Calcagno (FSTTCS 2001 and a few others) have found decidable subsets of the assertion language.
  - I will explore these and their implications on procedure 1 in future.
- Nevertheless, we can carry out hand-proofs of isolation.

Outline

1. Background
   - Hoare Logic
   - Intuition behind Separation Logic
2. The Mashup Isolation problem ?
   - Formal Definition of Mashups
   - Isolation Property
   - How does Separation Logic help ?
3. Basic Separation Logic
   - Assertion language and Inference rules
   - Solving the Isolation Problem
4. Separation Logic with Permissions
   - Assertion language and Inference rules
   - Solving the Isolation Problem
5. Ongoing and Future Work

Separation Logic with Permissions SL_2

Assertion language $\mathcal{A}_2$

$$P := B \mid emp \mid x.p \overset{a}{\mapsto} E \mid P * P \mid P \mathrel{-\!\!*} P \mid P \implies P \mid \exists x.P$$

where $a \subseteq \{r, w\}$.

Satisfaction of Assertions

- Assertions are on heap-stores and also on actions performed by programs.
- Define permission maps Σ as $Loc \rightharpoonup \mathbb{P} \rightharpoonup \{\{r\}, \{w\}, \{r, w\}\}$
- General form: $H, A, \Sigma \models_{\mathcal{A}_2} P$

Satisfaction of Assertions ($\mathcal{A}_2$)

- Points-to: $H, A, \Sigma \models_{\mathcal{A}_2} E_1.p \overset{a}{\mapsto} E_2 \quad iff$
  - $dom(H) = \{([\![E_1]\!]_{Exp}A, p)\} = dom(\Sigma)$
  - $H([\![E_1]\!]_{Exp}A).p = [\![E_2]\!]_{Exp}A$
  - $a = \Sigma([\![E_1]\!]_{Exp}A).p$
- Separating-Conjunction:
  $H, A, \Sigma \models_{\mathcal{A}_2} P_1 * P_2 \quad iff \quad \exists H_1, \Sigma_1, H_2, \Sigma_2 :$
  - $H_1, A, \Sigma_1 \models_{\mathcal{A}_2} P_1$
  - $H_2, A, \Sigma_2 \models_{\mathcal{A}_2} P_2$
  - $H_1, \Sigma_1 \bowtie H_2, \Sigma_2$
  - $H_1 \cup H_2 = H \bigwedge \Sigma_1 \cup \Sigma_2 = \Sigma$

  where $dom(H_1), \Sigma_1 \bowtie dom(H_2), \Sigma_2$ means that
  - $H_1 \cup H_2, \Sigma_1 \cup \Sigma_2$ are defined
  - $\forall l, p \in dom(\Sigma_1) \cap dom(\Sigma_2) : \Sigma_1(l) = \Sigma_2(l) = \{r\}$

Observe that $(x.p \overset{r}{\mapsto} E) * (x.p \overset{r}{\mapsto} E) \Leftrightarrow x.p \overset{r}{\mapsto} E$.

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Observe that $(x.p \overset{r}{\mapsto} E) * (x.p \overset{r}{\mapsto} E) \Leftrightarrow x.p \overset{r}{\mapsto} E$.

Axioms and Inference rules (SL_2)

$$\frac{y \textit{ not free in } E}{\{present(x) \wedge \exists y : E.p \overset{r}{\mapsto} y\} x = E.p \{\exists y : E[y/x].p \overset{r}{\mapsto} x\}} \quad []$$

$$\{E_1.p \overset{w}{\mapsto} \_ \wedge Wt(E_2)\} E_1.p = E_2 \{E_1.p \overset{w}{\mapsto} E_2 \wedge Wt(E_2)\} \quad []$$

$$\frac{y \textit{ not free in any } E_i}{\{present(x) \wedge Wt(\tilde{E_i}) \wedge emp\} x := \{\tilde{p}_i : \tilde{E}_i\} \{\exists y : (x : \{p_i \overset{r,w}{\mapsto} E_i[y/x]\})\}} \quad []$$

- There are “backwards” version for each these using $-\!\!\ast$.
- All other rules stay the same.

Soundness

$$[\![P]\!]_{\mathcal{A}_2} \stackrel{def}{=} \{H, A \mid \exists\Sigma : H, A, \Sigma \models_{\mathcal{A}_2} P\}$$

Validity $\models_{SL_1}$

$\{P\}C\{Q\}$ is *SL_1-valid* IFF: for all H, A, Σ, $H, A, \Sigma \models_{\mathcal{A}_2} P$,

1. $Safe(H, A, C)$
2. For all K, B, D such that $H, A, C \rightsquigarrow K, B, D$,
   a. $K, B \in [\![Q]\!]_{\mathcal{A}_2}$ if $D = normal$.
   b. For all (l, p, a, v) IF $(l, p, a, v) \in Acc^{heap}((H, A, C), (K, B, D))$ and $l, p \in dom(H)$ THEN $a \in \Sigma(l).p$

Provability $\vdash_{SL_2}$

$\{P\}C\{Q\}$ is *SL_2-provable* IFF: it can be derived using the inference rules of Separation logic and $\mathcal{A}_2$-valid assertions.

Soundness: $\vdash_{SL_2} \{P\}C\{Q\} \implies \models_{SL_2} \{P\}C\{Q\}$.

Solving the Isolation Problem

Case B: Any location-property pair that is read during the reduction of D_i on H, A_{mash} is at most read during the reduction of on D_j on H, A_{mash}

Procedure 2 (sufficient for case B)

1. Deduce specifications $\{P_1\}C_1\{Q_1\}, \ldots, \{P_n\}C_n\{Q_n\}$ in SL_2.
2. Show that $H, A \in [\![P_1 * \ldots * P_n * true]\!]_{\mathcal{A}_2}$ holds.

Theorem

Procedure 2 is sound.

Handling Case C

Case C: Any location-property pair that is read during the reduction of D_i on H, A_{mash} is at most written and restored during the reduction of on D_j on H, A_{mash}

- Difficult to handle using permissions, we have an invariant which is temporarily broken and then restored.

Separation Logic with Information Hiding: POPL 2004

- **Main Idea**: Consider a program using multiple procedures.
  - Each procedure will have certain internal resources only managed by it.
  - The program should be reasoned about independent of these internal resources.
- Introduced the "hypothetical frame rule"

$$\frac{\begin{array}{c}\vdash \{P_1 * R\}C_1\{Q_1 * R\} \\ \cdots \\ \vdash \{P_n * R\}C_n\{Q_n * R\} \\ \{P_1\}k\{Q_1\}[X_1], \ldots, \{P_n\}k\{Q_n\} \vdash \{P\}C\{Q\}\end{array}}{\vdash \{P * R\}let\ k_1 = C_1, \ldots, k_n = C_n\ in\ C\{Q * R\}}$$

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Our Procedure

- Our procedure is inspired by the hypothetical frame rule.
- Exact Assertion: R is exact iff for all stores S there is *unique* heap H such that $H, A \in [\![R]\!]_{\mathcal{A}_1}$.

Procedure 3 (sufficient for case C)

1. Deduce specifications of the form $\{P_1 * R\}C_1\{Q_1 * R\}, \ldots, \{P_n * R\}C_n\{Q_n * R\}$ in SL_1.
2. Show that R is exact.
3. Show that $H, A \in [\![P_1 * \ldots * P_n * R * true]\!]_{\mathcal{A}_1}$ holds.

Theorem

Procedure 3 is sound.

- Soundness of hypothetical frame rule used in the proof.
- *Ownership transfer* of R from C_1 *to* $\ldots$ *to* C_n.

Comparison with Authority Safety

Oakland 2010 paper:

- $Auth(H, A, C)$: Over-approximations of the set of actions performed during the reduction of C on H, A
- $AuthIsolation(H, A, C_1, \ldots, C_n)$: For all i, j, authority map of C_i does not contain a write action to a location where C_j reads from.
- Theorem: $AuthIsolation(H, A, C_1, \ldots, C_n) \implies Isolation(\mathcal{M}(H, A, C_1, \ldots, C_n))$

How does this relate to what we have done ?

- Semantically relates to solving Case B.
- Authority maps are usually computed by heap reachability analysis.
- Specifications are "more informative authority maps": More closely related to what is actually reached.

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Future Work: Migrating to *JavaScript*

Hopeful about the following features:

1. Deleting record properties: There are rules for *dispose*
2. Computable Properties x[E]: There are rules for handling pointer arithmetic (Reynolds, 2002).

Following seem tricky:

1. Dynamic addition of properties: Tempting to think of them as "resource allocation", but they are subtly different.
   - This is allocation of a particular resource and not a non-deterministically chosen one.
   - Frame rule might break !
   - May be some kind of "existence permissions" can help.
2. Prototype chains: How do we express what part of the chain is reached during property access ?

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Other Future directions

1. Think of better solutions for handling case C.
    - Explore if a permission based approach exists.
2. Formalize the notion of defensive consistency using separation logic.
    - Informally, defensively consistent functions are ones that are incorruptible by their clients.
    - Hypothetical frame rule can be useful.
3. Explore capability systems where meaning of a capability is specified using a specification rather than an authority map.
4. Formalize the right isolation property for mashups where each component is allowed to call methods defined by other components. Example: Yelp

Thank You