Scaling Up Hardware Accelerator Verification using A-QED with Functional Decomposition

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Abstract—Hardware accelerators (HAs) are essential building blocks for fast and energy-efficient computing systems. Accelerator Quick Error Detection (A-QED) is a recent formal technique which uses Bounded Model Checking for pre-silicon verification of HAs. A-QED checks an HA for self-consistency, i.e., whether identical inputs within a sequence of operations always produce the same output. Under modest assumptions, A-QED is both sound and complete. However, as is well-known, large design sizes significantly limit the scalability of formal verification, including A-QED. We overcome this scalability challenge through a new decomposition technique for A-QED, called A-QED with Decomposition (A-QED2). A-QED2 systematically decomposes an HA into smaller, functional sub-modules, called sub-accelerators, which are then verified independently using A-QED. We prove completeness of A-QED2; in particular, if the full HA under verification contains a bug, then A-QED2 ensures detection of that bug during A-QED verification of the corresponding sub-accelerators. Results on over 100 (biggy) versions of a wide variety of HAs with millions of logic gates demonstrate the effectiveness and practicality of A-QED2.

I. INTRODUCTION

Hardware accelerators (HAs) are critical building blocks of energy-efficient System-on-Chip (SoC) platforms [1]–[3]. Unlike general-purpose processors, HAs implement a set of domain-specific functions (e.g., encryption, 3D Rendering, deep learning inference), referred to as actions in this paper, for improved energy and throughput. Today’s SoCs integrate dozens of diverse HAs (e.g., 40+ HAs in Apple’s A12 mobile SoC [4]).

Unfortunately, the energy and throughput improvements enabled by HAs come at the cost of increased design complexity. Ensuring that a given SoC will behave correctly and reliably requires verifying each and every constituent HA. Furthermore, HAs must achieve short design-to-deployment timelines in order to meet the needs of a wide variety of evolving applications [5]. Using conventional formal verification techniques to verify HAs faces several key challenges. Manually crafting extensive design-specific formal properties or full abstract functional specifications can be time-consuming and error-prone [6], [7]. Moreover, scaling verification to large HAs (with millions of logic gates) is difficult or even infeasible using off-the-shelf formal tools.

A recent formal verification technique targeting HAs, Accelerator-Quick Error Detection (A-QED) [8], overcomes the first challenge above. A-QED is readily applicable for a popular class of HAs: loosely-coupled accelerators (LCAs) [9], [10] (i.e., HAs that are not integrated as part of a central processing unit (CPU), but via an SoC’s network-on-chip or a bus) that are also non-interfering. Non-interfering HAs produce the same result for a given action independent of their context within a sequence of actions (not to be confused with combinational circuits). In other words, the state of the accelerator does not affect future computations, and each computation is independent from previous computations. In contrast, computations of interfering HAs depend on state that is the result of previous computations. A-QED uses Bounded Model Checking (BMC) [11] to symbolically check sequences of actions for self-consistency. Specifically, it checks for functional consistency (FC), the property that identical inputs within a sequence of operations always produce the same outputs. It was shown that FC checks, together with response bound (RB) checks and single-action correctness (SAC) checks, provide a thorough verification technique for non-interfering LCAs [8]. However, despite its success in discovering bugs in moderately-sized HA designs, A-QED suffers from the scalability challenges of formal tools. For example, A-QED (backed by off-the-shelf formal verification tools) times out after 12 hours when run on NVDLA, NVIDIA’s deep-learning HA [12] with approximately 16 million logic gates.

In this paper, we present a new verification approach called A-QED with Decomposition (A-QED2) to address the scalability challenge. First, we introduce a new, more general formal model of HA execution, which captures both interfering and non-interfering LCAs. We then show how A-QED2 can decompose a large LCA into smaller sub-accelerators in such a way that both FC and RB checks can be directly applied to the sub-accelerators. Unlike conventional verification approaches based on decomposition, no new properties need to be devised to apply FC and RB to the decomposed sub-accelerators. Existing decomposition approaches can be leveraged to additionally check SAC of the sub-accelerators. A-QED2 is complementary to verification approaches that rely on design abstraction, which can be used to further improve scalability and to simplify the effort required for SAC checks on decomposed sub-accelerators.

This paper presents both a formal foundation of A-QED2 and an empirical evaluation that demonstrates its bug-finding capabilities in practice. We prove that A-QED’s completeness guarantees [8] continue to hold for A-QED2—if the full HA
under verification contains a bug, then A-QED$^2$ will detect that bug. Furthermore, we apply A-QED$^2$ to a wide variety of non-interfering LCAs (although our theoretical proofs apply to interfering LCAs as well): 109 different (buggy) versions of large open-source HAs of up to 200 million logic gates (including industrial HAs). Our empirical results focus on that does not generate false errors can be difficult and may be techniques, and various approaches to this problem have properties are targeted at specific types of bugs.

The RTL synthesized in HLS flows [17]. However, these derived developed to automatically derive and check properties from flows, where appropriate optimizations like pipelining and parallelism are instantiated. Such HLS-based HA design flows are becoming increasingly common in industry. However, A-QED$^2$ is not restricted to these specific HA design styles. Our empirical results show:

1. Off-the-shelf formal tools cannot handle large HAs with millions of logic gates, even when the HAs are expressed as high-level C/C++ designs. In our experiments, A-QED verification of many such HAs times out after 12 hours or runs out of memory.
2. A-QED$^2$ is broadly applicable to a wide variety of HAs and detects all bugs detected by conventional simulation-based verification. For very large HAs with several million (up to over 200 million) logic gates, A-QED$^2$ detects bugs in less than 30 minutes in the worst case and in a few seconds in most cases.
3. A-QED$^2$ is thorough – it detected all bugs that were detected by conventional (simulation-based) verification techniques. At the same time, A-QED$^2$ improves verification effort significantly compared to simulation-based verification – $\sim 5X$ improvement on average, with $\sim 9X$ improvement (one person month with A-QED$^2$ vs. 9 person months with conventional verification flows) for the large, industrial designs.

The rest of this paper is organized as follows. Sec. II presents related work. Sec. III presents a formal model of the accelerators targeted by A-QED$^2$ and our decomposition technique. Sec. IV details the A-QED$^2$ algorithms. Results are presented in Sec. V and Sec. VI concludes.

II. RELATED WORK

Conventional formal HA verification, e.g., [13]–[16], requires a specification, typically in the form of manually written, design-specific properties. These are then combined with a formal model of the design and handed to a formal tool, which attempts to prove the properties or find counter-examples. For the verification of latency-insensitive designs, an approach was developed to automatically derive and check properties from the RTL synthesized in HLS flows [17]. However, these derived properties are targeted at specific types of bugs.

Large design sizes have always been a challenge for formal techniques, and various approaches to this problem have been proposed. Among techniques to improve scalability are abstraction [18] and compositional reasoning (cf. [19]). The former removes details of the design, gaining scalability at the cost of possible false errors. Finding a scalable abstraction that does not generate false errors can be difficult and may be impossible in some cases. The latter uses assume-guarantee reasoning (e.g., [20]–[25]) and can be applied to decompose a large HA into smaller sub-modules. Importantly, the property $p$ of the HA to be verified must also be decomposed into properties of the sub-modules. The properties of the sub-modules are verified individually under certain assumptions about the behavior of the other sub-modules. If all the properties of the sub-modules hold under the respective assumptions, then it can be concluded that $p$ holds. However, finding the right properties for this decomposition can be very challenging.

Unlike for general compositional reasoning, the two main components of A-QED$^2$ (FC and RB) do not require decomposing properties. FC, in particular, leverages a universal self-consistency property. Self-consistency expresses the property that a design is expected to produce the same outputs whenever it is provided with the same inputs [26]. In A-QED$^2$, self-consistency is checked independently for each sub-module (sub-accelerator in our case). Importantly, these aspects of A-QED$^2$ do not require complex assumptions about the behavior of the other sub-modules.

It is challenging to establish general completeness guarantees for conventional formal verification techniques [27]–[31], since completeness depends on the set of properties being checked. Designer-guided approaches [32], [33] require manual effort. Automatic generation of properties is usually incomplete and depends on abstract design descriptions [34] or models [35], or analysis of simulation traces [36], which may be difficult. In contrast, we have general completeness results for A-QED$^2$.

A-QED$^2$ builds on A-QED [8] and leverages BMC [11], [37]. Similar approaches based on self-consistency have been successfully applied to other classes of hardware designs, such as processor verification (as symbolic quick error detection (S-QED) [38]–[43]), as well as to hardware security [44]–[49].

III. FORMAL MODEL AND THEORETICAL RESULTS

In this section, we introduce a formal model for HAs, define functional consistency (FC), single-action correctness (SAC), and responsiveness for the model, and show how these properties provide correctness guarantees. We then define a notion of functional composition for our model and show how the above properties can be applied in a compositional way.

Our formal model differs from the one in previous work [8] in several important ways. It allows multiple inputs to be provided simultaneously by explicitly modeling the notion of input batches. The HAs we consider are batch-mode accelerators as they process input batches and produce output batches. Modeling batches is useful because it more closely matches the interfaces of real HAs. Moreover, input batches enable intra-batch checks for FC checking, as we describe below. With intra-batch checks, only one input batch is used for FC checking. Intra-batch checks are more restricted than general FC checks. However, they are easier to set up and run in practice, and they are highly effective at finding bugs, as we demonstrate empirically.

Our model also explicitly separates control states and memory states. Control states represent control-flow information
such as, e.g., program counters in HLS models of HAs. Memory states represent all other state-holding elements, e.g., program variables.

In our model we distinguish starting and ending control states in which inputs are provided and the computed outputs are ready, respectively. This makes the formulation simpler and is also a better match for HLS designs written in a high-level language, which is our main target in the experimental evaluation. Further, our model enables us to formulate the notion of strong FC, which leads to a complete approach to bug-finding with only two input batches.

In previous work [8], a ready-valid protocol was used to model input/output transactions in RTL designs. In contrast, our focus is on HLS designs. Finally, we distinguish so-called relevant states, which are parts of the state space that can affect output values. This makes it possible to model interfering as well as non-interfering HAs. In our experiments we focus on non-interfering HAs.

Before presenting formal definitions, we illustrate terminology informally with an example of a non-interfering batch-mode HA as shown in Listing 1 (a slightly modified excerpt of an HA implementing AES encryption [50]).

Function fun of the HA has two sub-accelerators in lines 8-10 and 13-14 which are identified and verified by A-QED². Each sub-accelerator applies a certain operation to all inputs in an input batch of HA. In general, the batch size of an HA is the number of inputs in each batch, which is 256 for this HA. The first sub-accelerator \( ACC_1 \) processes an input batch provided via data and stores its output batch in buf. The second sub-accelerator \( ACC_2 \) takes its input batch from buf, where it also stores the output batch it produces. The control state of the HA is only implicitly represented by the program counter when executing function fun. Variables key and local_key are global and determine the relevant state of the HA on which the result of the encryption operation depends. The HA is non-interfering because key and local_key are left unchanged by \( ACC_1 \) and \( ACC_2 \). Constants BS, UF, and US are used in HLS to configure the generated RTL.

Listing 1: HA Example (AES Encryption)

```c
1 #define BS ((1) << 12) // BUF_SIZE
2 #define UF 2 // UNROLL_FACTOR
3 #define US BS/UF // UNROLL_SIZE
4
5 void fun(int data[BS], int buf[UF][US], int key[2]) {
6   int j, k;
7   // == ACC_1 START ==
8   for(j=0; j<UF; j++) {
9     for(k = 0; k < BS/UF; k++)
10       buf[j][k] = *(data + i*BS + j*US + k)^key[0];
11     // == ACC_1 END ==
12     // == ACC_2 START ==
13     for(j=0; j<UF; j++)
14       aes256_encrypt(local_key[j], buf[j]);
15     // == ACC_2 END ==
16   }
```

Definition 1. A batch-mode hardware accelerator (HA) is a finite state transition system \([57], [52]\) \( Acc := (b, A, D, O, S, s_0, c, F, S_M, T_O) \), where

- \( b \in \mathbb{N} \) with \( b \geq 1 \) is the batch size,
- \( A \) is a finite set of actions,
- \( D \) is a finite set of data values,
- \( O \) is a finite set of outputs,
- \( S = S_C \times S_M \) is the set of states consisting of control states \( S_C \) and memory states \( S_M = S_{In} \times S_{Out} \times S_R \times S_N \), where
  - \( S_{In} = (A \times D)^b \) are the input states,
  - \( S_{Out} = O^b \) are the output states,
  - \( S_R \) are the relevant states, and
  - \( S_N \) are the non-relevant states,
- \( s_{c,I} \in S_C \) is the unique initial control state, which defines the set \( S_I = \{s_{c,I}\} \times S_M \) of initial states,
- \( s_{c,F} \in S_C \) is the unique final control state, which defines the set \( S_F = \{s_{c,F}\} \times S_M \) of final states,
- \( S_{m,I} \) is the set of allowable initial memory states, which defines the set \( S_{CI} = \{s_{c,I}\} \times S_{m,I} \) of concrete initial states,
- and \( T : S \to S \) is the state transition function.

When referring to different HAs, e.g., \( Acc_0 \) and \( Acc_1 \), we use subscript notation to identify their components, e.g., \( Acc_0 := (b_0, A_0, D_0, O_0, S_0, s_{c,0}, s_{F,0}, S_{m,0}, T_0) \).

We use \( v = \langle v_1, \ldots, v_{|v|} \rangle \) to denote a sequence with elements denoted \( v_i \) and length \(|v|\). We concatenate sequences (and for simplicity of notation, single elements with sequences) using ‘·’, e.g., \( v = v_1 \cdot v' \), where \( v' = \langle v_2, \ldots, v_{|v|} \rangle \). We will sometimes identify a sequence \( v \) with the corresponding tuple, and we write \( v \in v \) to denote that \( v \) appears in \( v \). We denote the \( i \)-th element of a tuple \( t \) as \( t(i) \).

An HA \( Acc \) operates on a set \( I^b \) of input batches, where \( b \) is the batch size and \( I = A \times D \). An input batch \( in \in I^b \) has \( b \) batch elements, each consisting of a pair \((a, d)\) containing an action \( a \in A \) to be executed and data \( d \in D \) (the data on which action \( a \) operates).

A state \( s \in S \) of \( Acc \) with \( s = (s_c, s_m) \) consists of a control state \( s_c \in S_C \) and a memory state \( s_m \in S_M \). The control state \( s_c \) represents control-flow-related state (e.g., the program counter in an execution of a high-level model of \( Acc \)). In a run of \( Acc \), the control state starts at a distinguished initial state \( s_{c,I} \) and ends at a distinguished final state \( s_{c,F} \).

The memory state represents all other state-holding elements of \( Acc \) (including, e.g., global variables, local variables, function parameters, and memory elements). The memory state \( s_m = (s_{m,0}, s_{m,1}, s_r, s_n) \) is divided into four parts. The first part, \( s_{m,0} \in S_{In} \), contains the input to \( Acc \). More precisely, in a run of \( Acc \), the value of \( s_{m,0} \) in the initial state is considered the input for that run. Similarly, at the end of a run of \( Acc \), \( s_{m,1} \in S_{Out} \) contains the outputs for that run (i.e., the values computed by \( Acc \) based on the inputs present at the start of the run).

The relevant state \( s_r \) represents those state elements (other than \( s_{m,0} \)) that can influence the values of the outputs. Any part of the state that can affect the output value in at least one execution should be included in the relevant state. As an example of when this is needed, consider an encryption HA with actions for setting the encryption key and for encrypting data. The internal state that stores the key is part of the relevant state because it affects the way the output is computed from the
input. The non-relevant state $s_n$ is everything else. We write $\text{ctrl}(s)$, $\text{mem}(s)$, $\text{inp}(s)$, $\text{out}(s)$, $\text{rel}(s)$, and $\text{nrel}(s)$ to denote the components $s_c, s_m, s_in, s_out, s_r,$ and $s_n$, respectively. We overload the latter four operators to apply to memory states as well, and we lift the notion to sequences of states.

The set $S_I$ of initial states contains all states resulting from combining a memory state in $S_M$ with the unique initial control state $s_{c.I}$. The concrete initial states, $S_{CI}$, are a subset of $S_I$, and essentially represent the reset state(s) of the HA. They play a role in defining the reachable states (see Definition 3 below). The set $S_F$ of final states contains all states resulting from combining a memory state in $S_M$ with the unique final control state $s_{c.F}$. Finally, the transition function $T$ defines the successor state for any given state in $S$.

Given an input batch $\text{in} \in \mathcal{I}^b$, the HA produces an output batch $\text{o} \in \mathcal{O}^b$ as follows. Let $s_0 \in S_I$ be an initial state with $\text{inp}(s_0) = \text{in}$, and let $s = T(s_0) = \langle s_1, \ldots, s_k \rangle$ denote the sequence of $|s| = k$ successor states generated by the transition function $T$, where $s_i = T(s_{i-1})$ for $1 \leq i \leq k$, such that $s_k \in S_F$ is a final state (and no earlier states in $s$ are final states). We also assume, without loss of generality, that $\text{ctrl}(s_i) \neq s_{c.I}$ for $i > 0$. The final state $s_k$ holds the output batch $\text{out}(s_k) = o$ with $o \in \mathcal{O}^b$ that is produced for the input batch $\text{inp}(s_0) = \text{in}$. Given a sequence $s$, we write $\text{initsym}(s)$ and $\text{final}(s)$ to denote the subsequence of $s$ containing all initial and final states that occur in $s$, respectively.

Given a sequence of input batches, an HA generates a sequence of output batches based on concatenating executions for each input batch.

**Definition 2.** Let $\text{in}$ be a sequence of inputs with $n = |\text{in}|$, and let $s_0 \in S_I$. Then, $\text{StateSeq}(\text{in}, s_0)$ denotes the sequence of successor states of $s_0$ that result from executing $\text{in}$, which is defined as follows.

- Let $s'_0$ be the result of replacing $\text{inp}(s_0)$ with $\text{in}_1$ in $s_0$. Let $s' = s'_0 \cdot T(s'_0)$.

- If $|\text{in}| = 1$ then $\text{StateSeq}(\text{in}, s_0) = s'$.

- If $|\text{in}| > 1$, then
  - let $s_f = \text{final}(s')$ (which is unique),
  - let $s_i = (s_{c.I}, \text{mem}(s_f))$,
  - let $s'' = \text{StateSeq}(\langle \text{in}_2, \ldots, \text{in}_n \rangle, s_i)$.
  - Then, $\text{StateSeq}(\text{in}, s_0) = s' \cdot s''$.

In Definition 2 the state $s_i$ from which each subsequent input batch is executed is obtained from the final state $s_f$ produced from executing the previous input batch. Given an HA Acc, we write $\text{StateSeq}(\text{Acc, in}, s_0)$ to explicitly refer to the successor states of $s_0$ generated by Acc. If Acc is clear from the context, we omit it.

**Definition 3.** A state $s \in S$ is reachable if $s \in S_{CI}$ or if there exists a concrete initial state $s_0 \in S_{CI}$ and sequence $\text{in}$ of input batches such that $s \in \text{StateSeq}(\text{in}, s_0)$. A relevant state $s_r$ is reachable if $s_r = \text{rel}(s)$ for some reachable state $s$.

Note that the initial states $S_I$ are not necessarily all reachable.

Next, we define an abstract specification for an HA function. Note that we use this to define correctness, but one of the features of A-QED is that the specification is not needed for the main verification technique.

**Definition 4 (Abstract Specification).** For an HA Acc, let $\text{Spec} : I \times S_R \rightarrow O$ be an abstract specification function.

Definition 3 states that the value of an output computed by an HA is completely determined by the corresponding input and the relevant part of the memory state when the HA was started. Note that the inclusion of the relevant memory state makes the definition general enough to model interfering HAs.

To model non-interfering HAs, we can either make the output dependent on only the input batch, or require that the relevant state does not change in state transitions.

Based on the abstract specification, we define the functional correctness of an HA in terms of the output batches that are produced for given input batches as follows.

**Definition 5 (Functional Correctness).** An HA Acc is functionally correct with respect to an abstract specification $\text{Spec}$ if, for all concrete initial states $s_0 \in S_{CI}$ and all sequences in of input batches, if

- $\text{in} = \langle \text{in}_1, \ldots, \text{in}_n \rangle$,
- $s = \text{StateSeq}(\text{in}, s_0)$,
- $s_f = \text{initsym}(s) = \langle s_{1.1}, \ldots, s_{1.n} \rangle$,
- $o = \text{out}(\text{final}(s)) = \langle o_1, \ldots, o_n \rangle$,

then $\forall j \in [1 \ldots b]. o_n(j) = \text{Spec}(\text{in}_n(j), \text{rel}(s_{1,n}))$.

A bug is simply a failure of functional correctness.

As mentioned above, even without a formal specification, we can apply the core technique of A-QED. To do so, we leverage the concept of functional consistency, the notion that under modest assumptions, two identical inputs will always produce the same outputs.

**Definition 6 (Functional Consistency (FC)).** An HA Acc is functionally consistent if, for all concrete initial states $s_0 \in S_{CI}$ and for all sequences in of input batches, if

- $\text{in} = \langle \text{in}_1, \ldots, \text{in}_n \rangle$, $s = \text{StateSeq}(\text{in}, s_0)$,
- $s_f = \text{initsym}(s) = \langle s_{1.1}, \ldots, s_{1.n} \rangle$,
- $o = \text{out}(\text{final}(s)) = \langle o_1, \ldots, o_n \rangle$,

then $\forall i \in [1 \ldots n], j, j' \in [1 \ldots b].$

$\text{in}_i(j) = \text{in}_i(j') \land \text{rel}(s_{i,j}) = \text{rel}(s_{i,j'}) = o_i(j) = o_i(j').$

Definition 6 illustrates the need for the relevant designation for memory states. It essentially says that two inputs, even if started at different times and in different batch positions, should produce the same output, as long as the relevant part of the memory is the same when the two inputs are sent in. The following lemma is straightforward (see the online appendix [53] for proofs of this and other results).

**Lemma 1 (Soundness of FC).** If an HA is functionally correct, then it is functionally consistent.

Checking FC requires running BMC over multiple iterations of the HA and may be computationally prohibitive for large designs or for large values of $n$. Often, it is possible to verify a stronger property, which only requires checking consistency across two runs of the HA.
**Definition 7** (Strong FC). An HA $Acc$ is strongly functionally consistent if, for all reachable initial states $s_0, s'_0$ and input batches $in, in'$, if 
- $s = \text{StateSeq}(\langle in, s_0 \rangle)$, $s' = \text{StateSeq}(\langle in', s'_0 \rangle)$,
- $s_F = \text{final}(s) = \langle s_F \rangle$, $s_F' = \text{final}(s') = \langle s'_F \rangle$,
- $o = \text{out}(s_F) = \langle o \rangle$, $o' = \text{out}(s'_F) = \langle o' \rangle$,

then $\forall j, j' \in [1, b]$.

$$in(j) = in'(j') \land \text{rel}(s_0) = \text{rel}(s'_0) \rightarrow o(j) = o'(j').$$

The main difference between FC and strong FC is that the initial states $s_0$ and $s'_0$ can be any reachable states. In contrast to that, the initial state $s_0 \in SCI$ in the definition of FC is a concrete one. It is easy to see that strong FC implies FC, but the reverse is not true in general. This is because it may not be possible for two reachable initial states $s_0$ and $s'_0$ chosen in a strong FC check to both appear in a single sequence of states resulting from executing a sequence of input batches starting in a concrete initial state. Similar to previous work on A-QED for non-batch-mode HAs [8], FC checking relies on sequences of input batches to reach all reachable states from a concrete initial state. For strong FC checking, on the other hand, two individual input batches are sufficient because the two initial states $s_0$ and $s'_0$ can be arbitrarily chosen from the reachable states. Like FC, strong FC is a sound approach.

**Lemma 2** (Soundness of Strong FC). If an HA is functionally correct then it is strongly functionally consistent.

A challenge with using strong FC is that it requires starting with reachable initial states. However, we found that in practice (cf., Section V), it is seldom necessary to add any constraints on the initial states. This may seem surprising given the well-known problem of spurious counterexamples that arises when using formal to prove functional correctness without properly constraining initial states. There are at least two reasons for this. First, many HAs have less dependence on internal state (none for non-interfering HAs) than other kinds of designs. But second, and more importantly, FC is a much more forgiving property than design-specific correctness. Many designs are functionally consistent, even when run from unreachable states.

In fact, we believe that this is a natural outcome of good design and that designing for FC is a sweet spot in the trade-off between design for verification and other design goals. If designers take care to ensure FC, even from unreachable states, then strong FC is both sound and easy to formulate.

Even simpler versions of the checks above can be obtained by making them intra-batch checks. An HA is intra-batch functionally consistent if it is functionally consistent when $i = n = 1$. That is, intra-batch FC checks are based on sending a single input batch to the HA. Consequently, it is not necessary to identify and compare the relevant parts of the initial states (cf. Definition 5) as there is precisely one initial state being used. Similarly, an HA is intra-batch strongly functionally consistent if it is strongly functionally consistent when $s_0 = s'_0$ and $in = in'$. Again, only one input batch is sent to the HA and the relevant parts of the initial states are thus always equal. As we will show in Section V, intra-batch checks can be a very effective approach for cheaply finding bugs. Intra-batch checks are applicable only to batch-mode HAs; i.e., they are not applicable in the context of A-QED targeted at HAs processing sequences of single inputs [8] rather than input batches.

While functional consistency alone can find many bugs, it becomes a complete technique (i.e., it finds all bugs) by combining it with single-action checks.

**Definition 8** (Single-Action Correctness (SAC)). An HA $Acc$ is single-action correct (SAC) with respect to an abstract specification $Spec$ if, for every batch element $(a, d)$ and for every reachable relevant state $s_r$, there exists some reachable initial state $s$ such that $\text{inp}(s)(j) = (a, d)$ for some $j$, $\text{rel}(s) = s_r$, and $\text{out}(\text{final}(T(s)))(j) = \text{Spec}((a, d), s_r)$.

Essentially, SAC requires that for each action $a$, data $d$, and reachable relevant state $s_r$, we have checked that the result is computed correctly when starting from some reachable initial state $s$ whose relevant state matches $s_r$. For every batch element $(a, d)$ and $s_r$, it is sufficient to run a single check where we can choose $(a, d)$ to be at any arbitrary position $j$ in the batch $\text{inp}(s)$. Checking SAC does require using the specification explicitly, but these kinds of checks typically already exist in unit or regression tests. SAC may even be possible to verify using simulation. As we show in Section V many bugs can be discovered without checking SAC at all.

When formalizing single-action checks, we again advocate using an over-approximation for reachability and encourages the design of HAs with simple over-approximations for the set of reachable relevant states. For the encryption example we gave above, the set of reachable relevant states is just the set of valid keys, which should be easy to specify.

In earlier work, using a slightly different HA model, we showed that SAC and functional consistency ensure correctness only when the HA is strongly connected (SC), that is, when there exists a sequence of state transitions from every reachable state to every other reachable state. The same is true here.

**Lemma 3** (Completeness of SAC + FC + SC). If an HA is strongly connected and single-action correct and has a bug, then it is not functionally consistent.

However, strong functional consistency leads to an even stronger result.

**Lemma 4** (Completeness of SAC + Strong FC). If an HA is single-action correct and has a bug, then it is not strongly functionally consistent.

Finally, to address timeliness of results in addition to correctness, we define a notion of responsiveness for our model.

**Definition 9** (Responsiveness). An HA is responsive with respect to bound $n$ if, for all concrete initial states $s_0 \in SCI$, sequences $in$ of input batches, and input batches $in$, if 
- $s = \text{StateSeq}(\langle in, s_0 \rangle) = \langle s_0, \ldots, s_m \rangle$ and
- $s' = \text{StateSeq}(\langle in \cdot in, s_0 \rangle) = \langle s_0, \ldots, s_{m+l} \rangle$,

then $l \leq n$. 
A. Decomposition for FC Checking

We now show how FC of a decomposed design can be derived from FC of its parts. We first give conditions under which two HAs can be composed.

**Definition 10** (Functionally Composable). \( Acc_1 \) and \( Acc_2 \) are functionally composable if: (i) \( b_1 = b_2 \); (ii) \( O_1 = A_2 \times D_2 \); (iii) \( S_{C,1} \cap S_{C,2} = \emptyset \); (iv) \( S_{R,1} = S_{R,2} \); and (v) \( S_{N,1} = S_{out,2} \times S'_N \) and \( S_{N,2} = S_{in,1} \times S'_N \) for some \( S'_N \).

Note in particular that compositability requires that the outputs of \( Acc_1 \) match the inputs of \( Acc_2 \). We also require that the two HAs have isomorphic memory states, which is ensured by including \( S_{out,2} \) in the non-relevant states of \( Acc_1 \) and \( S_{in,1} \) in the non-relevant states of \( Acc_2 \). In order to map a memory state of \( Acc_1 \) to the corresponding memory state in \( Acc_2 \), we define a mapping function \( \alpha : S_{M,1} \to S_{M,2} \) as follows: \( \alpha(s_m) = (\text{out}(s_m), \text{nrel}(s_m)(1), \text{rel}(s_m), (\text{inp}(s_m), \text{nrel}(s_m)(2))) \). We next define functional composition.

**Definition 11** (Functional Composition, Sub-Accelerators). Given functionally composable HAs \( Acc_1 \) and \( Acc_2 \), we define the functional composition \( Acc_0 = Acc_2 \circ Acc_1 \) (\( Acc_1 \) and \( Acc_2 \) are called sub-accelerators of \( Acc_0 \)) as follows: \( b_0 = b_1 \), \( A_0 = A_1 \), \( D_0 = D_1 \), \( O_0 = O_2 \), \( S_{C,0} = S_{C,1} \cup S_{C,2} \), \( S_{M,0} = S_{M,1} \). \( s_{c,1,0} = s_{c,1,1} \), \( s_{c,F,0} = s_{c,F,2} \), \( S_{m,1,0} = S_{m,1,1} \). The transition function is defined as follows: \( T_0(s_c, s_m) = \)

1. \( (i) \) if \( s_c \in S_{C,1} \) and \( s_c \neq s_{c,F,1} \) then \( T_1(s_c, s_m) \);
2. \( (ii) \) if \( s_c \in S_{C,2} \) then \( T_2(s_c, \alpha(s_m)) \); and
3. \( (iii) \) if \( s_c = s_{c,F,1} \) then \( (s_{c,F,1}, \alpha(s_m)) \).

Definition 11 essentially states that an execution of \( Acc_0 = Acc_2 \circ Acc_1 \) is obtained by first running \( Acc_1 \) to completion, then passing the outputs of \( Acc_1 \) to the inputs of \( Acc_2 \), and then running \( Acc_2 \) to completion. As a variant of Definition 11, it is also possible to define functional composition where the sub-accelerators operate in parallel. This way, the sub-accelerators process non-overlapping parts of a given input batch and produce the respective non-overlapping parts of the output batch.

We now introduce a compositional version of FC.

**Definition 12** (Strong FC for Decomposition (FCD)). An HA \( Acc \) is strongly functionally consistent for decomposition (strongly FC) if it is strongly functionally consistent and, in addition to \( o(j) = o'(j) \), the property \( \text{rel}(s_F) = \text{rel}(s'_F) \) holds in the conclusion of the implication in Definition 4.

Note that strong FCD is stronger than strong FC. In order to stitch together results on sub-accelerators, we need to establish that not only the output but also the relevant memory state is the same after processing identical inputs. The following is clear from the definition.

**Corollary 1.** If an HA \( Acc \) is strongly FCD, then \( Acc \) is strongly FC.

We now show that composition preserves strong FC and then state our main result.

**Lemma 5** (Functional Composition and Strong FCD). Let \( Acc_0 = Acc_2 \circ Acc_1 \). If both \( Acc_1 \) and \( Acc_2 \) are strongly FCD then \( Acc_0 \) is strongly FCD.

**Theorem 1** (Completeness of A-QED^2). Let \( Acc_0, Acc_1, \) and \( Acc_2 \) be HAs such that \( Acc_0 = Acc_2 \circ Acc_1 \) and \( Acc_0 \) is single-action correct. If \( Acc_1 \) and \( Acc_2 \) are strongly FCD then \( Acc_0 \) is functionally correct.

Theorem 1 states that A-QED^2 is complete. That is, by contraposition, if an HA \( Acc_0 \) has a bug, i.e., it is not functionally correct, then either \( Acc_1 \) or \( Acc_2 \) is not strongly FCD, and thus the bug can be detected by A-QED^2.

Note that there is no corresponding soundness result. This is because it is possible to decompose a functionally consistent HA into functionally inconsistent sub-accelerators. However, as shown in Section V, this appears to be rare in practice, and here again we reiterate our position on design for verification and advocate that also sub-accelerators should be designed with functional consistency in mind.

Functional composition can easily be generalized to more than two sub-accelerators. Moreover, it can be applied recursively to further decompose sub-accelerators. If functional decomposition based on Definition 11 is not applicable to further decompose a sub-accelerator, then such a sub-accelerator can be decomposed using existing formal decomposition approaches, though these require significant manual effort. Our approach identifies conditions under which simple, automatable decomposition of FC checking is possible.

IV. A-QED^2 FUNCTIONAL DECOMPOSITION IN PRACTICE

We now present our implementation of A-QED^2, which builds on the theoretical framework of the previous section. We combine functional decomposition with checks for FC (dFC), SAC (dSAC), and responsiveness (dRB).

A. Decomposition for FC: dFC

dFC takes as input a non-interfering LCA design \( Acc \) (satisfying Definitions 1 and 2) together with designer-provided annotations (explained in this section). dFC decomposes \( Acc \) into sub-accelerators (following Definition 11). FC checks are run on the sub-accelerators and any counterexamples are reported. Note that the way in which \( Acc \) is actually decomposed into sub-accelerators has no influence on the completeness of A-QED^2 (Theorem 1). That said, FC checks may scale better for certain decompositions. While failing FC checks expose consistency issues at the sub-accelerator level, it is possible that they do not cause incorrect behaviors at the full \( Acc \) level. However, we did not observe any instances of this in our experiments.

Our dFC implementation relies on identifying batch operations in a given \( Acc \). A batch operation operates on a vector of inputs, applying some action to each input in order to produce a vector of outputs. The input to a batch operation could be an intermediate output batch of another sub-accelerator or an input batch to \( Acc \) itself. A batch operation produces either an
intermediate output batch which is subsequently processed by another sub-accelerator or an output batch of Acc itself.

We assume that Acc is expressed in a high-level language, specifically as a C/C++ program that implements sequential computation of Acc outputs from Acc inputs. Batch operations in the C/C++ program are identified by finding contiguous C/C++ statements called functional blocks that implement those batch operations. Each functional block represents a sub-accelerator.

We have developed a set of annotations by which the designer can help identify these functional blocks. Examples of such annotations are given in Listing 2, which extends Listing 1. It has two functional blocks corresponding to batch operations: lines 15-17 and 32-33.

Annotations are defined by particular keywords that are prefixed by "#" (and denoted in blue) in Listing 2. These annotations describe the compute and memory access patterns of the functional block as it transforms an input batch into an output batch. In practice, hardware designers already use similar annotations frequently, e.g., to express parallelization opportunities for HLS to generate efficient hardware. As a result, we expect manageable effort in creating such annotations to support dFC. The HLS research community is actively developing new techniques to automatically explore the HA design space and derive optimal design points together with opportunities for HLS to generate efficient hardware. As a result, we expect manageable effort in creating such annotations to support dFC.

From the annotations, we create sub-accelerators. For example, the annotations in Listing 2 generate two sub-accelerators: Acc1 corresponding to the functional block in Lines 15-17 with annotations in Lines 8-13 and 19-24, and Acc2 corresponding to the functional block in Lines 32-33 with annotations in Lines 26-30 and 35-39. For each sub-accelerator, we create an A-QED module for FC checking. It generates symbolic inputs for the sub-accelerator and symbolically executes the corresponding functional block in order to produce symbolic expressions for the outputs. For strong FC checks (Definition 6 and 7), the relevant states (Definition 1) must additionally be identified and explicitly constrained to be consistent across sub-accelerator calls processing two input batches. Identifying the relevant states is not necessary for intra-batch FC checks (discussed in the context of Lemma 2). For example, in sub-accumulator Acc1 in Listing 2, key[0] is a relevant state element (distinct from the batch input data). Between two calls of Acc1 during a strong FC check, key[0] must be consistent. In our implementation, we ignore reachability and allow all checks to start from fully symbolic initial states. This does not lead to spurious counterexamples in our experiments.

B. Decomposition for RB: dRB

The sub-accelerators for A-QED's RB checks (Definition 9) can be (and often are) different from those for FC because RB involves a much simpler check: some output is produced within the response bound n. We expect n to be provided by the designer for the top-level accelerator. We then use the same bound n for each sub-accelerator. The rationale is that if a sub-accelerator fails an RB check, then the full accelerator would also fail the same RB check.

For dRB, we generate a static single assignment (SSA) representation of the design. We then apply a sliding window algorithm to dynamically generate sub-accelerators. Lines of code in the SSA that fall within a certain window W form the sub-accelerator. Due to SSA form, the inputs of this sub-accumulator are variables that are never updated or assigned in W while the outputs are the variables which update variables outside W. The current size of W is given by the number of LOCs that fit in W, and it changes dynamically during a run of the algorithm to incorporate the largest sub-accelerator that will fit the BMC tool. Once the sub-accelerator is verified, W slides by δ LOCs (δ is a parameter) and adjusts its boundary to get the next largest sub-accelerator that can be verified.

We synthesize sub-accumulator using HLS (since some responsiveness bugs only manifest after HLS) and then run RB checks using BMC. The initial states of each generated

```c
// === ACC1 START ===
for (j = 0; j < UF; j++)
    buf[j][k] = (data[i] + j*BS + key[2] + k);
// === ACC1 END ===

// === ACC2 START ===
for (j = 0; j < UF; j++)
    buf[j][k] = (data[i] + j*BS + j*US + k)*key[0];
// === ACC2 END ===
```

1HAs expressed in Verilog or SystemC can be converted into C/C++, and then our dFC implementation can be applied. We do this in Sec. [V].

2Existing HLS tools (e.g., Xilinx Vivado HLS, Mentor Catapult HLS) can then optimize Acc, incorporate appropriate pipelining and parallelism, and produce Verilog for subsequent logic synthesis and physical design steps. Such HLS-based HA design flows are becoming increasingly common.

3See the online appendix [53] for details.
sub-accelerator are left unconstrained (i.e., fully symbolic) in order to analyze all possible behaviors. The specific size of \( W \) and its position in the SSA code change dynamically as dRB proceeds. dRB terminates when \( W \) reaches the end of the SSA code or if at any time an RB check fails.

C. Decomposition for SAC: dSAC

As mentioned above, and as will be shown in the next section, many bugs can be detected using only dFC and dRB. The advantage of this is that both of these checks can be run without any functional specification. dSAC completes the story, but at the cost of requiring specifications. We use standard functional decomposition techniques (essentially, writing preconditions, invariants, and postconditions) to decompose SAC checks. One feature of dSAC is that only a single input in a batch needs be checked—all other inputs in the batch can be set to constants (we use zero in our experiments). This makes both writing the properties and checking them much simpler. The non-input part of the initial state for each check is again kept fully symbolic for simplicity. If a sub-accelerator is too big, we further decompose it using finer-grained functional blocks.

V. EXPERIMENTAL RESULTS

We demonstrate the practicality and effectiveness of A-QED\(^2\) for 109 (buggy) versions of several non-interfering LCAs\(^2\), including open-source industrial designs \(^{\text{[12]}}\). We selected these designs for the following reasons:

- They cover a wide variety of HAs (neural nets, image processing, natural language processing, security). Most are too large for existing off-the-shelf formal tools.
- They have been thoroughly verified (painsstakingly) using state-of-the-art simulation-based verification techniques. Thus, we can quantify the thoroughness of A-QED\(^2\).
- With access to buggy versions, we did not have to artificially inject bugs. Bugs we encountered include incorrect initialization, incorrect memory accesses, incorrect array indexing, and unresponsiveness in HLS-generated designs.

Many of the designs were already available in sequential C or C++. We converted Verilog and SystemC designs into sequential C. To facilitate dFC, we manually inserted annotations (like those in Listing\(^{\text{[2]}}\)). For A-QED FC, we used CBMC for all designs originally represented in sequential C or C++. For designs in Verilog and SystemC, we used Cadence JasperGold (SystemC designs converted to Verilog via HLS). For A-QED\(^2\) FC and SAC checks, we used CBMC version 5.10 \(^{\text{[66]}}\). For A-QED and A-QED\(^2\) RB checks, we used Cadence JasperGold version 2016.09p002 on Verilog designs generated by the HLS tools used by the designers. Lastly, we used Frama-C \(^{\text{[67]}}\) to check for initialization and out-of-bounds bugs on the entire C/C++ designs. We ran all our experiments on Intel Xeon E5-2640 v3 with 128GB of DRAM.

Tables \(^{\text{[I]}}, \text{II}, \text{III}\) summarize our results. We present comparisons between A-QED\(^2\) (dFC, dRB, dSAC) and A-QED (FC, RB, SAC). Table \(^{\text{[I]}}\) also compares A-QED\(^2\) intra-batch FC vs. A-QED\(^2\) strong FC (cf. details in the online appendix \(^{\text{[53]}}\)).

Observation 1: HAs from various domains (including industry) show that non-interfering LCAs are highly common.

Observation 2: The vast majority of the studied HAs are too big for existing off-the-shelf formal verification tools, for both A-QED and conventional formal property verification.

Observation 3: Table \(^{\text{[I]}\text{I}}\) shows that A-QED\(^2\) intra-batch FC checks detected bugs inside sub-accelerators (with batch sizes \(> 1\)) very quickly—under a minute for almost all of the designs, and just over a minute for \(n_v\) large. For most batch-mode sub-accelerators—except two for each of the following four designs (accounting to eight sub-accelerators in total): grayscale64, grayscale32, mean128, and mean32—intra-batch dFC checks were easily completed using off-the-shelf formal tools. Strong FC checks incur more complexity. Hence, the formal tool timed out after 12 hours for 62 sub-accelerators when running strong FC checks, distributed across multiple designs. Empirically, we found that intra-batch FC checks detected all bugs that were detected by strong FC checks.

Observation 4: A-QED\(^2\) RB and A-QED\(^2\) SAC are also highly effective in detecting bugs inside sub-accelerators. For the first 11 designs (AES to gsm) in Table \(^{\text{[I]}\text{I}}\) we do not expect unresponsiveness bugs (confirmed by simulations). Hence, A-QED\(^2\) RB checks ran for 12 hours (for increasingly longer input sequences) without detecting unresponsiveness. For designs with RB bugs, A-QED\(^2\) RB checks on sub-accelerators were able to detect those in less than 11 minutes on average. For A-QED\(^2\) dSAC, we observed that a significant fraction (26 out of 46 bugs (56%)) of these bugs were also detected by A-QED\(^2\) FC checks. Thus, FC alone is effective at catching a wide variety of bugs.

Observation 5: A-QED\(^2\) detected all bugs that were detected by conventional (simulation-based) verification techniques. Further, all counterexamples produced from verifying sub-accelerators corresponded to real accelerator-level bugs. Compared with traditional simulation-based verification, we report a \(\sim 5X\) improvement in verification effort on the average, with a \(\sim 9X\) improvement for the large, industrial NVDLA designs. The overhead of inserting our annotations for dFC can be small compared to what designers already insert to optimize the design. For ISmartDNN, for example, the total number of annotations is 304, which is 2.8% of the total lines of code of the design. In the code of the HLS designs we considered, pragmas amount to 11% on average. We also observe a \(\sim 60X\) improvement in average verification runtime compared to conventional simulations\(^{\text{[5]}\text{[5]}}\).

VI. CONCLUSION

Our theoretical and experimental results demonstrate that A-QED\(^2\) is an effective and practical approach for verification.
TABLE I: Avg. RunTimes of FC checks for A-QED and A-QED\textsuperscript{2}. For A-QED\textsuperscript{2}, sub-accelerator counts are provided, including the Total count that resulted from dFC decomposition, the count with batch sizes greater than one (i.e., Parallel), the count (with batch sizes greater than one) for which FC checks were successful on 1 and 2 batches for intra-batch FC and strong FC respectively, and the count for which Bugs were detected by FC checks. For A-QED\textsuperscript{3} FC experiments, incomplete FC checks for a single batch in 12 hours (timeout) or exhibited out-of-memory (OOM) errors before timeout. Average runtimes result from dividing the time to detect all bugs by the number of bugs. \textsuperscript{1}keypair \cite{59}, \textit{gsm} \cite{60}, \textit{HLSCNN} \cite{61}, \textit{FlexNLP} \cite{62}, \textit{Dataflow} \cite{63}, and \textit{Opticalflow} \cite{64} all time out for A-QED\textsuperscript{3} FC and do not contain any sub-accelerators with batch size greater than one. One OOB bug was detected in \textit{gsm} and one initialization bug in keypair.

TABLE II: RB checks for A-QED and A-QED\textsuperscript{2}. For A-QED\textsuperscript{2}, sub-accelerator counts produced by dFC are provided, as in Table I. For A-QED\textsuperscript{2} RB checks did not complete even for an input sequence length of 1 within 12 hours (timeout). Sub-accelerators for which RB checks for at least input sequence length of 1 was completed are considered Complete. For the first 11 designs, from AES to \textit{gsm}, no bugs related to unresponsiveness were detected by traditional simulation-based verification. Results are omitted for \textit{nv_large} and \textit{nv_small}; responsiveness related bugs generally result from parallelism and pipelining, both of which were lost in our manual translation of NVDLA from Verilog to sequential C code.

TABLE III: SAC checks for A-QED\textsuperscript{2}. Sub-accelerator counts produced by dSAC are provided, as in Table I. A-QED\textsuperscript{2} SAC checks were performed on all sub-accelerators regardless of batch size, so \textbf{P} is omitted compared to Table I.

- Further A-QED\textsuperscript{2} scalability using abstraction.
- Extension of A-QED\textsuperscript{2} beyond sequential (C/C++) code to include concurrent programs.
- Effectiveness of A-QED\textsuperscript{2} for RTL designs (without converting them to sequential C/C++).
- Applicability of A-QED\textsuperscript{2} beyond functional bugs (e.g., to detect security vulnerabilities in HAs).
- Comparison of A-QED\textsuperscript{2} and conventional decomposition.
- Identifying conditions under which A-QED\textsuperscript{2} is sound.

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