Homomorphic Signatures for Polynomial Functions

Dan Boneh and David Mandell Freeman

Stanford University, USA

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Homomorphic Signatures

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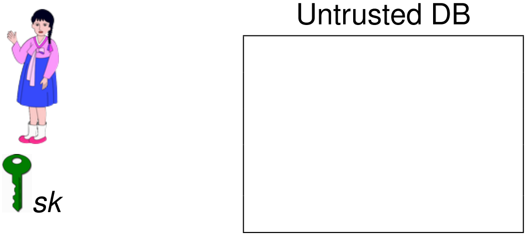

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Homomorphic signatures allow users to delegate computation while ensuring *integrity*.

sk

signed → grades

Untrusted DB

Student	Score	Sig
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Becky	73	σ_2
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σ_1 = signature on ("grades", 91, "Adam")

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→ 87.3, σ

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What properties do we want the derived signature σ to have?

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2. Unforgeability: no adversary can produce a σ^* that authenticates a different mean for the "grades" data.
3. Length efficiency: σ is short.
4. Privacy: σ reveals nothing about data other than the mean.

More generally: $\mathcal{F}$-homomorphic signatures

As introduced by [JMSW02]:

- $\mathcal{F}$ is a set of “admissible” functions on messages.
- τ is the name of the file or data set
  (prevents mixing of data from different sets)
- Given *pk*, admissible function $f \in \mathcal{F}$, and signatures on

  $$(\tau, m_1, 1), \ \ldots, \ (\tau, m_k, k)$$

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Observation [JMSW02]

Secure homomorphic signatures for $\mathcal{F} = \{\text{linear functions}\}$ cannot exist.

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Our modification: authenticate the function.

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Theorem [BFKW09,GKKR10,BF11]

Secure homomorphic signatures for $\mathcal{F} = \{\text{linear functions}\}$ do exist (under certain assumptions).

Applications

What are homomorphic signatures good for?

$\mathcal{F}$	Application
Linear functions	Mean Fourier transform Network coding

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Arbitrary circuits	Non-linear estimators and regression Data mining (decision trees, SVM, etc.)

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Specifically, we construct secure, length-efficient, $\mathcal{F}$-homomorphic signatures for

$$\mathcal{F} = \{\text{polynomials of bounded degree with small coefficients}\}$$

Related Concepts

Computationally Sound Proofs [M00]:

Server computes a short proof of knowledge that for given (f, y)

$$\exists\ (\vec{m}, \sigma) \text{ s.t. } \begin{cases} y = f(\vec{m}) \quad \text{and} \\ \text{Verify}(pk, \vec{m}, \sigma) = 1. \end{cases}$$

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- Homomorphic signatures allow third party verification.

Application: Least Squares Fits

Least squares fits — the basics

For a data set $\{(x_i, y_i)\}_{i=1}^{k}$, the degree d least squares fit is a polynomial

$$f(x) = c_0 + c_1 x + \cdots + c_d x^d$$

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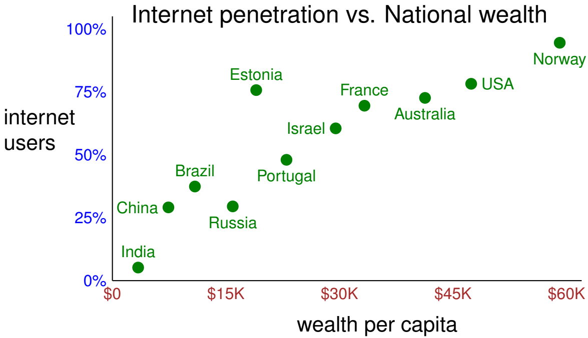

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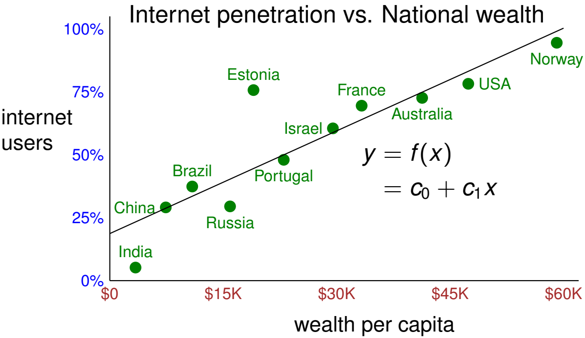

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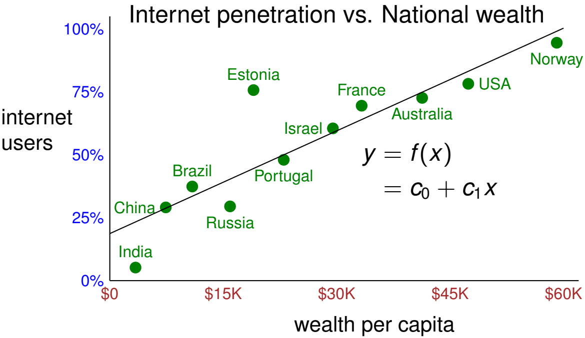
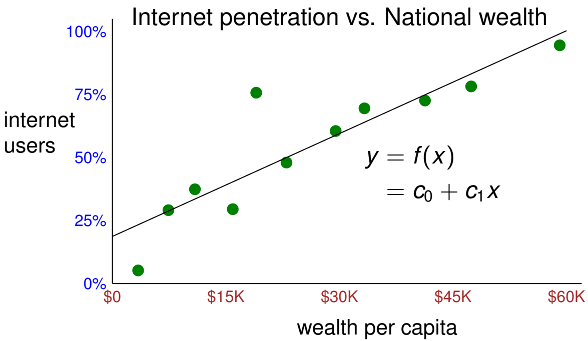


Formula:

$$\vec{c} = (X^t X)^{-1} X^t \vec{y}$$

$\vec{c}$ = vector of coefficients of $f(x)$,
X = Vandermonde matrix of x values,
$\vec{y}$ = vector of y values.

Authenticating a least-squares fit (x-values only)



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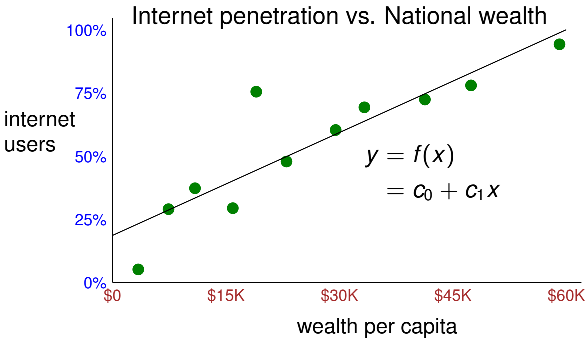

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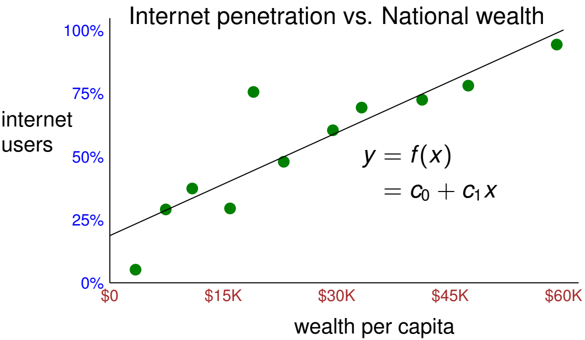

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- However: $\det(X^t X) \cdot c_j$ are polynomial functions of x and y.

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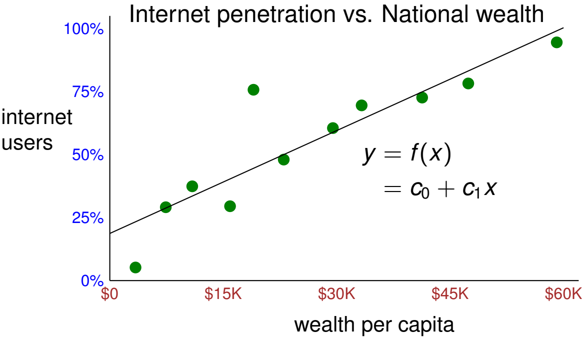

Formula: $\det(X^t X) \cdot c_j =$ polynomial in $\{x_i, y_i\}$

- United Nations stores signed data on server using polynomially homomorphic signature.

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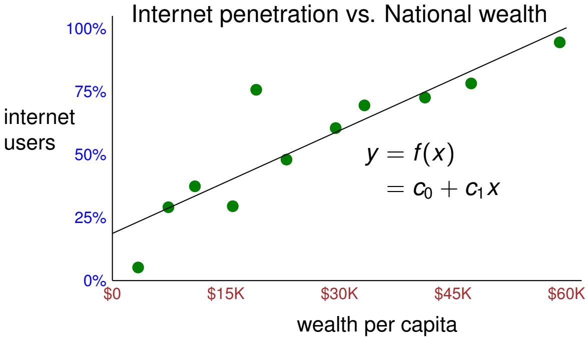

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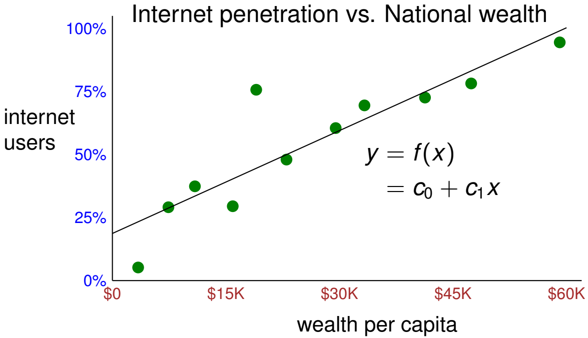

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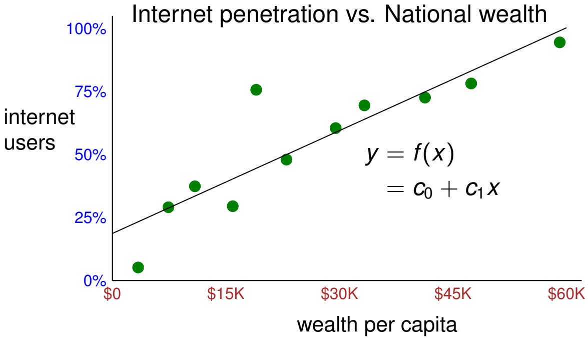

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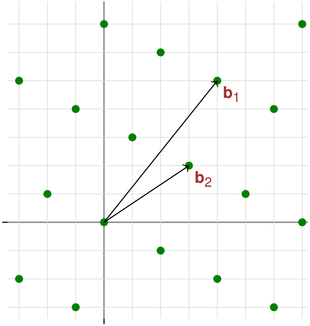
- Linear fit can be computed using degree 3 polynomials.

Homomorphic Signatures:
Our Construction

Building block: GPV Signatures

pk: $\Lambda \subset \mathbb{Z}^n$ a lattice (full-rank additive subgroup), defined by "bad" basis.



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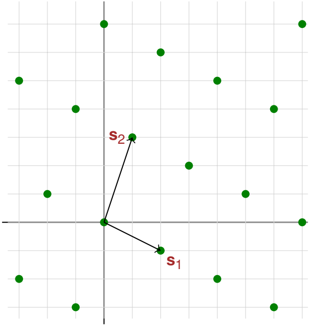

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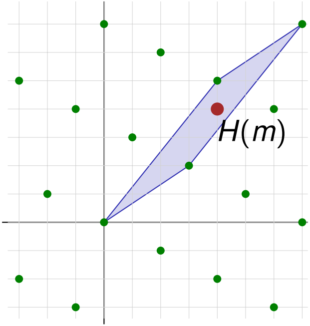

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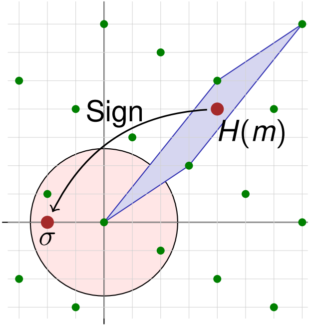

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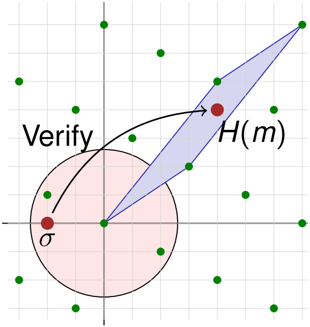

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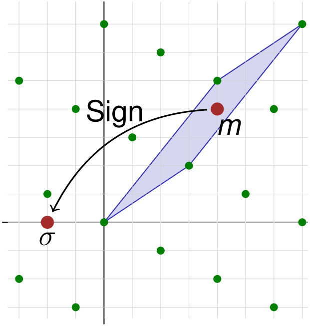

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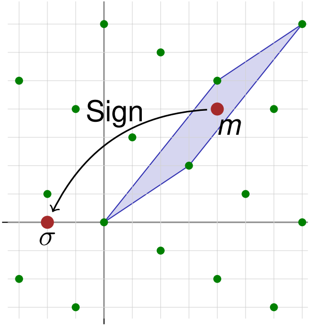

What if we encode m in $\mathbb{Z}^n/\Lambda$ directly?
Then signatures are linearly homomorphic:

$$(\sigma_1 + \sigma_2) \text{ is short}, \quad (\sigma_1 + \sigma_2) \bmod \Lambda = m_1 + m_2$$

so $\sigma_1 + \sigma_2$ authenticates $m_1 + m_2$!

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- Malicious DB outputs $18.0 = s_1 - s_2$ and signature $\sigma = \sigma_1 - \sigma_2$.

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- Malicious DB outputs $18.0 = s_1 - s_2$ and signature $\sigma = \sigma_1 - \sigma_2$.
- σ authenticates 18, but 18 is not the mean!

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Use a second lattice to authenticate functions:

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If "encoding" $\omega(\cdot)$ is linear, (i.e., $\omega(f) + \omega(g) = \omega(f + g)$)
signatures are linear on the space of functions.

"Intersection method" binds messages to functions

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Messages $m \in \mathbb{Z}^n/\Lambda_1$, functions f encoded as $\omega(f) \in \mathbb{Z}^n/\Lambda_2$.

Define signature as simultaneous GPV signature on $(m, \omega(f))$.

- $\text{Sign}(m) =$ short vector in $(\Lambda_1 + m) \cap (\Lambda_2 + \omega(f))$.
  $sk =$ “good” basis of $\Lambda_1 \cap \Lambda_2$.

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Messages $m \in \mathbb{Z}^n/\Lambda_1$, functions f encoded as $\omega(f) \in \mathbb{Z}^n/\Lambda_2$.

Define signature as simultaneous GPV signature on $(m, \omega(f))$.

- Sign(m) $=$ short vector in $(\Lambda_1 + m) \cap (\Lambda_2 + \omega(f))$.
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  2. $\sigma \bmod \Lambda_1 = m$,
  3. $\sigma \bmod \Lambda_2 = \omega(f)$.
- Evaluate$(f, (\sigma_1, \ldots, \sigma_k)) = f(\sigma_1, \ldots, \sigma_k)$ for linear f.
  If σ_i = Sign(m_i), output authenticates $f(m_1, \ldots, m_k)$.

Homomorphic signatures for polynomial functions

Linearly homomorphic scheme: messages in $\mathbb{Z}^n/\Lambda_1$, functions encoded in $\mathbb{Z}^n/\Lambda_2$, signatures in $\mathbb{Z}^n$.

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- Same construction now authenticates polynomial functions on messages.
- Length of signature vector grows with polynomial degree
  $\Rightarrow$ degree must be bounded to ensure security.

Security

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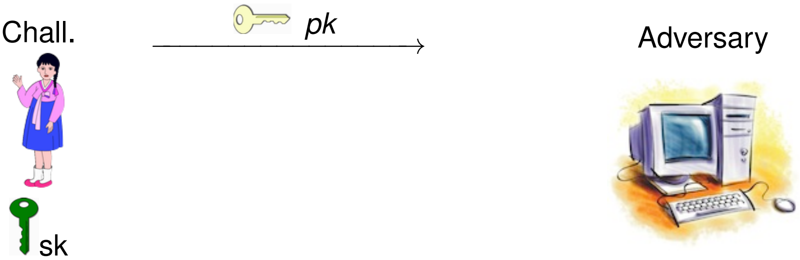

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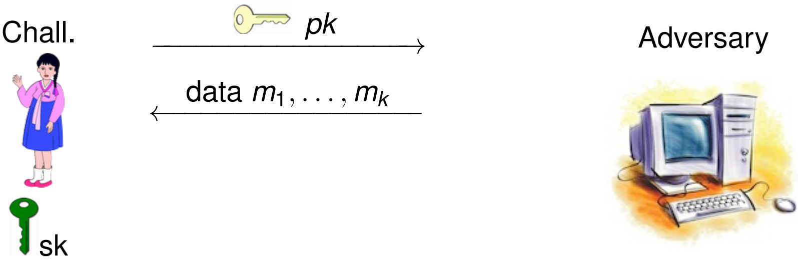

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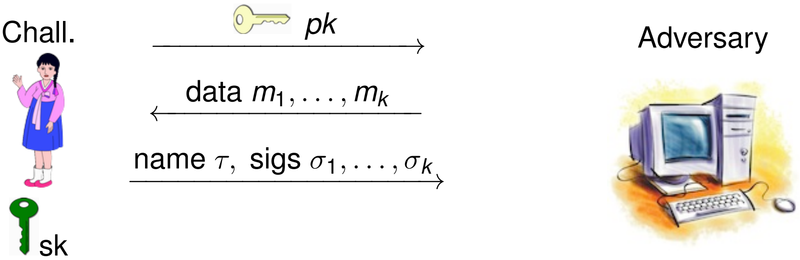

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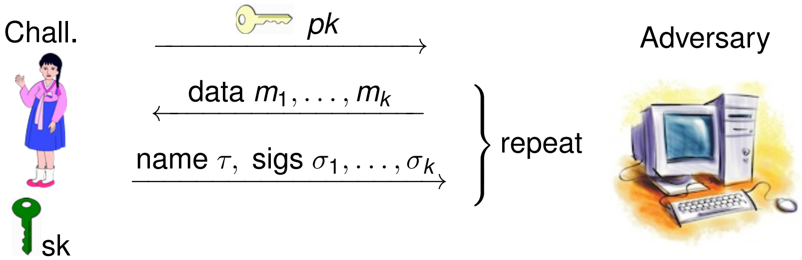

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pk →

← data $m_1, \ldots, m_k$

name τ, sigs $\sigma_1, \ldots, \sigma_k$ →

} repeat

← forgery τ^*, m^*, σ^*, f

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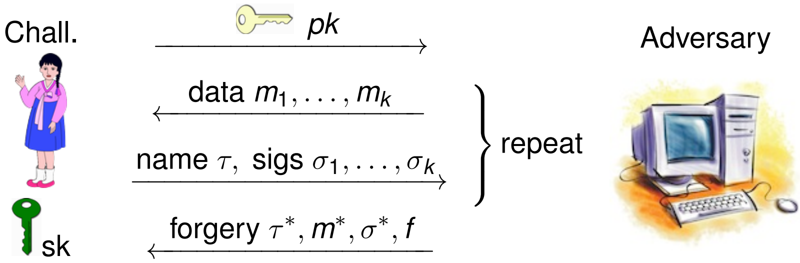

Adversary wins if f admissible, σ^* verifies for (τ^*, m^*, f), and

1. τ^* not obtained in response to a query, or
2. $\tau^* = \tau$ for query $(m_1, \ldots, m_k)$, and $m^* \neq f(m_1, \ldots, m_k)$.

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  Finding short vectors is as hard as solving worst-case lattice problems [A96,MR04,GPV08].
- Polynomial system: use ideal lattices proposed for homomorphic encryption [SV10].

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Privacy property: derived signature on $f(m_1, \ldots, m_k)$ reveals nothing about $m_1, \ldots, m_k$ beyond value of f.

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$$\vec{m} = (m_1, \ldots, m_k), \qquad \vec{m}' = (m'_1, \ldots, m'_k)$$

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Our linearly homomorphic signatures are private.

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Thank you!