Improved Security for Linearly Homomorphic Signatures: A Generic Framework

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Problem: Computing on Authenticated Data

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Untrusted DB

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$\xrightarrow[\text{grades}]{\text{signed}}$

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Adam	91	σ_1
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$\vdots$	$\vdots$	$\vdots$
Kevin	84	σ_k

σ_1 = signature on
(“grades”, 91, “Adam”)

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- Security: no adversary can authenticate a different mean.
- Length efficiency: σ is short.

Solution: Homomorphic Signatures

Messages $(m_1, \ldots, m_k)$ grouped together into *files*, identified by a randomly chosen *tag* τ.

- $\mathsf{KeyGen}(n) \to \mathsf{pk}, \mathsf{sk}$
- $\mathsf{Sign}_{\mathsf{sk}}(\tau, m_i, i) \to$ signature σ_i on ith message
- $\mathsf{Eval}_{\mathsf{pk}}(\tau, (\sigma_1, \ldots, \sigma_k), f) \to$ signature σ on $f(m_1, \ldots, m_k)$
- $\mathsf{Verify}_{\mathsf{pk}}(\tau, m, \sigma, f) \to 1$ iff $m = f(m_1, \ldots, m_k)$

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Linearly homomorphic signatures:

- messages m_i are vectors in $\mathbb{Z}^n$ or $\mathbb{F}_q^n$
- functions f are linear combinations.
- applications: mean, Fourier transform, regression models, network coding.

Linearly Homomorphic Signatures: State of the Art

Scheme	Built on	Assumption	Vectors in
[BFKW09]	BLS signatures	CDH in bilinear groups	$\mathbb{F}_p^n$ (large p)
[GKKR10]	RSA signatures	RSA	$\mathbb{Z}^n$
[BF11a,b]	GPV signatures	worst-case lattice problems	$\mathbb{F}_p^n$ (small p)

(Orange = random oracle model)

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[AL11]	Lewko-Waters IBE	nonstandard, decisional assumptions in bilinear groups	$\mathbb{Z}_N^n$
[CFW11]	"adaptive pseudo-free groups"	strong RSA	$\mathbb{Z}^n$
[CFW12]		q-SDH in bilinear groups; strong RSA	$\mathbb{F}_p^n$; $\mathbb{F}_e^n$

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Missing: Weak assumptions in the standard model!

Our Contribution (1)

Generic framework for converting (ordinary) signatures to linearly homomorphic signatures.

- Applies to signature schemes with certain "pre-homomorphic" properties.
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Instantiations:

Scheme	Assumption (in standard model)
[W05]	CDH in bilinear groups
[BB04b]	q-SDH in bilinear groups
[GHR99]	strong RSA
[HW09b]	RSA

Our Contrubution (2)

Stronger security model for homomorphic signatures.

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$\longrightarrow$

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tag τ_F, sigs $\sigma_1^F, \ldots, \sigma_k^F$

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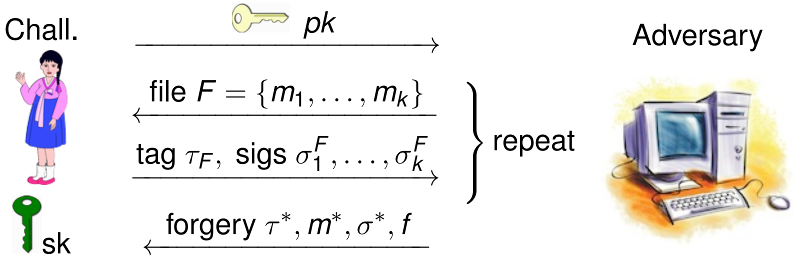

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Our schemes are secure against the stronger adversary.

Previous RSA Construction [GKKR10]

Homomorphic hash: fix public $h_1, \ldots, h_n \in \mathbb{Z}_N^*$;
for vector $\mathbf{v} \in \mathbb{Z}^n$, define

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$$\sigma = (t_i \cdot H_{\text{hom}}(\mathbf{v}_i))^{1/e} \bmod N \qquad (t_i \text{ public}).$$

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Homomorphic: If σ_1, σ_2 are signatures on $\mathbf{v}_1, \mathbf{v}_2$, then

$$\begin{aligned} \sigma_1 \cdot \sigma_2 &= \left(t_1 \cdot H_{\text{hom}}(\mathbf{v}_1) \cdot t_2 \cdot H_{\text{hom}}(\mathbf{v}_2)\right)^{1/e} \\ &= (t_1 t_2 \cdot H_{\text{hom}}(\mathbf{v}_1 + \mathbf{v}_2))^{1/e} \end{aligned}$$

authenticates $\mathbf{v}_1 + \mathbf{v}_2$ for the function $f(x, y) = x + y$.

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authenticates $\mathbf{v}_1 + \mathbf{v}_2$ for the function $f(x, y) = x + y$.

- t_i must be different for each file to prevent mixing.
- Secure if $t_i = R(i, \tau)$ produced by a random oracle.

Removing the Random Oracle

Instead of RSA sigs, use [GHR99]:

$$\text{Sign}(m) = g^{1/H(m)} \bmod N.$$

- g public, H hashes to odd primes.
- secure in standard model under *strong RSA assumption*:
  - Given (g, N), find any $(e, g^{1/e} \bmod N)$.

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Our idea: to sign ith vector $\mathbf{v}_i$ for file τ, compute:

$$\sigma = \Big(\underbrace{g^{1/H(\tau)}}_{\sigma_1}, \underbrace{(t_i \cdot H_{\text{hom}}(\mathbf{v}))^{1/H(\tau)}}_{\sigma_2}\Big) \qquad (t_i \text{ public}).$$

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To verify (σ_1, σ_2) on vector $\mathbf{w}$ for function $f(\vec{x}) = \sum c_i x_i$:

1. Check that $\sigma_1^{H(\tau)} = g$.
2. Check that $\sigma_2^{H(\tau)} = \prod t_i^{c_i} \cdot H_{\text{hom}}(\mathbf{w})$

Homomorphic Property

$$\text{Sign}(\tau, \mathbf{v}_i) \to \Big(\underbrace{g^{1/H(\tau)}}_{\sigma_1}, \underbrace{(t_i \cdot H_{\text{hom}}(\mathbf{v}))^{1/H(\tau)}}_{\sigma_2} \Big).$$

$\text{Verify}(\tau, \mathbf{w}, (\sigma_1, \sigma_2), f)$ with $f(\vec{x}) = \sum c_i x_i$:

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Homomorphic: If $(\sigma_1, \sigma_2), (\sigma_1, \sigma_2')$ are signatures on $\mathbf{v}_1, \mathbf{v}_2$, then

$$\sigma_2 \cdot \sigma_2' = (t_1 t_2 \cdot H_{\text{hom}}(\mathbf{v}_1 + \mathbf{v}_2))^{1/H(\tau)}$$

so $(\sigma_1, \sigma_2 \cdot \sigma_2')$ authenticates $\mathbf{v}_1 + \mathbf{v}_2$ for $f(x, y) = x + y$.

Security

Forgery is a valid signature σ^* on (τ^*, m^*, f) with

$$m^* \neq f(\text{messages in file w/ tag } \tau^*).$$

Two types:

1. τ^* not obtained in response to a query, or
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Type 2 forgery breaks an RSA assumption:

- strong RSA if H is [GHR99] hash function.
- RSA if H is a random oracle.
- RSA if H is [HW09b] hash function.

Proof Sketch for Type 2 Forgery

Consider a *weak* adversary that submits files $F_\ell = \{\mathbf{v}_1^\ell, \ldots, \mathbf{v}_k^\ell\}$ for $\ell = 1, \ldots, q$ and receives pk, tags τ_ℓ, and signatures

$$\sigma = \left(g^{1/H(\tau)}, (t_i \cdot H_{\mathrm{hom}}(\mathbf{v}))^{1/H(\tau)}\right).$$

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1. Given RSA challenge g, choose $\ell^* \stackrel{R}{\leftarrow} \{1, \ldots, q\}$ and set

$$x = g^{\prod_\ell H(\tau_\ell)}, \qquad y = g^{\prod_{\ell \neq \ell^*} H(\tau_\ell)}.$$

   - Simulator can compute $x^{1/H(\tau_\ell)}$ for all ℓ.
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3. Generalize using *homomorphic chameleon hash*.

Generalization to Other Signatures

Construction works for any signatures of the form

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1. [GS02/BB04a/W05]: $\text{sk} = \alpha$, $\text{Sign}(m) = (g^r, g^\alpha H(m)^r)$
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2. [BB04b]: $\mathsf{sk} = \alpha$, $\mathsf{Sign}(m) = g^{1/(m+\alpha)}$.
   - Secure under q-SDH assumption in bilinear group $\mathbb{G}$.

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2. [BB04b]: $\mathsf{sk} = \alpha$, $\mathsf{Sign}(m) = g^{1/(m+\alpha)}$.
   - Secure under q-SDH assumption in bilinear group $\mathbb{G}$.

Generalized homomorphic signature on ith vector $\mathbf{v}_i$ for file τ is

$$\sigma = \left(g^{f(\mathsf{sk},\tau,r)}, \sigma_2, (t_i \cdot H_{\mathsf{hom}}(\mathbf{v}_i))^{f(\mathsf{sk},\tau,r)}\right).$$

For details see full version (IACR eprint 2012/060).

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Thank you!