Problem 1. Suppose that \(v_1, v_2, v_3, v_4 \in V\) are linearly independent. Decide whether

- \(v_1\) and \(v_1 + v_2\) are linearly independent?
- \(v_1, v_1 - v_2, v_2 - v_3, v_3 - v_4\) are linearly independent?
- \(v_1 - v_2, v_2 - v_3, v_3 - v_4, v_4 - v_1\) are linearly independent?

Problem 2. Suppose \(v_1, \ldots, v_k\) are linearly independent and \(w \notin \text{Span}(v_1, \ldots, v_k)\). Prove that then \(v_1 + w, v_2 + w, \ldots, v_k + w\) are also linearly independent.

Problem 3. Find a basis of the following vector space:

\[
U = \{(z_1, \ldots, z_4) \in \mathbb{C}^4 : 2z_1 = z_2, z_3 + iz_4 = 0\}.
\]

Prove that your choice is indeed a basis. Then extend it to a basis of \(\mathbb{C}^4\).

Problem 4. Is there a basis of \(P_3(\mathbb{R})\) consisting of polynomials of degree 3 only?

Problem 5. Suppose \(u_1, \ldots, u_k\) is a basis of \(U\) and \(w_1, \ldots, w_\ell\) is a basis of \(W\). Prove that \(u_1, \ldots, u_k, w_1, \ldots, w_\ell\) is a basis of \(U + W\) if and only if it is a direct sum.

Bonus problem. Prove that the vector space of continuous real-valued functions on \([0,1]\) is not finite-dimensional.