Please try to solve the homework on your own. Discussions are okay but make your own effort.

**Problem 1.** Prove from the Cluster Expansion Lemma (not using Shearer’s lemma!) that
\[ p(1 + ed) \leq 1 \]
is a sufficient condition to avoid all bad events in the symmetric case.

**Problem 2.** True or false? In the Moser-Tardos algorithm, for a fixed \( t > 1 \) and \( i \in [n] \), the probability that the \( t \)-th resampled event is \( E_i \) is at most \( \Pr[E_i] \). Prove this or find a counterexample.

**Problem 3.** Let the dependency graph be a tree \( T \), with (an arbitrarily chosen) root \( r \). Prove that \( p \) satisfies Shearer’s conditions, if and only if there are parameters \( z_v \in (0, 1) \) such that
\[ p_v = z_v \prod_{w \in C(v)} (1 - z_w), \]
where \( C(v) \) are the children of \( v \) (neighbors not on the path to \( r \)).
*Hint:* Find an expression for \( z_v \) in terms of Shearer’s polynomials.

**Problem 4.** Consider a 4-partite graph on \( V_0 \cup V_1 \cup V_2 \cup V_3 \) such that \( e(V_i, V_{i+1 \text{ mod 4}}) \geq \rho |V_i||V_{i+1 \text{ mod 4}}| \) for each \( i \). Prove that if \( \rho \geq 1/\sqrt{2} \), then there must exist \( C_4 \) as a subgraph with one vertex in each \( V_i \).

*Bonus question:* Can you identify the optimal threshold \( \rho \) for this question?