Homework 2
Local Search Heuristics

Problem 2-1. The Maximum Acyclic Subgraph Problem: We are given a *directed* graph $G(V, E)$. The goal is to find the *acyclic* subgraph $G'(V, E')$ (where $E' \subseteq E$) such that $|E'|$ is as large as possible. The vertices of any acyclic subgraph can be ordered on a horizontal line so that all directed edges go from left to right. Therefore, the goal is to find an ordering of the vertices in V so that the number of edges from E which go from left to right is maximized. This problem is therefore also refered to as the Linear Ordering problem.

1. Show a simple greedy 0.5 approximation to this problem that runs in time $O(|E|)$.
2. Consider the local search heuristic that starts with an arbitrary ordering of the vertices, and keeps swapping the positions of two vertices if it increases the number of edges going from left to right. Show that this produces a 0.5 approximation.
3. Consider the local search heuristic that picks one vertex and places it at some other position in the ordering (without changing the relative locations of the other vertices) if this improves the number of edges going from left to right. Show that this also produces a 0.5 approximation.
4. Give examples to show that the performance of the above algorithms cannot be better than 0.5.

Problem 2-2. Show that the local search procedure of switching which side of the cut any vertex belongs to, cannot produce better than a 0.5 approximation for Max-Cut.