LTL Fonts Proof Sheet

An LTL formula φ is defined by the following grammar:

$$
\begin{aligned}
\varphi ::= {} & p \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \vee \varphi \mid \varphi \rightarrow \varphi \mid \varphi \leftrightarrow \varphi \mid \forall x\, \varphi \mid \exists x\, \varphi \\
& \mid \Box\varphi \mid \Diamond\varphi \mid \bigcirc\varphi \mid \varphi\,\mathcal{U}\,\varphi \mid \varphi\,\mathcal{W}\,\varphi \mid \boxminus\varphi \mid \diamondminus\varphi \mid \ominus\varphi \mid \circleddash\varphi \mid \varphi\,\mathcal{S}\,\varphi \mid \varphi\,\mathcal{B}\,\varphi,
\end{aligned}
$$

where p is an assertion and x is a variable.

The truth of an LTL formula φ at position n of an infinite sequence σ of states is denoted $\sigma, n \models \varphi$ and defined, by induction on the structure of φ, as follows:

- $\sigma, n \models p$, for p an assertion, if p holds at state $\sigma[n]$;
- $\sigma, n \models \neg\varphi$ if $\sigma, n \not\models \varphi$;
- $\sigma, n \models \varphi \wedge \psi$ if both $\sigma, n \models \varphi$ and $\sigma, n \models \psi$;
- $\sigma, n \models \varphi \vee \psi$ if either $\sigma, n \models \varphi$ or $\sigma, n \models \psi$, or both;

  $\vdots$

- $\sigma, n \models \Box\varphi$ if $\sigma, i \models \varphi$ for all $i \geq n$;
- $\sigma, n \models \Diamond\varphi$ if there exists $i \geq n$ such that $\sigma, i \models \varphi$;
- $\sigma, n \models \bigcirc\varphi$ if $\sigma, (i+1) \models \varphi$;
- $\sigma, n \models \varphi\,\mathcal{U}\,\psi$ if there exists $i \geq n$ such that $\sigma, i \models \psi$ and $\sigma, j \models \varphi$ for all $j \in [n, j)$;
- $\sigma, n \models \varphi\,\mathcal{W}\,\psi$ if either $\sigma, n \models \varphi\,\mathcal{U}\,\psi$ or $\sigma, n \models \Box\varphi$;
- $\sigma, n \models \boxminus\varphi$ if $\sigma, i \models \varphi$ for all $i \in [0, n]$;
- $\sigma, n \models \diamondminus\varphi$ if there exists $i \in [0, n]$ such that $\sigma, i \models \varphi$;
- $\sigma, n \models \ominus\varphi$ if $n > 0$ and $\sigma, (i-1) \models \varphi$;
- $\sigma, n \models \circleddash\varphi$ if either $n = 0$ or $\sigma, (i-1) \models \varphi$;
- $\sigma, n \models \varphi\,\mathcal{S}\,\psi$ if there exists $i \in [0, n]$ such that $\sigma, i \models \psi$ and $\sigma, j \models \varphi$ for all $j \in (j, n]$;
- $\sigma, n \models \varphi\,\mathcal{B}\,\psi$ if either $\sigma, n \models \varphi\,\mathcal{S}\,\psi$ or $\sigma, n \models \boxminus\varphi$.

The strict versions of the operators are defined as follows:

$$
\begin{aligned}
\widehat{\Box}\varphi &\equiv \bigcirc\Box\varphi & \widehat{\boxminus}\varphi &\equiv \circleddash\boxminus\varphi \\
\widehat{\Diamond}\varphi &\equiv \bigcirc\Diamond\varphi & \widehat{\diamondminus}\varphi &\equiv \ominus\diamondminus\varphi \\
\varphi\,\widehat{\mathcal{U}}\,\psi &\equiv \bigcirc(\varphi\,\mathcal{U}\,\psi) & \varphi\,\widehat{\mathcal{S}}\,\psi &\equiv \ominus(\varphi\,\mathcal{S}\,\psi) \\
\varphi\,\widehat{\mathcal{W}}\,\psi &\equiv \bigcirc(\varphi\,\mathcal{W}\,\psi) & \varphi\,\widehat{\mathcal{B}}\,\psi &\equiv \circleddash(\varphi\,\mathcal{B}\,\psi)
\end{aligned}
$$

Some examples:

$p \rightarrow \Diamond q$	$\Box(p \rightarrow \Diamond q)$	$\Box\Diamond q$
$\Diamond\Box q$	$(\neg q)\,\mathcal{W}\,p$	$\Box(p \rightarrow q\,\widehat{\mathcal{W}}\,r)$
$\Box\exists u\,\big(x = u \wedge \bigcirc(x = u+1)\big)$	$\forall u\,\Box\big(x = u \rightarrow \widehat{\Diamond}(y = u)\big)$	$\exists b.\, b \wedge \Box(b \leftrightarrow \neg\bigcirc b) \wedge \Box(p \rightarrow b)$
$\Box(q \rightarrow \diamondminus p)$	$p \Rightarrow q_m\,\mathcal{W}\,q_{m-1} \ldots q_1\,\mathcal{W}\,q_0$	$p\,\mathcal{U}\,q \sim \Diamond(q \wedge \widehat{\boxminus} p)$

Operator pairs:

$\Box\Box p$	$\Box\Diamond p$	$\Box\bigcirc p$	$\Box(p\,\mathcal{U}\,q)$	$\Box(p\,\mathcal{W}\,q)$	
$\Box\boxminus p$	$\Box\diamondminus p$	$\Box\ominus p$	$\Box\circledtilde p$	$\Box(p\,\mathcal{S}\,q)$	$\Box(p\,\mathcal{B}\,q)$
$\Diamond\Box p$	$\Diamond\Diamond p$	$\Diamond\bigcirc p$	$\Diamond(p\,\mathcal{U}\,q)$	$\Diamond(p\,\mathcal{W}\,q)$	
$\Diamond\boxminus p$	$\Diamond\diamondminus p$	$\Diamond\ominus p$	$\Diamond\circledtilde p$	$\Diamond(p\,\mathcal{S}\,q)$	$\Diamond(p\,\mathcal{B}\,q)$
$\bigcirc\Box p$	$\bigcirc\Diamond p$	$\bigcirc\bigcirc p$	$\bigcirc(p\,\mathcal{U}\,q)$	$\bigcirc(p\,\mathcal{W}\,q)$	
$\bigcirc\boxminus p$	$\bigcirc\diamondminus p$	$\bigcirc\ominus p$	$\bigcirc\circledtilde p$	$\bigcirc(p\,\mathcal{S}\,q)$	$\bigcirc(p\,\mathcal{B}\,q)$
$\Box p\,\mathcal{U}\,\Box q$	$\Diamond p\,\mathcal{U}\,\Diamond q$	$\bigcirc p\,\mathcal{U}\,\bigcirc q$	$(p\,\mathcal{U}\,q)\,\mathcal{U}\,(r\,\mathcal{U}\,s)$	$(p\,\mathcal{W}\,q)\,\mathcal{U}\,(r\,\mathcal{W}\,s)$	
$\boxminus p\,\mathcal{U}\,\boxminus q$	$\diamondminus p\,\mathcal{U}\,\diamondminus q$	$\ominus p\,\mathcal{U}\,\ominus q$	$\circledtilde p\,\mathcal{U}\,\circledtilde q$	$(p\,\mathcal{S}\,q)\,\mathcal{U}\,(r\,\mathcal{S}\,s)$	$(p\,\mathcal{B}\,q)\,\mathcal{U}\,(r\,\mathcal{B}\,s)$
$\Box p\,\mathcal{W}\,\Box q$	$\Diamond p\,\mathcal{W}\,\Diamond q$	$\bigcirc p\,\mathcal{W}\,\bigcirc q$	$(p\,\mathcal{U}\,q)\,\mathcal{W}\,(r\,\mathcal{U}\,s)$	$(p\,\mathcal{W}\,q)\,\mathcal{W}\,(r\,\mathcal{W}\,s)$	
$\boxminus p\,\mathcal{W}\,\boxminus q$	$\diamondminus p\,\mathcal{W}\,\diamondminus q$	$\ominus p\,\mathcal{W}\,\ominus q$	$\circledtilde p\,\mathcal{W}\,\circledtilde q$	$(p\,\mathcal{S}\,q)\,\mathcal{W}\,(r\,\mathcal{S}\,s)$	$(p\,\mathcal{B}\,q)\,\mathcal{W}\,(r\,\mathcal{B}\,s)$
$\boxminus\Box p$	$\boxminus\Diamond p$	$\boxminus\bigcirc p$	$\boxminus(p\,\mathcal{U}\,q)$	$\boxminus(p\,\mathcal{W}\,q)$	
$\boxminus\boxminus p$	$\boxminus\diamondminus p$	$\boxminus\ominus p$	$\boxminus\circledtilde p$	$\boxminus(p\,\mathcal{S}\,q)$	$\boxminus(p\,\mathcal{B}\,q)$
$\diamondminus\Box p$	$\diamondminus\Diamond p$	$\diamondminus\bigcirc p$	$\diamondminus(p\,\mathcal{U}\,q)$	$\diamondminus(p\,\mathcal{W}\,q)$	
$\diamondminus\boxminus p$	$\diamondminus\diamondminus p$	$\diamondminus\ominus p$	$\diamondminus\circledtilde p$	$\diamondminus(p\,\mathcal{S}\,q)$	$\diamondminus(p\,\mathcal{B}\,q)$
$\ominus\Box p$	$\ominus\Diamond p$	$\ominus\bigcirc p$	$\ominus(p\,\mathcal{U}\,q)$	$\ominus(p\,\mathcal{W}\,q)$	
$\ominus\boxminus p$	$\ominus\diamondminus p$	$\ominus\ominus p$	$\ominus\circledtilde p$	$\ominus(p\,\mathcal{S}\,q)$	$\ominus(p\,\mathcal{B}\,q)$
$\circledtilde\Box p$	$\circledtilde\Diamond p$	$\circledtilde\bigcirc p$	$\circledtilde(p\,\mathcal{U}\,q)$	$\circledtilde(p\,\mathcal{W}\,q)$	
$\circledtilde\boxminus p$	$\circledtilde\diamondminus p$	$\circledtilde\ominus p$	$\circledtilde\circledtilde p$	$\circledtilde(p\,\mathcal{S}\,q)$	$\circledtilde(p\,\mathcal{B}\,q)$
$\Box p\,\mathcal{S}\,\Box q$	$\Diamond p\,\mathcal{S}\,\Diamond q$	$\bigcirc p\,\mathcal{S}\,\bigcirc q$	$(p\,\mathcal{U}\,q)\,\mathcal{S}\,(r\,\mathcal{U}\,s)$	$(p\,\mathcal{W}\,q)\,\mathcal{S}\,(r\,\mathcal{W}\,s)$	
$\boxminus p\,\mathcal{S}\,\boxminus q$	$\diamondminus p\,\mathcal{S}\,\diamondminus q$	$\ominus p\,\mathcal{S}\,\ominus q$	$\circledtilde p\,\mathcal{S}\,\circledtilde q$	$(p\,\mathcal{S}\,q)\,\mathcal{S}\,(r\,\mathcal{S}\,s)$	$(p\,\mathcal{B}\,q)\,\mathcal{S}\,(r\,\mathcal{B}\,s)$
$\Box p\,\mathcal{B}\,\Box q$	$\Diamond p\,\mathcal{B}\,\Diamond q$	$\bigcirc p\,\mathcal{B}\,\bigcirc q$	$(p\,\mathcal{U}\,q)\,\mathcal{B}\,(r\,\mathcal{U}\,s)$	$(p\,\mathcal{W}\,q)\,\mathcal{B}\,(r\,\mathcal{W}\,s)$	
$\boxminus p\,\mathcal{B}\,\boxminus q$	$\diamondminus p\,\mathcal{B}\,\diamondminus q$	$\ominus p\,\mathcal{B}\,\ominus q$	$\circledtilde p\,\mathcal{B}\,\circledtilde q$	$(p\,\mathcal{S}\,q)\,\mathcal{B}\,(r\,\mathcal{S}\,s)$	$(p\,\mathcal{B}\,q)\,\mathcal{B}\,(r\,\mathcal{B}\,s)$

Upper case versions of the font (right column):

$\Box P \rightarrow \bigcirc\Diamond Q$	$\Box P \rightarrow \bigcirc\Diamond Q$
$\Box\exists u\,\big(x = u \wedge \bigcirc(x = u+1)\big)$	$\Box\exists u\,\big(x = u \wedge \bigcirc(x = u+1)\big)$