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BOOK REVIEWS

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Tree Network and Planar Rectilinear Location Theory. *By A. J. W. Kolen.* Stichting Mathematisch Centrum, Amsterdam, the Netherlands, 1986. viii + 85 pp. 12.50 Dfl. ISBN 90-6196-300-1. Centre for Mathematics and Computer Science, CWI Tract 25.

The conceptual problem considered in location theory is to decide the optimal configuration of placing supply objects (facilities) in a metric space given with demand objects (where users are located). Concrete problems are defined by specifying the space, objects, optimality criteria, constraints, etc. The book considers location theory from the points of view of combinatorial optimization and computational complexity. Two fields of location theory are discussed: location on graphs (concentrating on tree-graphs) and location in the plane (concentrating on the ℓ_1 metric). There is a distinction between "center" problems and "median" problems. In the former, we are interested in minimizing the usage cost incurred by the least favored user, while in the latter the efficiency is measured by the average usage cost per user. Another cost is related to setting up the supplies. This cost usually does not affect the objective function but is subject to a budget constraint.

Multifacility problems on general graphs are usually NP-hard. There has been a lot of work done on tree-graphs, where most of these problems have polynomial-time algorithms. The author studies properties of trees from which duality results can be derived with respect to covering problems. For example, subtrees of a tree are similar to intervals on the real line in the sense that they have the following Helly property: if every two members of a family of subtrees intersect, then the entire family has a nonempty intersection. It then follows that the intersection graph of subtrees of a tree is chordal, that is, every cycle of length four or more has an edge connecting two vertices not adjacent in the cycle.

In Chapter 1 the author shows how certain duality properties can be useful for designing algorithms. For example, they sometimes tell us the structure of a set which is guaranteed to contain the optimal value. In such a case we can search the set, probing candidate values by solving a dual problem. Consider, for example, the p-center problem. The input consists of a graph with edge-lengths d_{ij} and vertex weights w_i. We have to find a set of p points $x_k (k = 1, \cdots, p)$ on the edges of the graph, so as to minimize the maximum weighted distance $w_i d(i, x_k)$ between any vertex i and its respective nearest point x_k. Here the optimal value of the problem has the form $w_i w_j d_{ij}/(w_i + w_j)$. On the other hand, we have the following covering problem: given a radius r, find the minimum number q such that there exist q points $x_k (k = 1, \cdots, q)$ with property that for every vertex i there is a point x_k at a distance not greater than r/w_i from i. If the covering problem can be solved efficiently, then the p-center problem can also be solved efficiently using an algorithm for the covering problem to perform a search for the optimal value. At least in the case of a tree this approach has led to fast algorithms.

In Chapter 2 the author considers problems with different setup costs. The simple plant location problem on a graph is the following: given a graph with edge-lengths d_{ij}, vertex-weights w_i and vertex-costs c_j, find a set of vertices $j_1, \cdots, j_k$ so as to minimize the sum of the setup costs $\sum_\nu c_{j_\nu}$ plus the weighted sum $\sum_i w_i d_{ij_\nu}$ of transportation costs from each vertex i to the respective nearest vertex j_ν. This problem can be formulated as a set-covering problem, that is, given a matrix A of zeros and ones, find a (0, 1)-vector x with a minimum number of ones such that $Ax \geqq e$ (e denotes a vector of ones). The interesting case is when the latter can be solved efficiently. Here the author introduces a class of set-covering problems that have a "standard greedy form," that is, A does not contain the matrix $\binom{11}{10}$ as a submatrix. This class can be solved by the greedy algorithm. In the application to location problems on trees we have to permute the rows and columns first to bring the matrix into standard greedy form.

In Chapters 3 and 4 the author discusses location problems with interfacility communication costs. For example, in the problem of p-median with mutual communication there exist costs associated with the distances between facilities and we have to find locations so as to minimize the total of a weighted sum of these distances plus a weighted sum of the distances clients have to travel to the respective nearest facility. This problem also is studied on trees and in the plane with the ℓ_1 metric.

In Chapter 5 the author suggests how Farkas' Lemma on linear inequalities can be used in designing algorithms for rectilinear location problems. The lemma, however, seems a bit too powerful for the problems discussed. Chapter 6 introduces the notion of a totally balanced matrix. This is a matrix of zeros and ones that does not have a square submatrix B such that B has no identical columns and its rows and columns sums are equal to two. It is shown that the totally balanced matrices are precisely those matrices that can be permuted into matrices in standard greedy form.

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