Lecture 3: (Hyper)Properties, Robustness and Property-Preserving Compilers

CS350

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Properties and Hyperproperties

- Formalise any security property
- Established theory with practical applications

Recommended reading:

- Schneider. 2000. Enforceable security policies.
- Alpern and Schneider. 1985. Defining liveness.
- Clarkson and Schneider. 2010. Hyperproperties.

Security Properties

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    - component-context interactions $\alpha?\alpha!\cdots$
    - code-environment interaction $read\ v; write\ v$

We use t abstractly now, though mostly: $t = \overline{\Theta}$

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This is unlike program equivalence:

- properties talk a single program

Examples

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  NRW: *the program does not send on the network after reading a file*
  $\vdash read\Theta$ and $\vdash send\Theta'$ are abstract predicates
- GS: $\{t \mid \vdash req\Theta_i \Rightarrow \vdash resp\Theta_j$ where $j > i\}$
  GS: *the program eventually responds to the requests*

Safety and Liveness

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- but, Safety = weak secrecy: we don't leak a fresh k to $\mathfrak{C}$

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- Describe safety by the so-called set of bad prefixes
- In the following: m is a finite trace t (a finite $\overline{\Theta}$) aka a prefix
- NRW-dual:
  $\{m \mid \Theta < \Theta'. \vdash read\Theta \wedge \vdash send\Theta'\}$

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- Properties = sets of traces
- capture a single run (the trace) of any program (the set)
  Hyperproperties = $\mathbf{sets}$ of $sets\ of\ traces$
- capture multiple runs (the $sets\ of\ traces$) of any program (the $\mathbf{sets}$)

Example: NonInterference

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- NI: two different high inputs result in the same low outputs
- high = secret, low = public
- a set of traces tells all the behaviours of the same program with different high inputs

NI :

$$\left\{ \{t_1, t_2\} \;\middle|\; \begin{array}{l} \forall t_1, t_2 \in \{t_1, t_2\}. \\ \quad \text{if } \mathtt{inputs}(t_1) =_L \mathtt{inputs}(t_2) \\ \quad \text{then } \mathtt{outputs}(t_1) =_L \mathtt{outputs}(t_2) \end{array} \right\}$$

Example: Average Response Time < 1

$$\text{ART}: \left\{ \{t \cdots\} \;\middle|\; \texttt{mean}\left(\bigcup_{t \in \{t \cdots\}} \texttt{response\_time}(t) \right) < 1 \right\}$$

where $\texttt{response\_time}(\cdot)$ looks in trace t and checks time between $\texttt{req}(\cdot)$ and $\texttt{resp}(\cdot)$

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- $\vdash P : \pi \overset{\text{def}}{=}$ if $P \leadsto t$ then $t \in \pi$

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- Hyperproperty $H = \{\{t\}\}$
- $\vdash P : H \stackrel{\text{def}}{=} \texttt{Behav}(P) \in H$

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So we want our program P to satisfy NRW, GS, NI or ART: $\forall \mathfrak{C}. \mathfrak{C}[P]$, so $\Theta = \mathfrak{C}[P]$

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Reminiscent of contextual equivalence!

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- $\vdash_R P : H \overset{\text{def}}{=} \forall \mathfrak{C}.\texttt{Behav}(\mathfrak{C}[P]) \in H$

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- so they can trivially invalidate any property
- we must filter events and consider only those generated by P

Example: Robust Safety

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- $\vdash_R P : \pi \stackrel{\text{def}}{=} \forall \mathfrak{C}.$ if $\mathfrak{C}[P] \rightsquigarrow t$ then $t \in \pi$

Example: Robust Safety

- $\pi \in \mathit{Safety}$
- $\vdash_R P : \pi \stackrel{\text{def}}{=} \forall \mathfrak{C}.$ if $\mathfrak{C}[P] \rightsquigarrow t$ then $t \in \pi$
- dually: $\{m\} :: \pi \in \mathit{Safety}$
- $m \leq t$ = m is a prefix of t
- $\vdash_R P : \{m\} \stackrel{\text{def}}{=} \forall \mathfrak{C}.$ if $\mathfrak{C}[P] \rightsquigarrow t$ then $\nexists m \in \{m\}.m \leq t$

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- can this hold robustly?
- we need a fair context in our setup: a context that will interact with us
- avoid DOS: the attacker wants to violate our code, not starve it

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Q: can we preserve them through compilation?

Yes!

Robust Compilation

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2. s

Q: ca n?

Assumptions:

- same alphabet of traces between S and T (I/O or syscalls)
- we lift this (partially) later

Example: Robust Property Preservation

- Assume the source has a property robustly

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$$RTP : \forall \pi.\ \forall \mathsf{P}.\ (\forall \mathfrak{C}\ t.\ \mathfrak{C}[\mathsf{P}] \rightsquigarrow t \Rightarrow t \in \pi) \Rightarrow (\forall \mathfrak{C}\ t.\ \mathfrak{C}[[\![\mathsf{P}]\!]] \rightsquigarrow t \Rightarrow t \in \pi)$$

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\begin{aligned}
RSP : & \forall \pi \in \mathit{Safety}.\ \forall \mathsf{P}. \\
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We want equivalent criteria that are easy to prove

Example: Robust Property Preservation #2

$$RTP : \forall \pi.\ \forall \mathsf{P}.\ (\forall \mathfrak{C}\ t.\ \mathfrak{C}[\mathsf{P}] \rightsquigarrow t \Rightarrow t \in \pi) \Rightarrow$$
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$$PFRTP : \forall \mathsf{P}.\ \forall \mathfrak{C}.\ \forall t.\ \ \mathfrak{C}[[\![\mathsf{P}]\!]] \rightsquigarrow t \Rightarrow \exists\ \mathfrak{C}.\ \mathfrak{C}[\mathsf{P}] \rightsquigarrow t$$

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Intuition

If any trace in the target is also done in the source, and the source has the property, so does the target.

$$\exists\ \mathfrak{C}.\ \mathfrak{C}[\mathsf{P}] \rightsquigarrow t$$

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Example: Robust Safety Preservation #2

Intuition

Safety is defined dually as a set of bad prefixes

If any prefix done in the target is also done in the source and the source has the safety property, that prefix is not bad, so the target also has the safety property

$\exists \mathfrak{C}. \mathfrak{C}[\mathrm{P}] \rightsquigarrow m$

Relating RTP and RSP

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$$RHP : \forall H.\ \forall \mathsf{P}.\ (\forall \mathfrak{C}.\, \mathtt{Behav}\,(\mathfrak{C}[\mathsf{P}]) \in H) \Rightarrow$$
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$$PFRHP : \forall \mathsf{P}.\ \forall \mathfrak{C}.\ \exists \mathfrak{C}.\ \mathtt{Behav}\,(\mathfrak{C}[\llbracket \mathsf{P} \rrbracket]) = \mathtt{Behav}\,(\mathfrak{C}[\mathsf{P}])$$
$$PFRHP : \forall \mathsf{P}.\ \forall \mathfrak{C}.\ \exists \mathfrak{C}.\ \forall t.\ \mathfrak{C}[\llbracket \mathsf{P} \rrbracket] \rightsquigarrow t \iff \mathfrak{C}[\mathsf{P}] \rightsquigarrow t$$

Spot the 2 Differences

$$PFRTP : \forall \mathsf{P}.\ \forall \mathfrak{C}.\ \forall t.\ \ \mathfrak{C}[\llbracket \mathsf{P} \rrbracket] \rightsquigarrow t \Rightarrow \exists\, \mathfrak{C}.\, \mathfrak{C}[\mathsf{P}] \rightsquigarrow t$$

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- Quantifier ordering

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- Quantifier ordering
- Implication

Spot the 2 Differences

Intuition

- Quantifier ordering: lifts to sets of traces since a $\mathfrak{C}$ in PFRHP works for a set of traces
- Implication: a single implication means refinement, so the target can have more behaviours. Co-implicaiton means no refinement, we need the exact same traces to ensure inclusion in the H

Example: Robust Hypersafety Preservation

$$PFRHSP: \ \forall \mathsf{P}.\ \forall \mathfrak{C}.\ \forall \{m\}.$$

$$\{m\} \leq \texttt{Behav}\left(\mathfrak{C}\left[\llbracket \mathsf{P} \rrbracket\right]\right) \Rightarrow \exists \mathfrak{C}.\ \{m\} \leq \texttt{Behav}\left(\mathfrak{C}\left[\mathsf{P}\right]\right)$$

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Where $\leq$ means *all* prefixes of $\{m\}$ are extended by the behaviour of the (compiled) program

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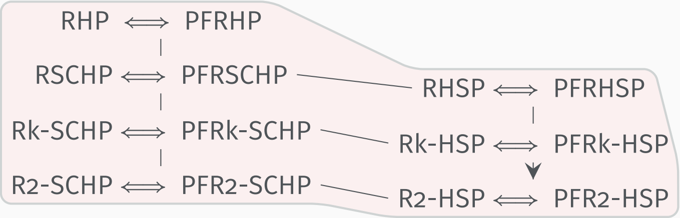
- Hyperliveness: not present: RHLP collapses with RHP

Robust Compilation (RC) Diagram



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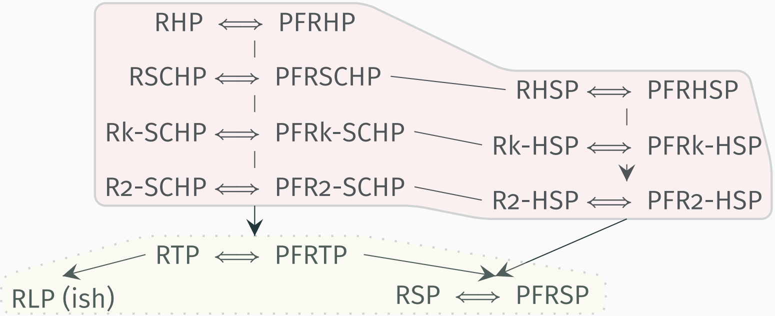

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- FAC is only relational
- both are robust
- FAC is only as precise as the equivalence
- RC do not preserve abstractions beyond the related security (hyper)property

Proving RC

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Recall $\Rightarrow$ for FAC (contrapositive):

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Recall $\Rightarrow$ for FAC (contrapositive):

$\forall \mathsf{P}_1, \mathsf{P}_2$

$$\exists \mathfrak{C}. \mathfrak{C}[[\![\mathsf{P}_1]\!]]\Uparrow \not\Longleftrightarrow \mathfrak{C}[[\![\mathsf{P}_2]\!]] \Rightarrow \exists \mathfrak{C}. \mathfrak{C}[\mathsf{P}_1]\Uparrow \not\Longleftrightarrow \mathfrak{C}[\mathsf{P}_2]\Uparrow$$

Proving RC

Backtranslation!

- generate a $\mathfrak{C}$ starting from what we have

Reca

$\forall P_1,$

$\exists \mathfrak{C}.\mathfrak{C}$ ... $[P_2]\Uparrow$

Proving RC

Backtranslation!

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- $\mathfrak{C}$, t for PFRTP

Reca

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$\exists \mathfrak{C}. \mathfrak{C}$ … $[P_2]\Uparrow$

Proving RC

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Backtranslation!

- generate a $\mathfrak{C}$ starting from what we have
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- $\mathfrak{C}$, m for PFRSP

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- $\mathfrak{C}$, {m} for PFRHSP

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  - can be precise BT
  - or approximate BT (intuitively analogous to trace-based BT)
- BT is not the inverse of compilation

Reca

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Conclusion

We have seen:

- Properties and Hyperproperties: to formalise a program having a securty property
- Robust compilation criteria, which preserve classes of (hyper)properties
- Backtranslation-equivalent Robust compilation criteria

Suggested Reading

- Approximate CBT: will be presented in *Fully-Abstract Compilation by Approximate Back-Translation*
- Precise CBT: will be presented in *Fully Abstract Compilation via Universal Embedding*
- RC: Abate *et al.*. *Journey Beyond Full Abstraction* (formerly: *Exploring Robust Property Preservation for Secure Compilation*)