CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2000

Assignment #5 - Due date: Wednesday, 5/17/00

CS 154N STUDENTS: I assume you have started attending the class by now. On this homework, you are **not** required to solve problems 1 and 3.

READING ASSIGNMENT: We are almost done with Chapter 8 in the reader. Although Section 8.5 was not covered in the class, we strongly recommend that you read this too. We will be covering Chapter 9 in the next week or two. (In the Hopcroft-Ullman textbook, these correspond to Chapters 7 and 8.)

Problem Set

Problem 1. *[20 points]*
Using the pumping lemma for context-free languages, prove that the following language is not context-free.

$$L = \{0^i 1^j \mid j = i^2\}$$

Problem 2. *[20 points]* Consider the following operation on languages:

$$\text{INIT}(L) = \{x \mid \text{for some } y, \text{ the string } xy \text{ is in } L\}.$$

Prove that context-free languages are closed under the INIT operation. (**Remarks:** There are several ways to establish such results: a PDA construction, a grammar construction, or using known closure properties. We ask that you describe your construction in complete detail but require only a brief justification of the correctness of your construction.)

Problem 3. *[20 points]* Consider the (deterministic) Turing machine M given by

$$M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, B\}, \delta, q_0, B, \{q_2\})$$

which has exactly four transitions defined in it, as described below.

1. $\delta(q_0, a) = (q_0, B, R)$
2. $\delta(q_0, b) = (q_1, B, R)$
3. $\delta(q_1, b) = (q_1, B, R)$
4. $\delta(q_1, B) = (q_2, B, R)$

(a). Specify the execution trace of M on the two input strings abb and bb, i.e., provide the complete sequence of ID's for both inputs.
(b). Provide a regular expression for the language of this Turing machine.
(c). Suppose we added the following transition to the above machine.

$$\delta(q_1, a) = (q_0, B, R)$$

Provide a regular expression for the language of the resulting Turing machine.

Problem 4. *[20 points]* Let M be a non-deterministic Turing machine with two tapes which has the following restrictions on its transitions and the way it can use its tapes. The first tape is a *read-only* tape and its head can never move left, that is the head can either be stationary or move right during each transition. The second tape has the restriction that if the machine moves the tape head to the left, then it must write a blank B before moving left. Initially, the first tape contains the input w followed by blanks and the head is positioned at the left-most tape cell; the second tape is completely blank initially, and its head is positioned at the left-most tape cell.

Characterize the class of languages which can be accepted by such Turing machines and provide a brief justification for your answer.

Problem 5. *[20 points]* Consider an NTM N whose tape is 2-way infinite, i.e., infinite in both directions. Initially, the tape is completely blank except for one special tape cell which contains the \$ symbol; also, the tape head is scanning an arbitrary blank cell.
(a). Provide the transitions which will enable N to halt with its head scanning the \$ symbol. (Since N is an NTM, it is sufficient if there is some sequence of transitions which achieves this.)
(b). Explain briefly how you would modify your answer to (a) if the TM was restricted to being deterministic.

Suggested Exercise 1. We suggest you try to solve Exercise 6.5(b) on page 142 of Hopcroft-Ullman after looking at the solution to Exercise 6.5(a) on page 144. This is **not** a part of this homework.