CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2009

Assignment #4 - Due date: Tuesday, 5/12/09

Problem 1. [15 points] Recall that a normal PDA removes the top symbol from the stack just before each transition, although of course it can push this symbol back onto the stack if desired. Define a "push-back" PDA (PB-PDA) to be a PDA which during each transitions is *required* to push back the current stack-top symbol before making the transition. (Or, equivalently, it never pops the stack-top symbol in the first place.)

What is the class of languages that can be accepted as a *final state language* by PB-PDAs? Justify your answer.

Problem 2. [25 points]
Consider the PDA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, F)$ where $Q = \{q_0, q_1, q_2\}$, $\Sigma = \{a, b\}$, $\Gamma = \{Z_0, A, B\}$, $F = \emptyset$, and δ defined as follows:

$$\begin{aligned}
\delta(q_0, a, Z_0) &= \{(q_1, BBZ_0)\} \\
\delta(q_0, b, Z_0) &= \{(q_2, AAZ_0)\} \\
\delta(q_1, \epsilon, Z_0) &= \{(q_0, \epsilon)\} \\
\delta(q_1, b, B) &= \{(q_1, \epsilon)\} \\
\delta(q_1, a, Z_0) &= \{(q_1, BBZ_0)\} \\
\delta(q_1, b, Z_0) &= \{(q_2, AAZ_0)\} \\
\delta(q_2, \epsilon, Z_0) &= \{(q_0, \epsilon)\} \\
\delta(q_2, a, A) &= \{(q_2, \epsilon)\} \\
\delta(q_2, a, Z_0) &= \{(q_1, BBZ_0)\} \\
\delta(q_2, b, Z_0) &= \{(q_2, AAZ_0)\}
\end{aligned}$$

(a). [5 points] Give an execution trace (using instantaneous descriptions) of the PDA M showing that input string *abbbaa* is in $N(M)$.
(b). [10 points] Describe the *empty stack* language $N(M)$ for this machine.
(c). [10 points] Suppose we were to make q_0 the only final state. How would the resulting *final state* language $L(M)$ differ from your answer to part (b) above?

Problem 3. [20 points] Show that the following language is not context-free.

$$L = \{a^n b^m a^n b^m \mid n, m \geq 0\}$$

Problem 4. [20 points] Solve Exercise 7.2.1(c) on page 286 of the text book.

Problem 5. [20 points] Solve Exercise 7.4.1(b) on page 307 of the textbook.

Reading Assignment: You should finish reading Chapters 6 and 7. Next week we are moving on to Chapter 8.