CS 154 - Introduction to Automata and Complexity Theory

Spring Quarter, 2009

Information on Midterm Exam

Midterm Exam. The midterm examination will be held from 3:15–4:30 pm on Thursday, April 30 at Hewlett 200 (TCSEQ 200).

> **Please note that the exam will be held in Hewlett 200 (TCSEQ 200) and not in the regular lecture room.**

You need to be present and seated by 3:10 pm so that we can distribute the exam in an orderly fashion. At exactly 4:30 pm, when the TAs announce that the time is up, everyone should stop writing. Those who continue writing will be *severely* penalized.

The exam will cover the material presented in class up to Tuesday, April 28. The exam will be open-book and open-notes, i.e., you will be allowed to consult any of the class handouts, your notes, and the textbook. You are not permitted to refer to any other source during the exam. The TAs will be present outside the room to provide any clarifications on the exam questions.

SCPD Students. If you are a remote SCPD student, we strongly recommend that you come to campus for the exam. Of course, we will also send the exam to you via SCPD. In case the exam hasn't arrived at your site a couple of hours before the designated time, please send us email.

Midterm Review Session. The discussion section on Friday May 1 and Monday May 4 are cancelled. Instead we will be holding a midterm review session on Wednesday April 29:

- 11:00-11:50am, Wednesday, April 29, Skilling 193 [on TV]

Special Office Hours . In addition to the usual office hours, the TAs will hold special office hours to help you prepare for the midterm. These are summarized below:

Wed, Apr 29: 9:00–12:00am [Kostas, Gates B26B]]

Wed, Apr 29: 12:00–1:00pm [Gary, Gates B26A]

Wed, Apr 29: 4:15–5:15pm [Gary, Gates B26A]

Thu, Apr 30: 12:00–3:00pm [Nelson, Gates 459]

Please note that the office hours for Friday May 1, Monday May 4, and Tuesday May 5 are cancelled.

Solutions for Homework 3. First, note that we will not be assigning any homework on Tuesday April 28 to give you time to prepare for the midterm. We will be handing out solutions for Homeworks 1/2 before the midterm. Unfortunately, we cannot handout solutions for Homework 3 prior to the midterm given that some people may be exercising their late credit till Friday May 1.

SAMPLE MIDTERM EXAM

Instructions: Answer all questions in the space provided. You have a total of 75 minutes and the total number of points is 150, so this should give you some idea of how much time to spend on each question. We recommend that if you cannot solve a particular problem, move on to the next one. It is important that you be brief in your answers and ensure that it fits in the space provided. *You will lose points for a complicated solution, even if it is correct.*

Problem 1. [30 points] Decide if the following statements about languages over $\{0, 1\}$ are TRUE or FALSE. *You must give a brief explanation of your answer to receive full credit.*

(a). The class of ϵ-NFAs with only one final state can accept all possible regular languages.
(b). The language of an NFA with only one state must be finite.
(c). The intersection of a non-regular language and a regular language cannot be regular.
(d). If L^* is regular, then L must be regular too.
(e). If L_1 and L_2 are both non-regular languages, then $L_1 \cup L_2$ must also be non-regular.

Solution: **(a).** TRUE – Follows from our construction of ϵ-NFAs from regular expressions, which produced only ϵ-NFAs with one final state.
(b). FALSE – The infinite language Σ^* has an NFA with only one state, which is both the start state and final state, with self-loops on all symbols from the alphabet.
(c). FALSE – Take the regular language to be the empty language. Its intersection with any language is also empty, and hence regular.
(d). FALSE – For any non-regular language L containing 0 and 1, it must be the case that $L^* = \Sigma^*$, which is regular. A concrete example is the language L_{pal} containing all palindromes.
(e). FALSE – Take any non-regular language L_1 and take L_2 to be its complement. Then their union is Σ^* which is regular.

Problem 2. [20 points] Consider the following language over the alphabet $\Sigma = \{0, 1\}$.

$$L = \{0^i 1^j \mid i \leq j \leq 2i \text{ and } i \geq 0\}$$

This is the set of strings where all the 0's come before all the 1's, and the number of 1's is at least the number of 0's but no more than *twice* the number of 0's.
Is the language $L(G)$ regular? Prove your answer.

Solution: We show that L is not regular using the pumping lemma. Assume that L is regular. By the pumping lemma, we get a constant n. We choose $w = 0^n 1^{2n}$ which is clearly in L and of length at least n. Therefore, by the pumping lemma, we get a decomposition of w into xyz. Since $|xy| \leq n$ and $|y| > 0$, it must be that xy contains only 0's. Let $|x| = a$ and $|y| = b$. Now, we have that $x = 0^a$, $y = 0^b$, and $z = 0^{n-a-b}1^{2n}$. Choosing $k = 0$, we get that

$$xy^k z = xz = 0^{n-b}1^{2n}.$$

Since $2n > 2(n - b)$, it cannot be the case that $xz \in L$ since it has more than twice as many 1's as 0's. This gives a contradiction. Thus, L is not regular.

Problem 3. [40 points] Consider the following two languages

$$L_1 = \{a^r b^s \mid r, s \geq 0 \text{ and } s = r^2\}$$

and

$$L_2 = \{a^r b^s \mid r, s \geq 0 \text{ and } s \neq r^2\}.$$

a). [20 points] Show that the language L_1 is not regular using the Pumping Lemma.

Solution: We assume that the language L_1 is regular and apply the Pumping Lemma. Given the constant n, we select the string $w = a^n b^{n^2}$; clearly, $w \in L_1$. Now, we are guaranteed a decomposition $w = xyz$ with $|xy| \leq n$ and $|y| > 0$. It must be the case that x and y contain only the symbol a and, for some $i \geq 0$ and $j \geq 1$, that $x = a^i$, $y = a^j$, and $z = a^{n-i-j} b^{n^2}$. Therefore, the string $xy^k z = a^{n+(k-1)j} b^{n^2}$ must belong to L_1 for all choices of k. Picking any value of k other than 1, we get a contradiction since the resulting string is not of the form $a^r b^{r^2}$ and therefore cannot belong to L_1. It follows that L_1 is not regular.

b). [20 points] Now, prove that the language

$$L_2 = \{a^r b^s \mid r, s \geq 0 \text{ and } s \neq r^2\}.$$

is not regular. (*Hint:* Use the fact that L_1 is non-regular and employ the closure properties for regular languages.)

Solution: Suppose that the language L_2 is regular. The language L_1 can be expressed in terms of the two regular languages L_2 and the language of the regular expression a^*b^*, as follows $L_1 = \overline{L_2} \cap L(a^*b^*)$. Since regular languages are closed under complement and intersection, it must be the case that L_1 is regular. But this contradicts the claim in part (a), implying that it cannot be the case that L_2 is regular.

Problem 4. [30 points] Consider the following two context-free languages over $\Sigma = \{0, 1, 2\}$.

$$L_1 = \{2^m 0^n 1^n \mid m, n \geq 0\} \qquad L_2 = \{0^n 1^n 2^m \mid m, n \geq 0\}$$

For L_1 we provide the grammar $G_1 = (V_1, T, P_1, S_1)$ with $T = \{0, 1, 2\}$, $V_1 = \{S_1, A, B\}$, and productions P_1 as below. For L_2 we provide the grammar $G_2 = (V_2, T, P_2, S_2)$ with $T = \{0, 1, 2\}$, $V_2 = \{S_2, C, D\}$, and the productions P_2 as below.

Productions P_1	Productions P_2
$S_1 \rightarrow AB$	$S_2 \rightarrow CD$
$A \rightarrow \epsilon \mid 2A$	$C \rightarrow \epsilon \mid 0C1$
$B \rightarrow \epsilon \mid 0B1$	$D \rightarrow \epsilon \mid D2$

a). **[5 points]** Consider the context-free grammar $G_3 = (V_3, T, P_3, S_3)$ with $T = \{0, 1, 2\}$, $V_3 = \{S_3, S_1, A, B, S_2, C, D\}$, and the productions: $P = P_1 \cup P_2 \cup \{S_3 \rightarrow S_1 \mid S_2\}$. Express the language $L_3 = L(G_3)$ in terms of L_1 and L_2.
b). **[10 points]** Show that the language L_3 is ambiguous.
c). **[15 points]** Provide a non-ambiguous grammar for L_3.

Solution:

a). The correct answer is: $L_3 = L_1 \cup L_2$. You will not get any points if you do not explicitly use L_1 and L_2 in your answer.

b). The simplest solution is to show that the string ϵ has two leftmost derivations:

$$S \Rightarrow S_1 \Rightarrow AB \Rightarrow B \Rightarrow \epsilon$$

$$S \Rightarrow S_2 \Rightarrow CD \Rightarrow D \Rightarrow \epsilon$$

There are several other ways to see this too, e.g., the derivation of strings of the form $0^n 1^n$ or the strings of the form 2^m, both which lie in L_3.

c).

$$\begin{aligned} S &\rightarrow \epsilon \mid A \mid B \mid AB \mid BA \\ A &\rightarrow 2 \mid 2A \\ B &\rightarrow 01 \mid 0B1 \end{aligned}$$

Problem 5. [30 points] Consider the language $L = \{a^i b a^{2i} \mid i \geq 0\}$. You are to construct an NPDA $M = (Q, \Sigma, \Gamma, \delta, q_0, Z_0, \phi)$ such that $N(M) = L$; note, we are looking for the **empty stack language** here. The starting point for your construction should be the following two transitions:

1. $\delta(q_0, a, Z_0) = (q_0, XX)$.
2. $\delta(q_0, a, X) = (q_0, XXX)$.

(a). [20 points] Give all *additional* transitions needed to completely define the machine M.

(b). [10 points] Provide a context-free grammar $G = (V, T, P, S)$ such that $L(G) = L$.

Solution:

(a).

$$\delta(q_0, b, Z_0) = \{(q_1, \epsilon)$$

$$\delta(q_0, b, X) = \{(q_1, X)$$

$$\delta(q_1, a, X) = \{(q_1, \epsilon)$$

(b).

$$S \rightarrow aSaa \mid b$$