CS 154: Introduction to Automata and Complexity Theory
Spring 2009

Assignment 2 Solutions
April 21, 2009

Problem 1

(a) Exercise 3.2.6(c): The set of prefixes of strings in L.

(b) Exercise 3.2.6(d): The set of substrings of strings in L.

Problem 2

(a) (10 + 0)* (ϵ + 1 + 11) (01 + 0)* - the first and third sub-expressions ensure that there are no consecutive 1s, while the second allows for the presence of a single pair of consecutive 1s.

(b) (1)* (0 + 111*)* (1)* - while the expression can begin and end with any number of 1s, a 0 must be followed by either zero 1s or at least two 1s before another 0.

Problem 3

(a) $R_{33}^{(0)} = a + b + c + \epsilon$
$R_{12}^{(0)} = a$
$R_{12}^{(4)} = a(ba)^*$
$R_{11}^{(0)} = \epsilon$
$R_{11}^{(4)} = (ab)^*$

(b) $(ab)^*c$

Problem 4

Assume L is regular and apply the Pumping Lemma.
By the P.L. there exists some $n > 0$.
We choose $w = 0^n 1^{2n}$ which is in L and also satisfies $|w| \geq n$.
By the P.L. $w = xyz$ such that $|xy| \leq n$ and $y \neq \epsilon$. Note that all x and y contain only 0s. Let $|x| = a$ and $|y| = b$, then $0 < b \leq n$.
We choose $k = 2 : xy^2z \in L$ by P.L., but $xy^2z = 0^{n+b}1^{2n}$ and b is at least 1. Since $2n < 2(n+b)$, this string cannot be in L. Thus we have a contradiction, hence L cannot be regular. □

Problem 5

Assume L is regular and apply the Pumping Lemma.
By the P.L. there exists some $n > 0$.
We choose $w = 0^{n^3}$ which is in L and also satisfies $|w| \geq n$ since $n^3 > n$ for all $n > 0$.
By the P.L. $w = xyz$ such that $|xy| \leq n$ and $y \neq \epsilon$. Note that all x, y and z contain only 0s. Let

$|x| = a$ and $|y| = b$, then $0 < b \leq n$.
We choose $k = 2 : xy^2z \in L$ by P.L., but $xz = 0^{n^3+b}$. But $n^3 + b$ cannot be a perfect cube since the bounds on b imply the following:

$$n^3 < n^3 + b \leq n^3 + n < n^3 + 3n^2 + 3n + 1 = (n+1)^3$$

Note that $n < 3n^2 + 3n + 1$ holds for all $n \geq 0$ because the quadratic $3n^2 + 2n + 1$ has a negative discriminant. So $n^3 + b$ is strictly between the values of two consecutive perfect cubes and cannot be a perfect cube itself. Thus we have a contradiction, hence L cannot be regular. □

Problem 6

(a) $b^l c^m$: $x = \epsilon$, $y = b^n$ and $z = b^{l-n}c^m$. It is easy to see that the first two conditions are satisfied. Since we have no a, there are no constraints on the string, and condition 3 is satisfied as well.

$ab^m c^m$: $x = \epsilon$, $y = a$ and $z = b^m c^m$. It is easy to see that the first two conditions are satisfied. For $i = 0$, there are no constraints. For $i = 1$, we already have the same powers of b and c. For all other i, we have strings with no constraints because the power of a is greater than 1. Thus, condition 3 is also satisfied.

$a^2 b^l c^m$: $x = \epsilon$, $y = a^2$ and $z = b^l c^m$. It is easy to see that the first two conditions are satisfied. For $i = 0$, there are no constraints. For all other i, we have strings with no constraints because the power of a is greater than 1. Thus, condition 3 is also satisfied.

$a^3 b^l c^m$: $x = \epsilon$, $y = a^3$ and $z = b^l c^m$. It is easy to see that the first two conditions are satisfied. For $i = 0$, there are no constraints. For all other i, we have strings with no constraints because the power of a is greater than 1. Thus, condition 3 is also satisfied.

(b) In general, it is not true that if $A \Rightarrow B$ then $B \Rightarrow A$.

(c) **Solution 1:** Assume that L is regular. Consider the language $L_1 = \{ab^*c^*\}$ — clearly, this is a regular language. Now, consider the language $L_2 = L \cap L_1 = \{ab^m c^m | m \geq 0\}$. Since L_1 is regular, it follows that if L is regular then L_2 must also be regular by the closure of regular languages under intersection. But we can prove via the pumping lemma that L_2 is not regular, so it follows that L cannot be regular. The proof that L_2 is not regular is similar to the pumping lemma proof that $\{0^n 1^n | n \geq 0\}$ is not regular and is omitted.

Solution 2: We know that if a language L is regular, then its reverse L^R must also be regular.

$$L^R = \{c^m b^l a^k \mid m, l, k \geq 0, k = 1 \Rightarrow l = m\}$$

Now assume L^R is regular and apply the Pumping Lemma. By the P.L. there exists some $n > 0$. We choose $w = c^n b^n a$ which is in L^R and also satisfies $|w| \geq n$. By the P.L. $w = xyz$ such that $|xy| \leq n$ and $y \neq \epsilon$. Note that all x and y contain only cs. Let $|y| = m$, then $0 < m \leq n$. We choose $k = 2 : xy^2z \in L$ by P.L., but $xy^2z = c^{n+m}b^n a$ and m is at least 1. Since $n + m \neq n$, this string cannot be in L^R. Thus we have a contradiction, hence L^R cannot be regular. □