PUMPING LEMMA FOR CFL

CONSIDER $L_1 = \{a^n b^n \mid n \geq 1\}$

$L_2 = \{a^n b^{n+m} c^m \mid n, m \geq 1\}$

$L_3 = \{a^n b^{2n} c^n \mid n \geq 1\}$

QUESTION WHICH ARE CFL'S?

CLAIM
$\begin{cases} \bullet\ L_1, L_2 \text{ ARE CFL'S} \\ \bullet\ L_3 \text{ IS NOT CFL'S} \end{cases}$

INTUITION L_3 IS NOT CFL BECAUSE NUMBER OF b'S DEPENDS ON ITS CONTEXT, BOTH TO ITS LEFT AND RIGHT

HOWEVER $L_2 = \{a^n b^n . b^m c^m \mid n, m \geq 1\}$ IS CFL.

PUMPING LEMMA

LET L BE CFL.

THEN $\exists$ CONSTANT m SUCH THAT

FOR ALL $z \in L$ WITH $|z| > m$

CAN WRITE $z = uvwxy$ FOR WHICH

a) $|vwx| \leq m$

b) $|vx| > 0$

c) $\forall i \geq 0,\ uv^i w x^i y \in L$.

PROOF — NEXT CLASS

OBSERVE APPLYING THIS IS SIMILAR TO REGULAR L.

REMARK FOR SOME NON-CFL'S, PUMPING LEMMA IS NO USE AND WE NEED STRONGER RESULTS SUCH AS OGDEN'S LEMMA.

APPLICATION WE WILL SHOW L_3 IS NOT CFL

$L_3 = \{a^n b^{2n} c^n \mid n \geq 1\}$

STEP 1 ASSUME L_3 IS CFL AND APPLY P.L.

STEP 2 GET CONSTANT $m > 0$

STEP 3 CHOOSE $z = a^m b^{2m} c^m$

STEP 4 GET u, v, w, x, y SUCH THAT

$z = uvwxy$

AND $|vwx| \leq m$, $|vx| > 0$

STEP 5 WE CHOOSE $i = 0$ AND CLAIM

$uv^0wx^0y \notin L$ (CONTRADICTION)

WHY?

OBSERVE $|vwx| \leq m$ IMPLIES THAT EITHER vwx HAS NO c'S OR IT HAS NO a'S

CASE I [vwx HAS NO c'S]

LET $z' = uwy = uv^0wx^0y$

$\Rightarrow |z'| = |z| - |vx| = 4m - |vx| < 4m$

SINCE $|vx| > 0$

BUT z' HAS m c'S, AND SO MORE THAN 1/4 OF ITS SYMBOLS ARE c'S.

BY DEFN $z' \in L$ HAS EXACTLY 1/4 c'S

THUS $z' \notin L$

CASE II SIMILAR ARGUMENT.

CS154

WEEK 5 (b)

H.O. #18