Lemma (Pumping Lemma)

Given a context-free language \( L = \{ u v^r w \mid u, v, w \in \{0,1\}^* \text{ and } |v| > 0 \} \), let \( G = (V, \Sigma, R) \) be any CFG grammar for \( L \).

Let \( \epsilon = u v^r w \) be any string in \( L \).

Given \( \epsilon \), let \( (a, b, c) \in \{0,1\} \times \{0,1\} \times \{0,1\} \) such that

\[
|a| = x, \quad |b| = y, \quad |c| = z
\]

where \( x + y + z = |\epsilon| > 0 \).

Then by Pumping Lemma, there exists a string \( \epsilon = u v^r w \) in \( L \), such that

\[
u \epsilon = u v^r w = \epsilon
\]

and

\[
|u v| > 1
\]

Consider \( z \) the smallest integer such that

\[
x + y + z = |\epsilon|
\]

for such \( z \), let

\[
|u| = x, \quad |v| = y, \quad |w| = z
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