The Hiring Problem:

Going Beyond Secretaries

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The Secretary Problem

Interview n candidates for a position one at a time. After each interview decide if the candidate is the best.

Goal: maximize the probability of choosing the best candidate.

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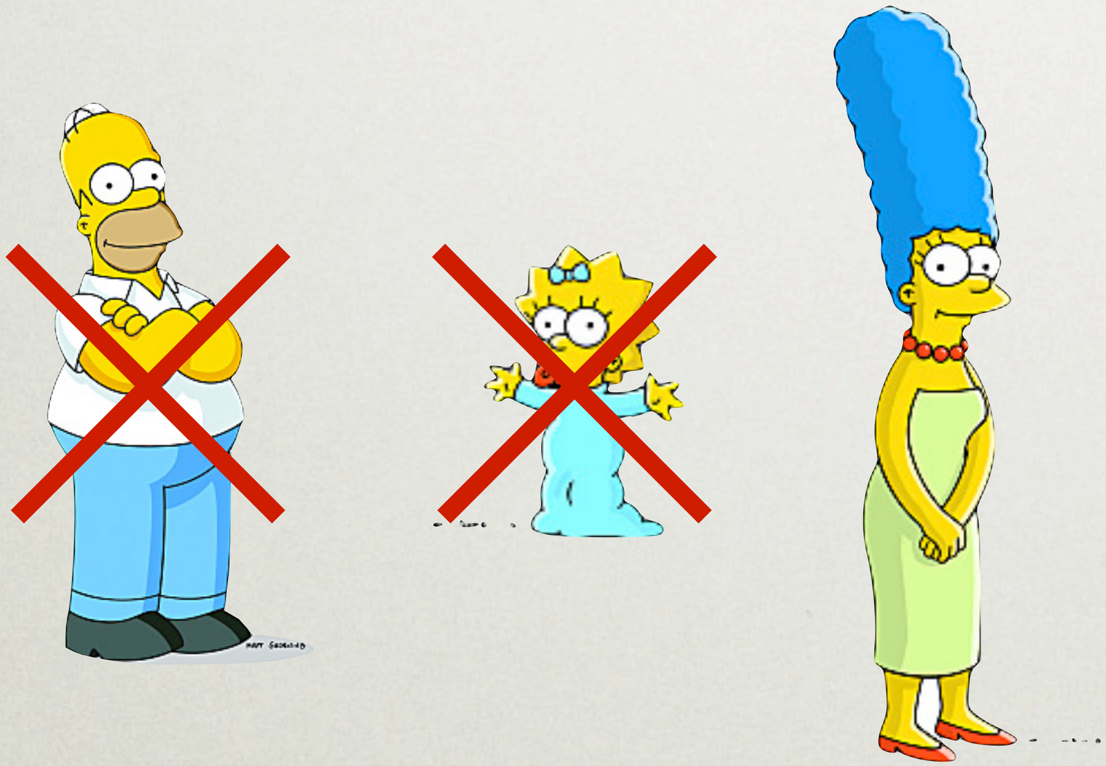

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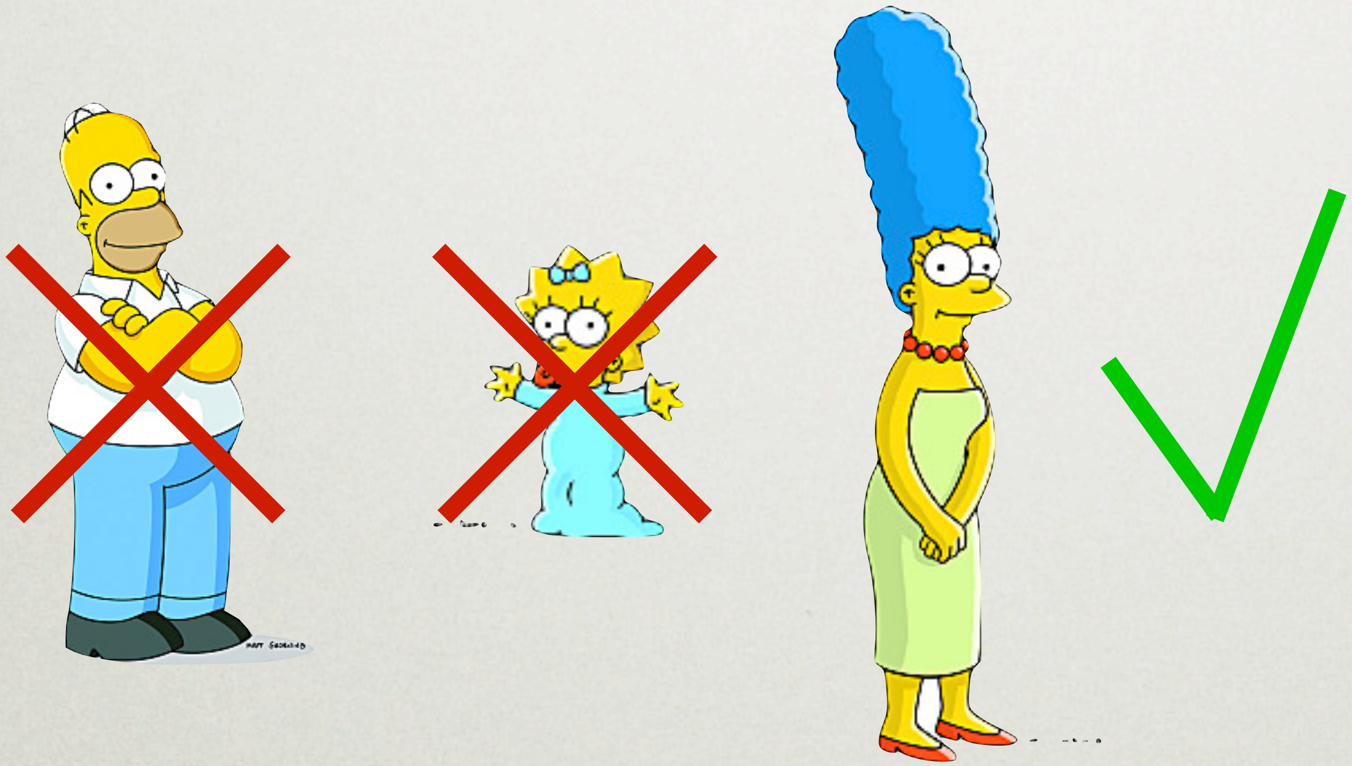

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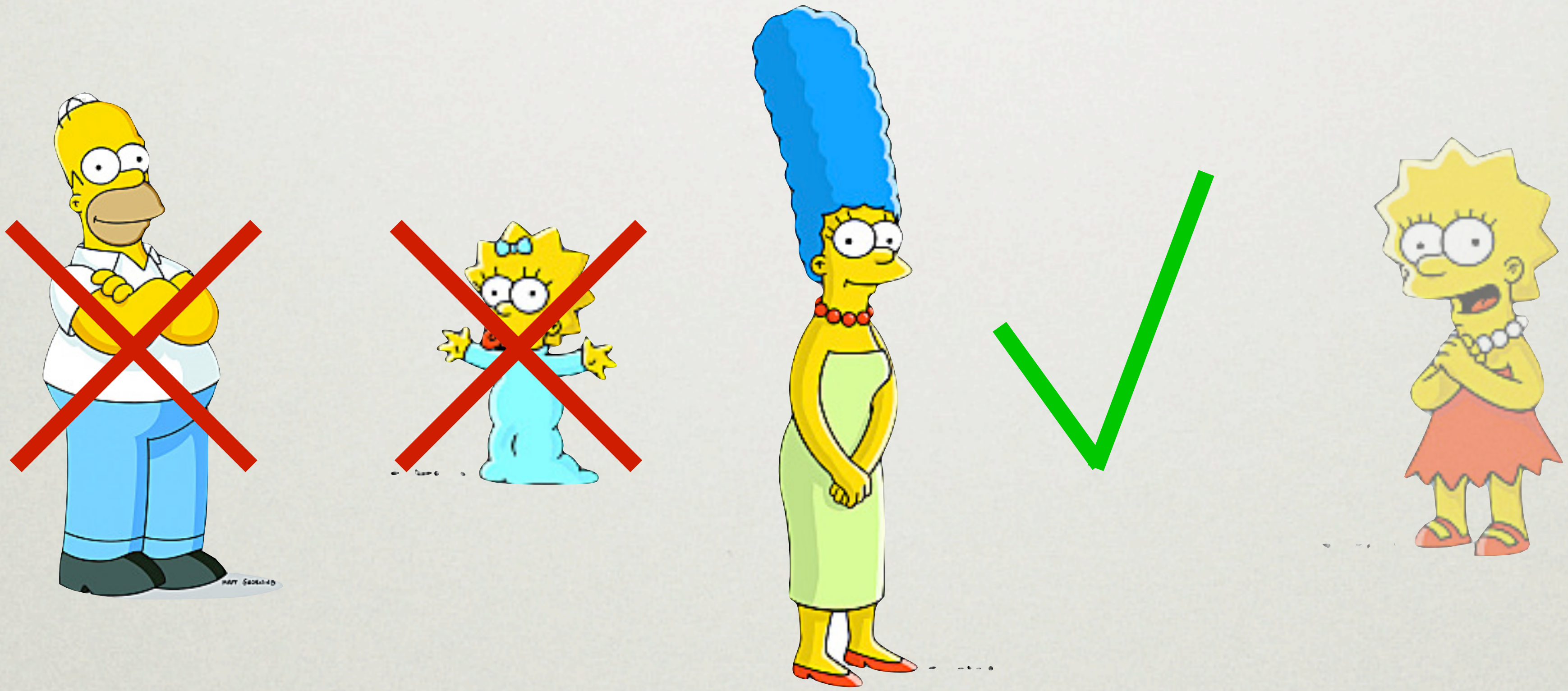

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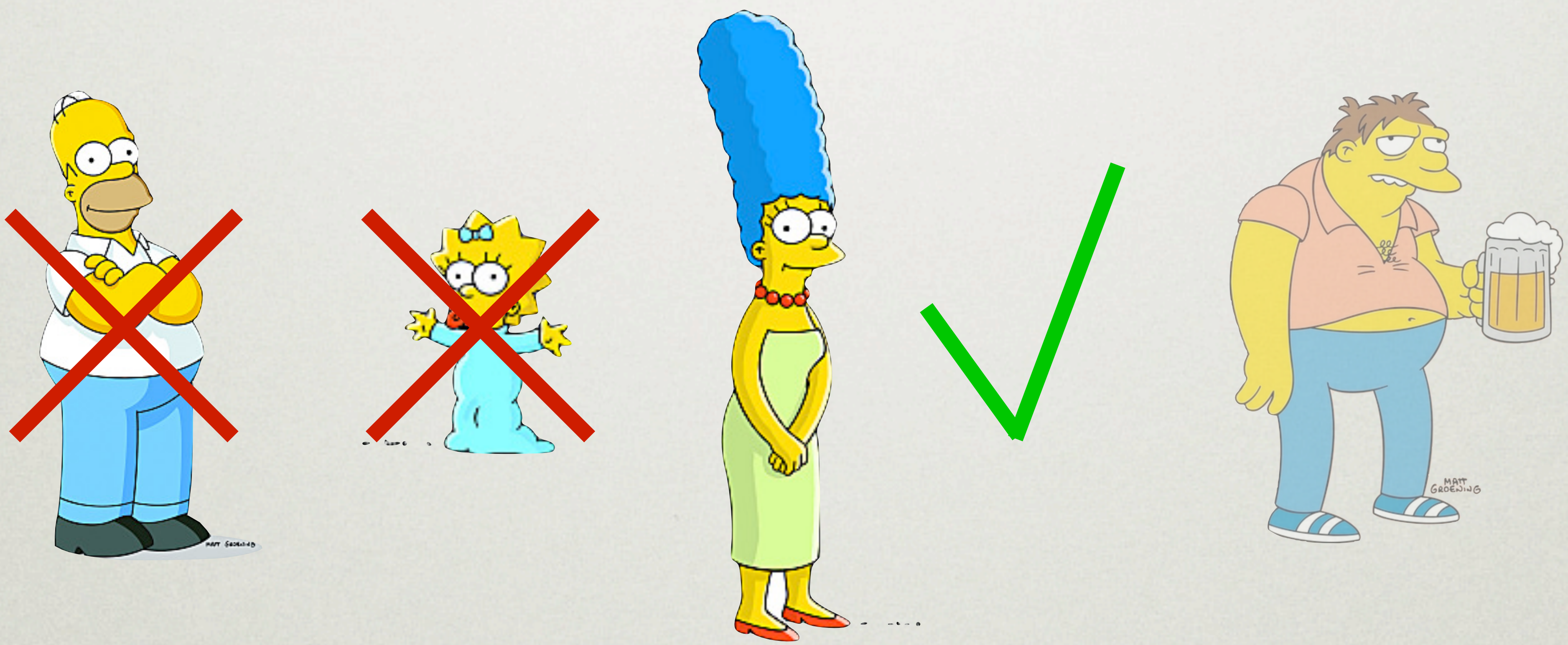

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This is not about hiring secretaries, but about decision making under uncertainty.

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- Want to hire good employees
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Many potential objectives.

- Explore the tradeoff between number of interviews & the average quality.

The Hiring model

Candidates arrive one at a time.

Assume all have iid uniform(0,1) quality scores - For applicant i denote it by i_q.

(Can deal with other distributions, not this talk)

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During the interview:

- Observe i_q
- Decide whether to hire or reject

Strategies

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Lake Wobegon Strategies:

> "Lake Wobegon: where all the women are strong, all the men are good looking, and all the children are above average"

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Lake Wobegon Strategies:

- Hire above the average (mean or median)

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Lake Wobegon Strategies:

- Hire above the average

Side note: [Google Research Blog - March '06]:

- "... only hire candidates who are above the mean of the current employees..."

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Set a threshold t, hire if $i_q \geq t$.

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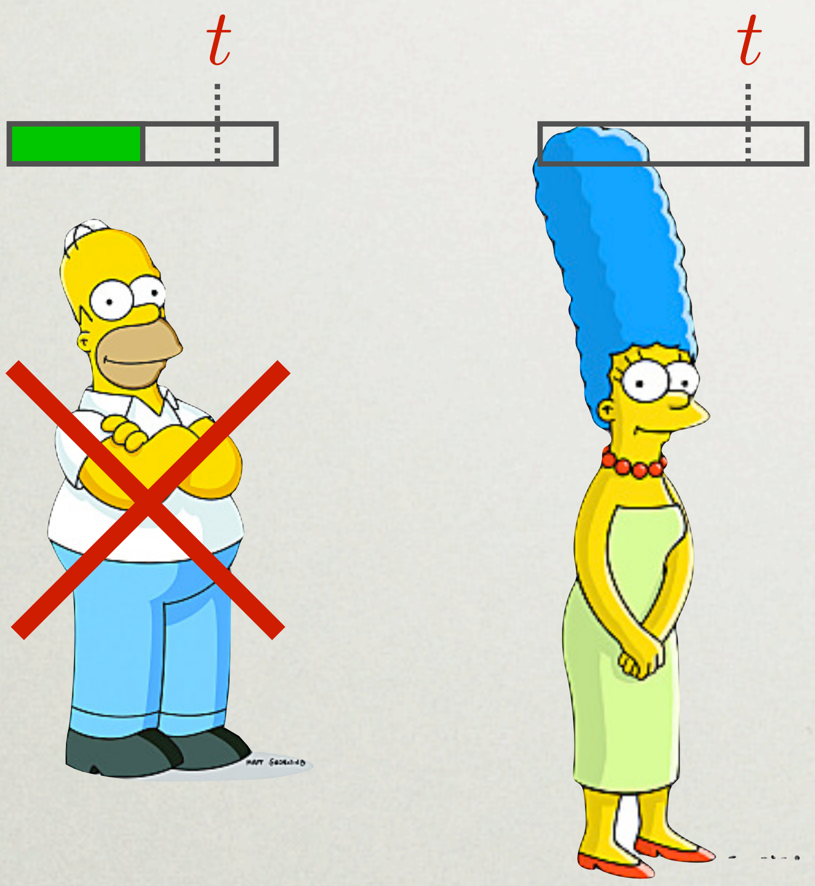

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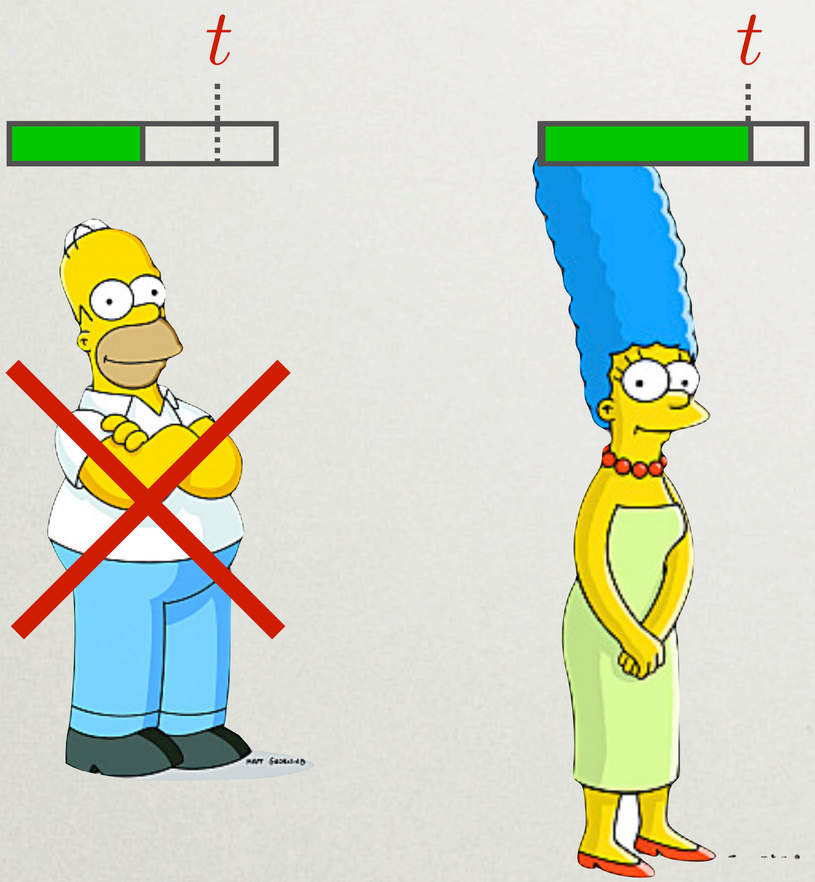

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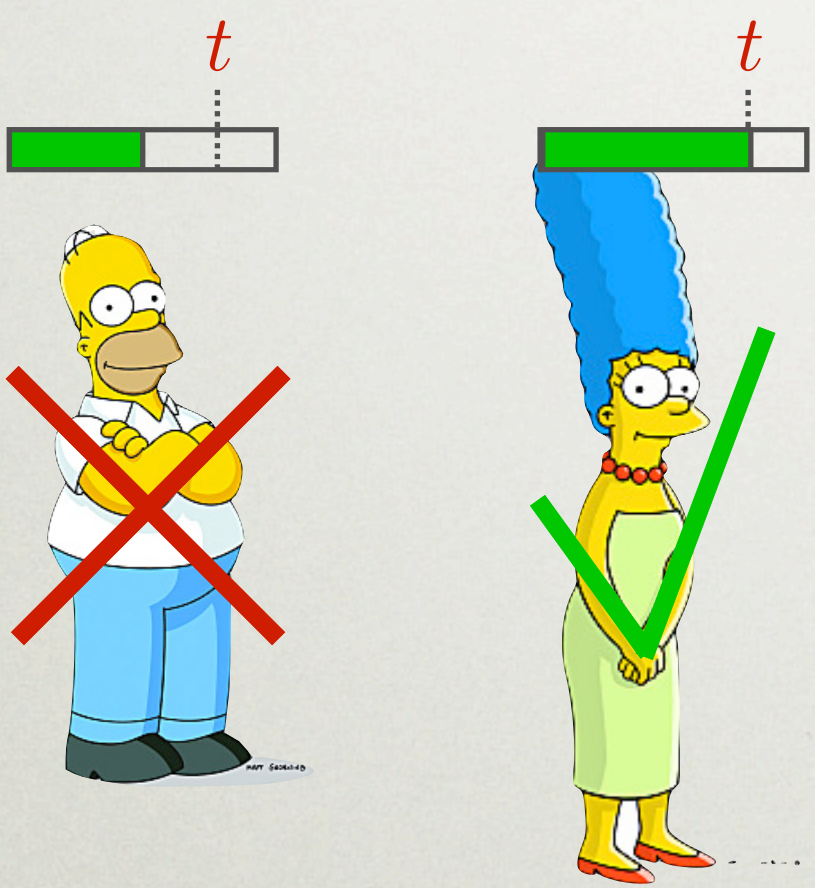

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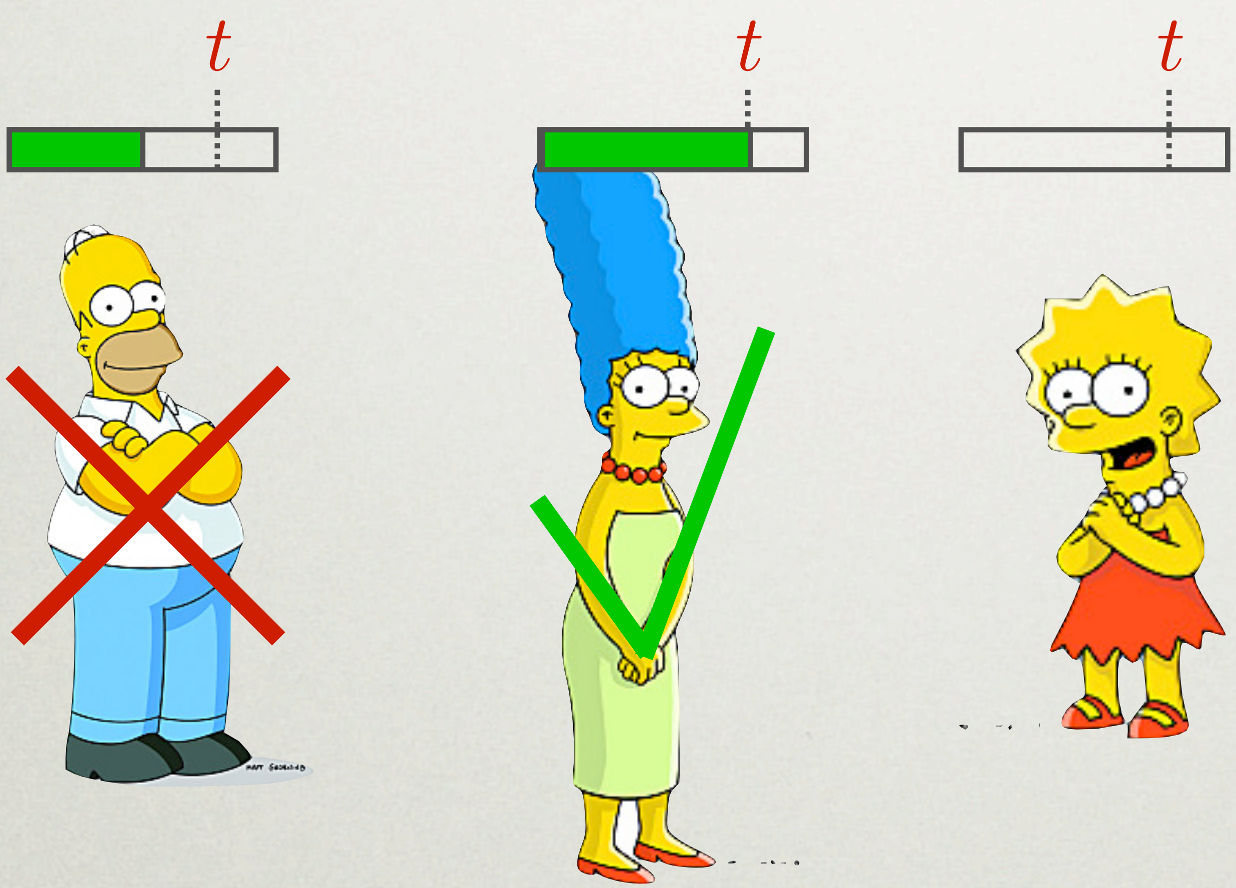

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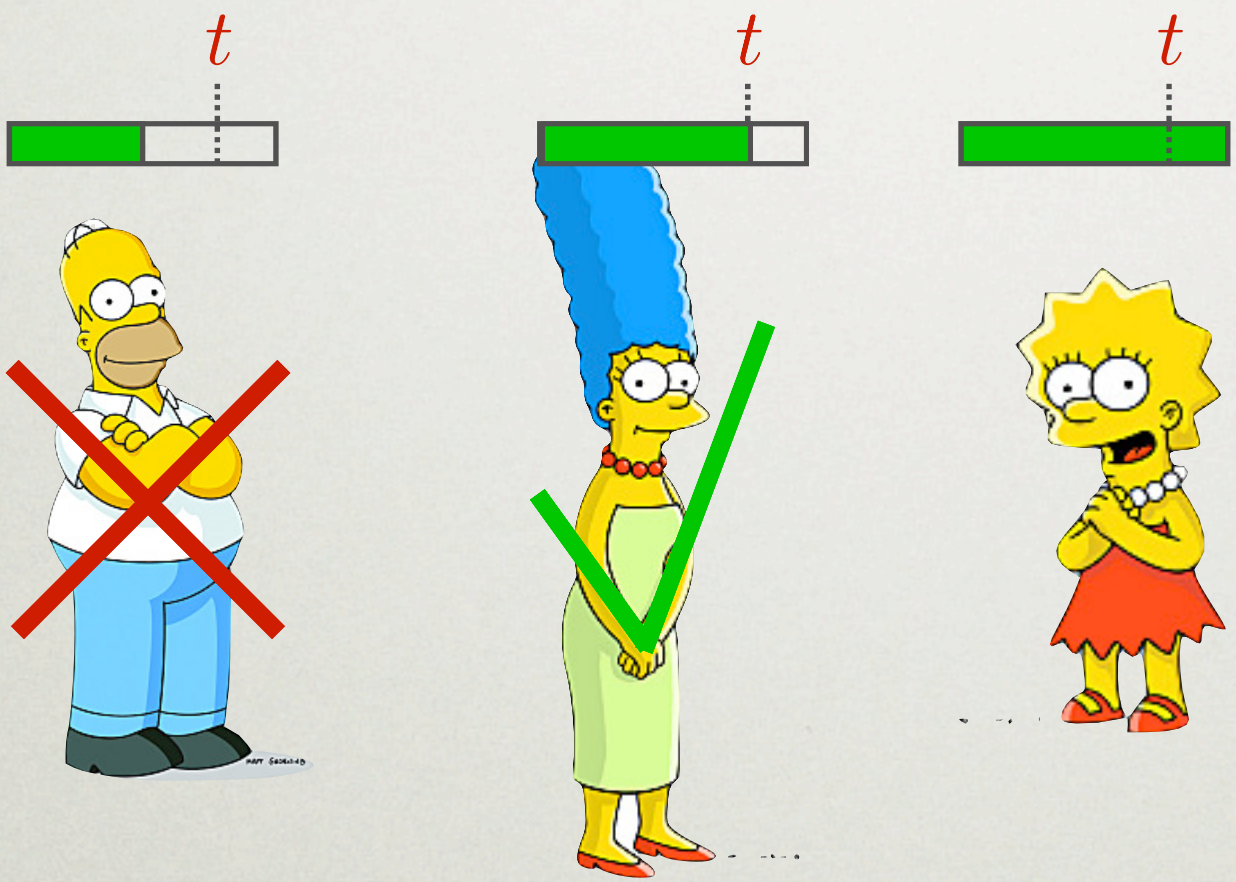

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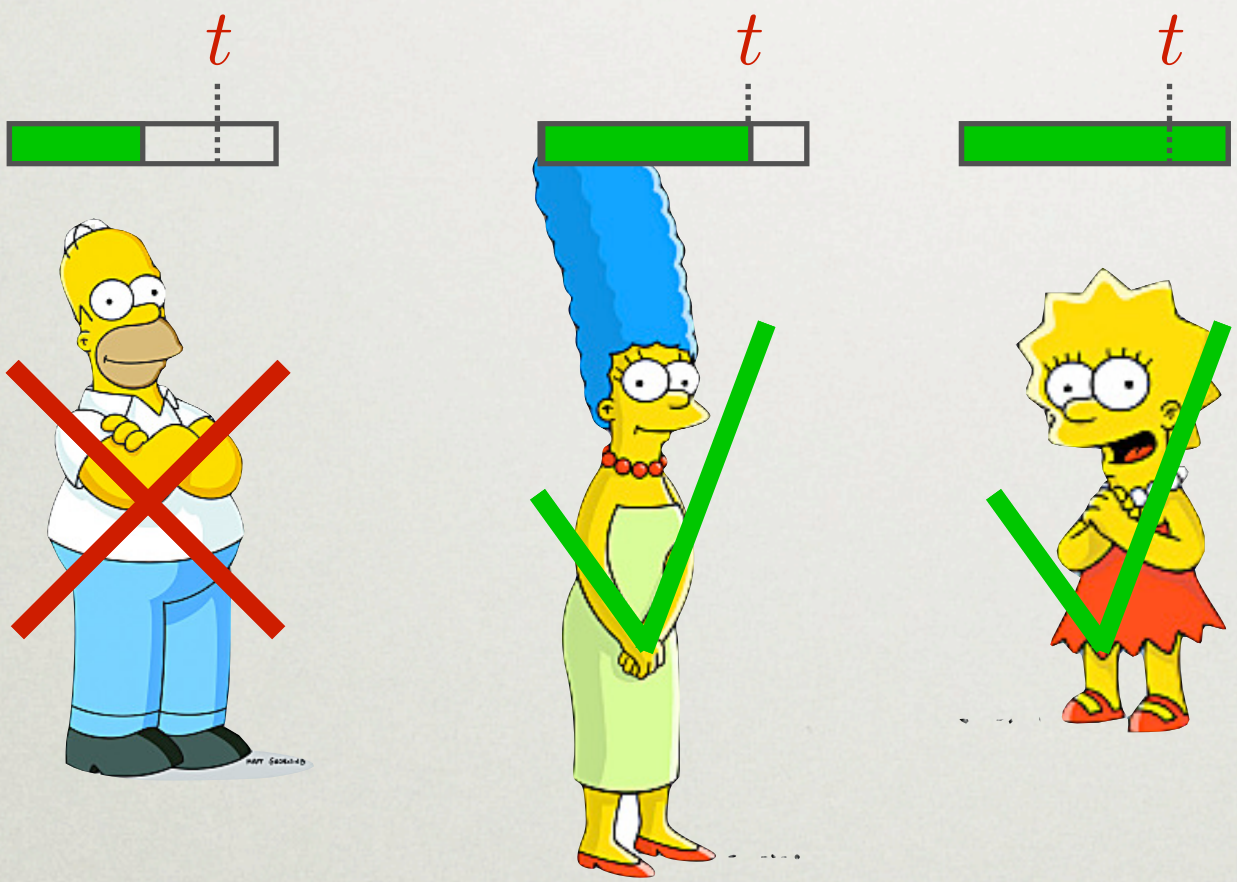

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Easy to see that average quality approaches $\frac{1+t}{2}$

Hiring rate $\frac{1}{1-t}$.

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Quality stagnates and does not increase with time.

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Hire only if better than everyone already hired.

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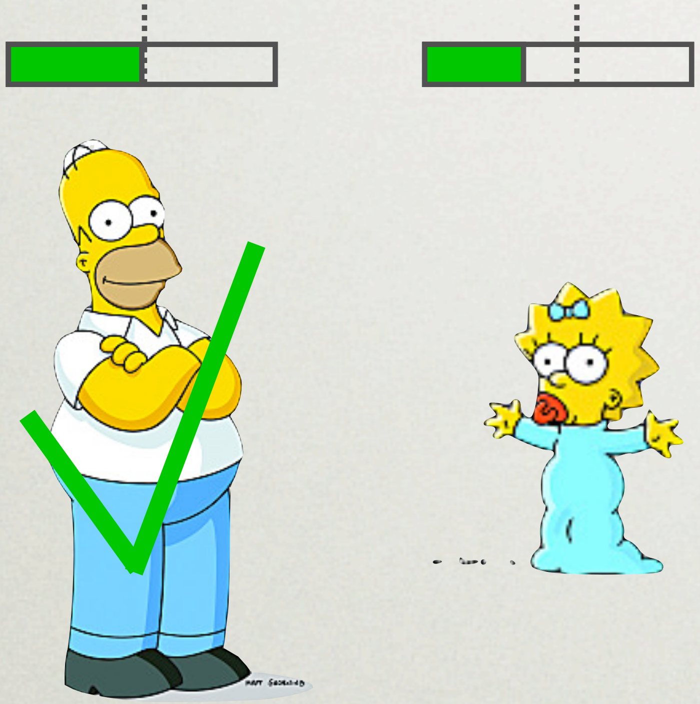

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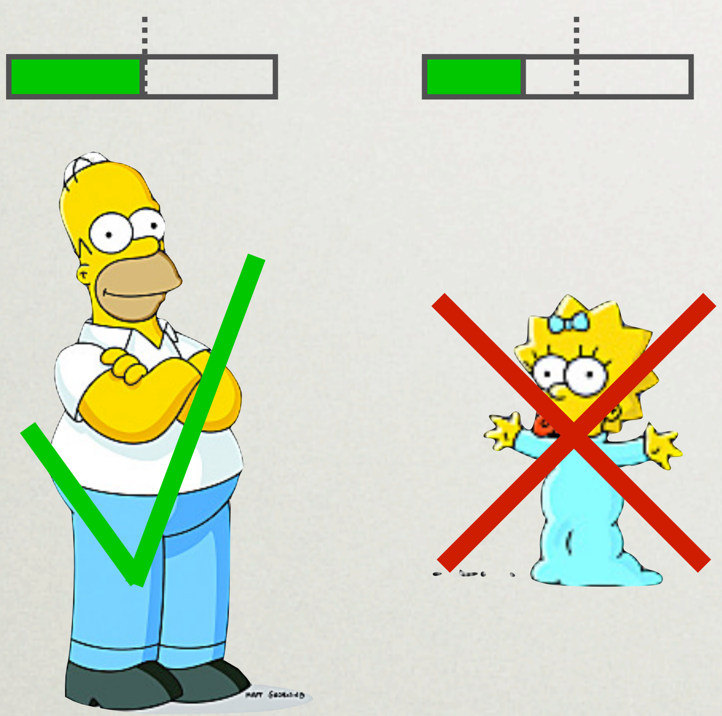

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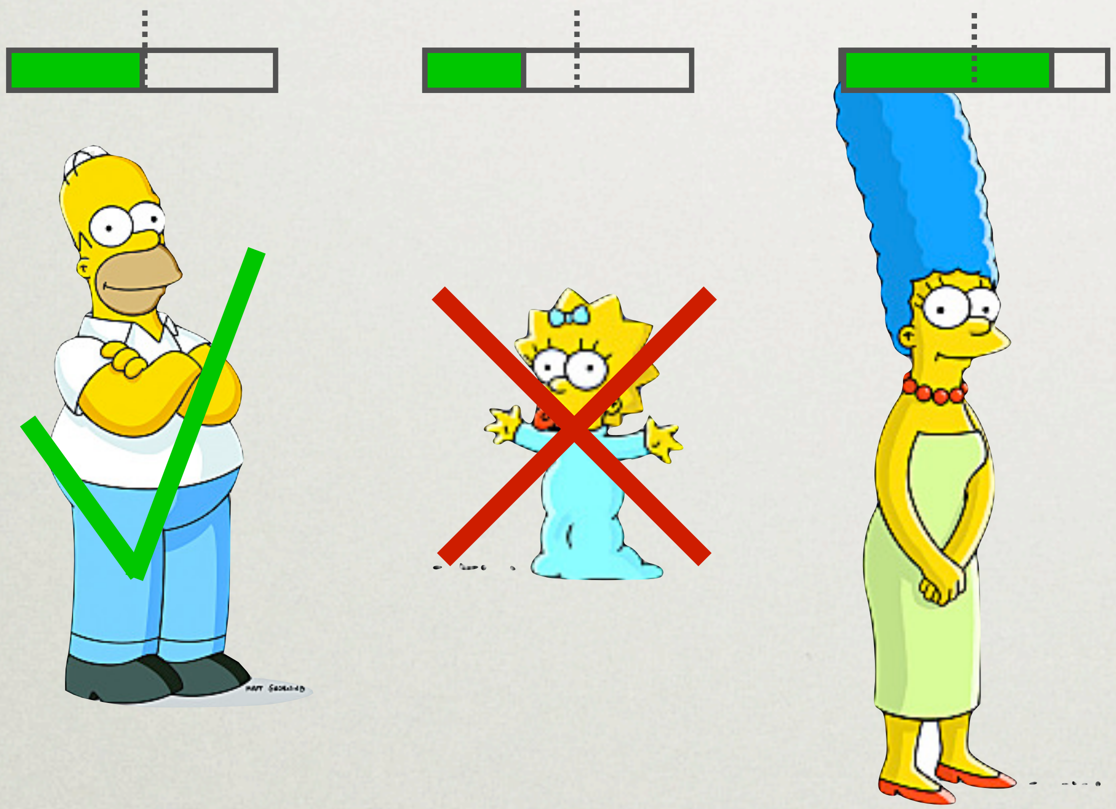

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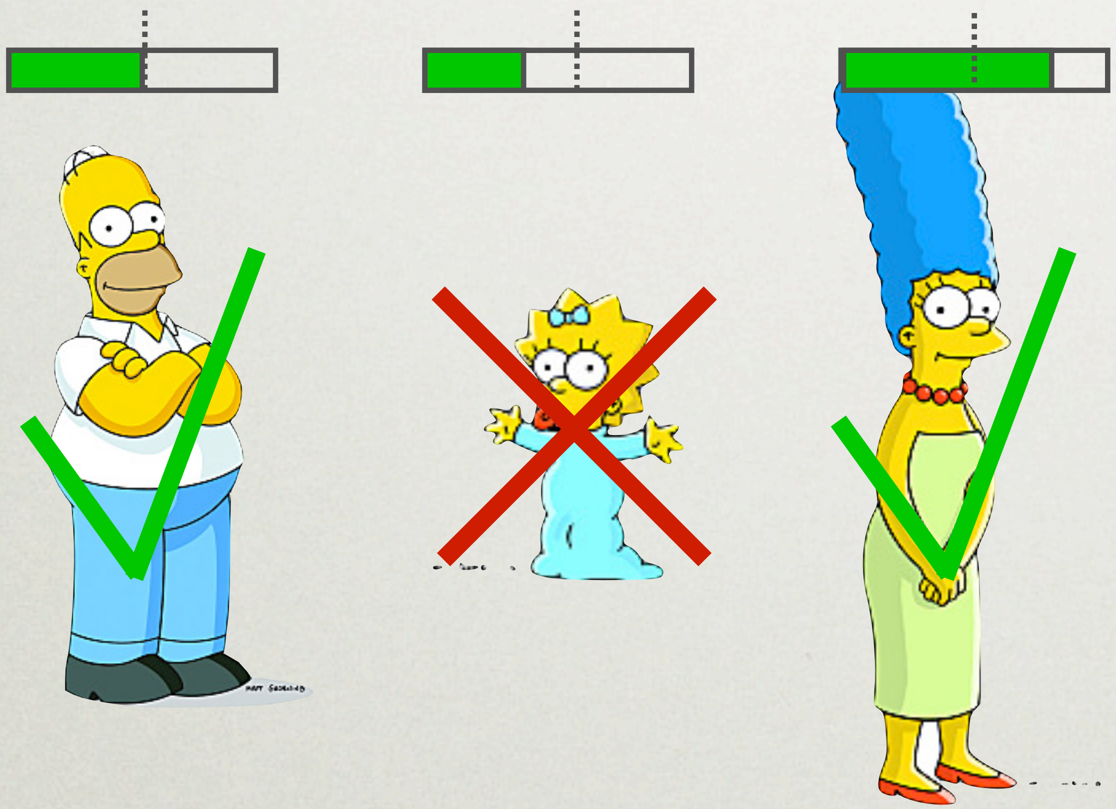

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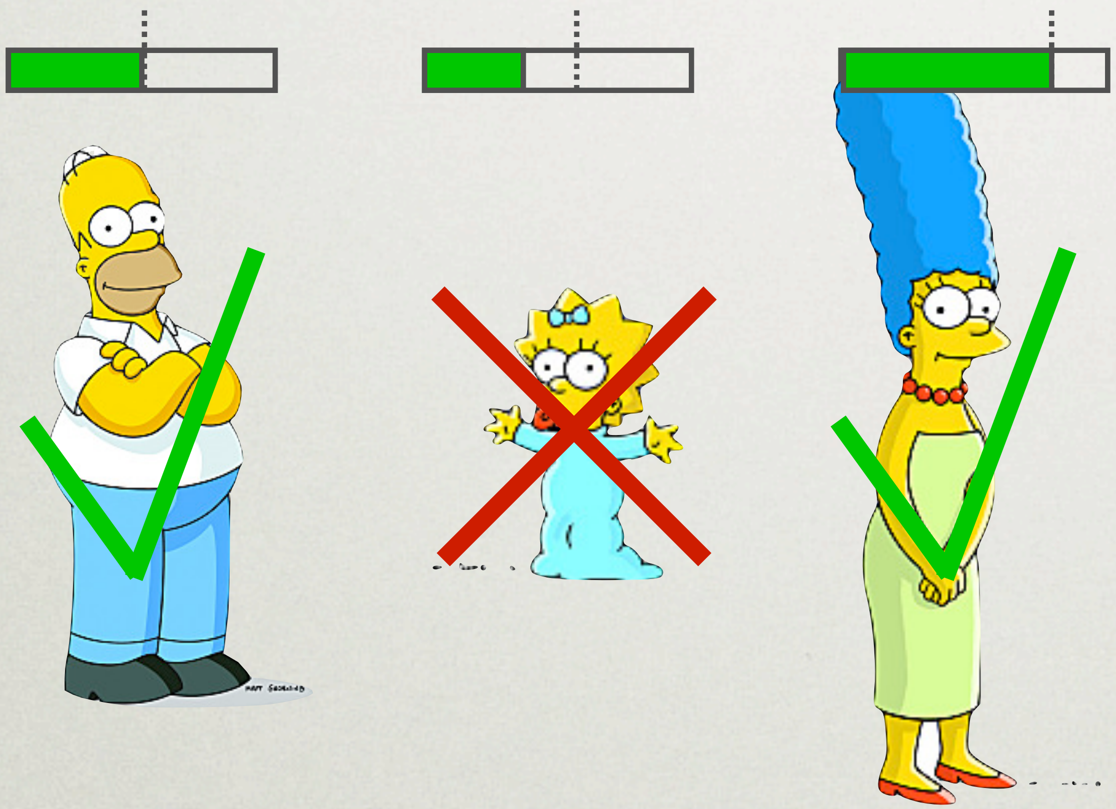

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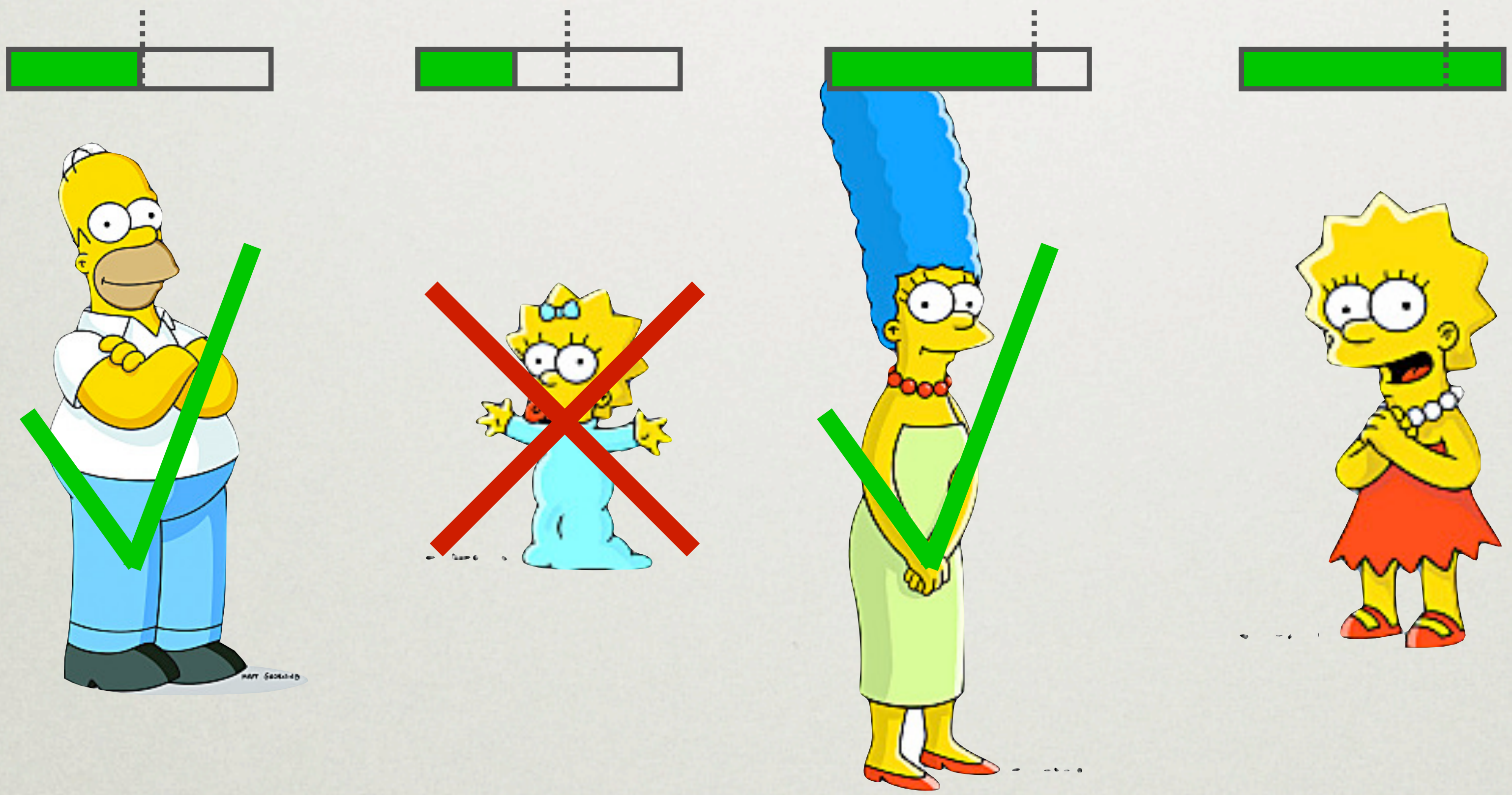

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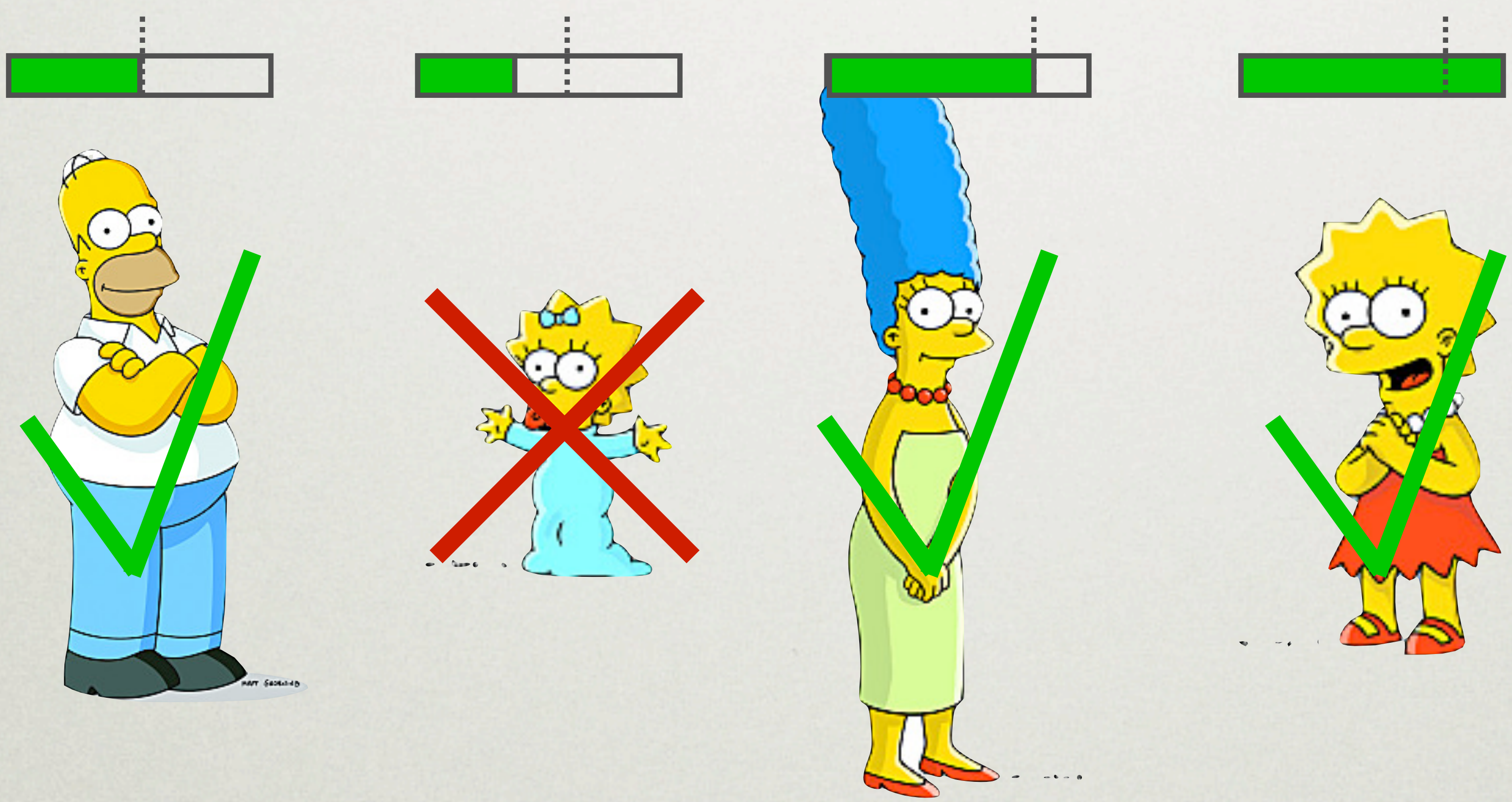

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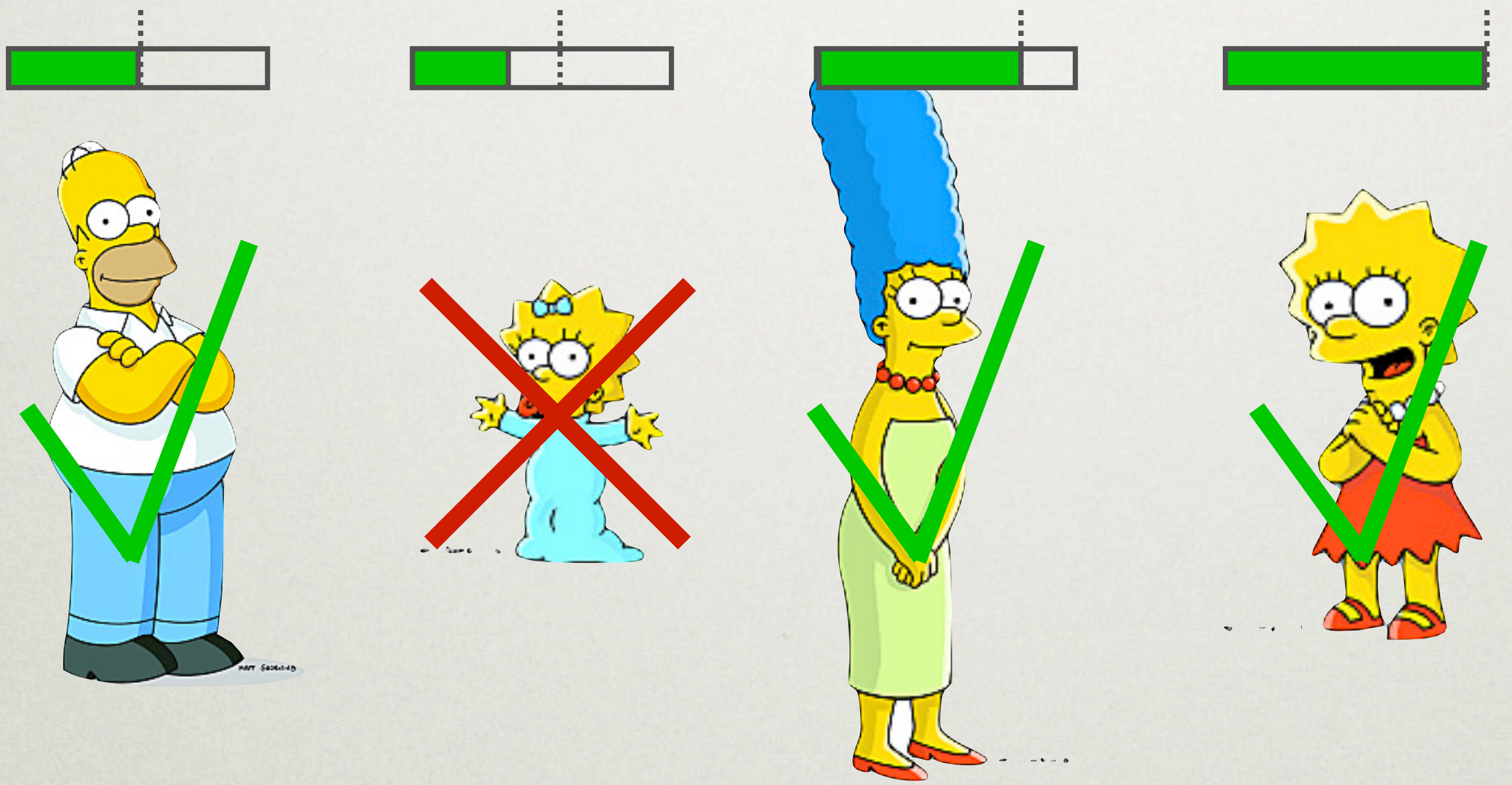

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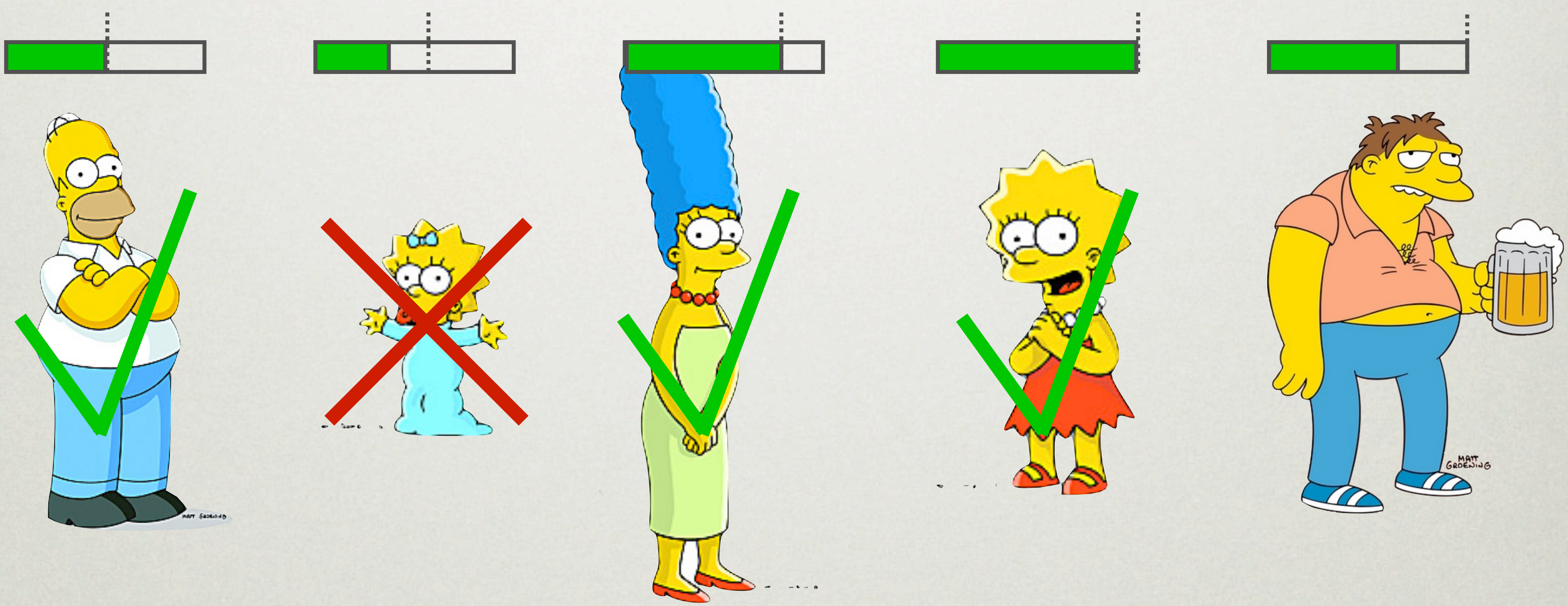

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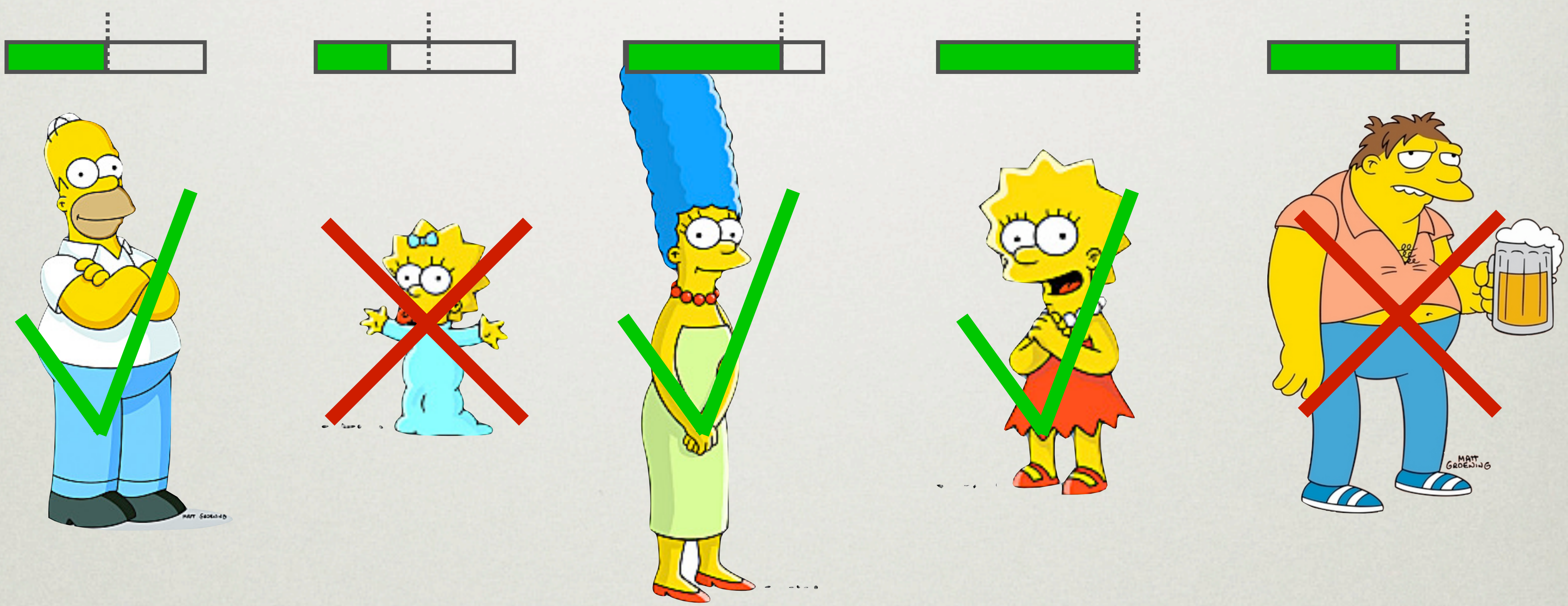

Maximum Analysis

Start with employee of quality q

Let h_i be the i-th candidate hired

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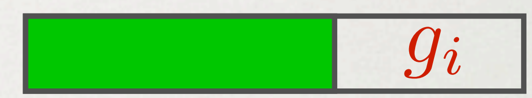
Focus on the gap: $g_i = 1 - (h_i)_q$



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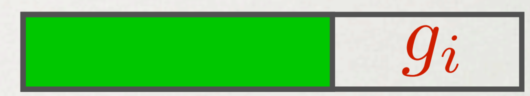

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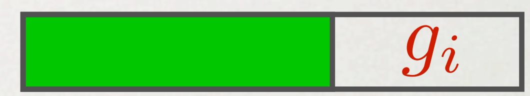

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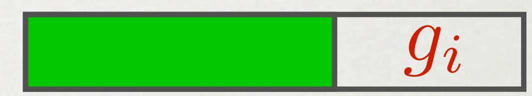

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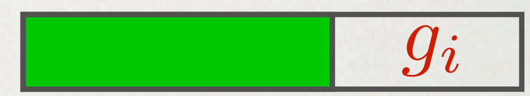

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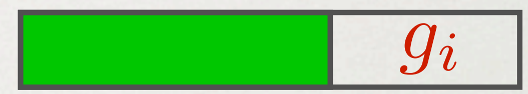

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Very high quality!

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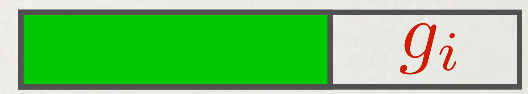

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Extremely slow hiring!

Lake Wobegon Strategies

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Average quality after n hires: $1 - \frac{1}{\sqrt{n}}$

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Surprising:

Tight concentration is not possible

Hiring above mean converges to a log-normal distribution

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Therefore: $$g_{n+1} \sim \frac{n+1}{n+2} g_n + \frac{1}{n+2} Unif(0, g_n)$$

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$$= g_n \left(1 - \frac{Unif(0,1)}{n+2}\right)$$

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Expand: $$g_{n+t} = g_n \prod_{i=1}^{t} \left(1 - \frac{Unif(0,1)}{n+i+1}\right)$$

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Therefore:

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$$E[g_n] = (1-q) \prod_{i=1}^{n} \left(1 - \frac{1}{2(i+1)}\right) = \Theta(\frac{1}{\sqrt{n}})$$

Hiring Above Mean

Conclusion:

Average quality after n hires: $1 - \Theta(\frac{1}{\sqrt{n}})$

Time to hire n employees: $\Theta(n^{3/2})$

Very weak concentration results.

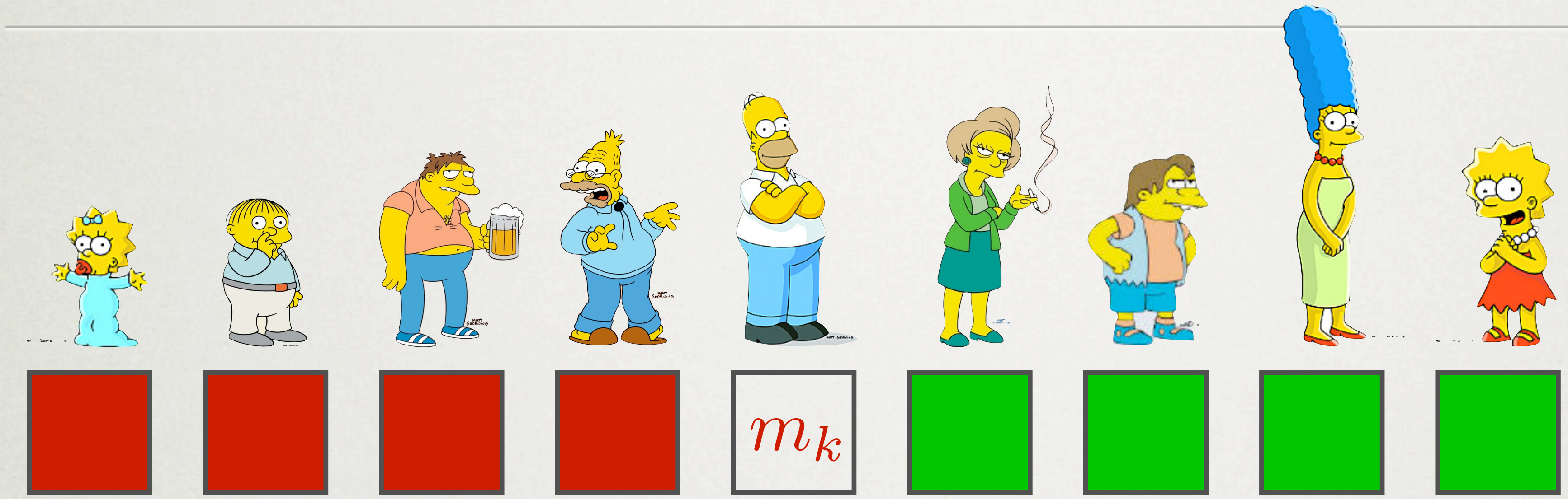
Hiring Above Median

Start with employee of quality q

When we have $2k+1$ employees.

- Compute median m_k of the scores
- Hire next 2 applicants with scores above m_k

Hiring Above Median



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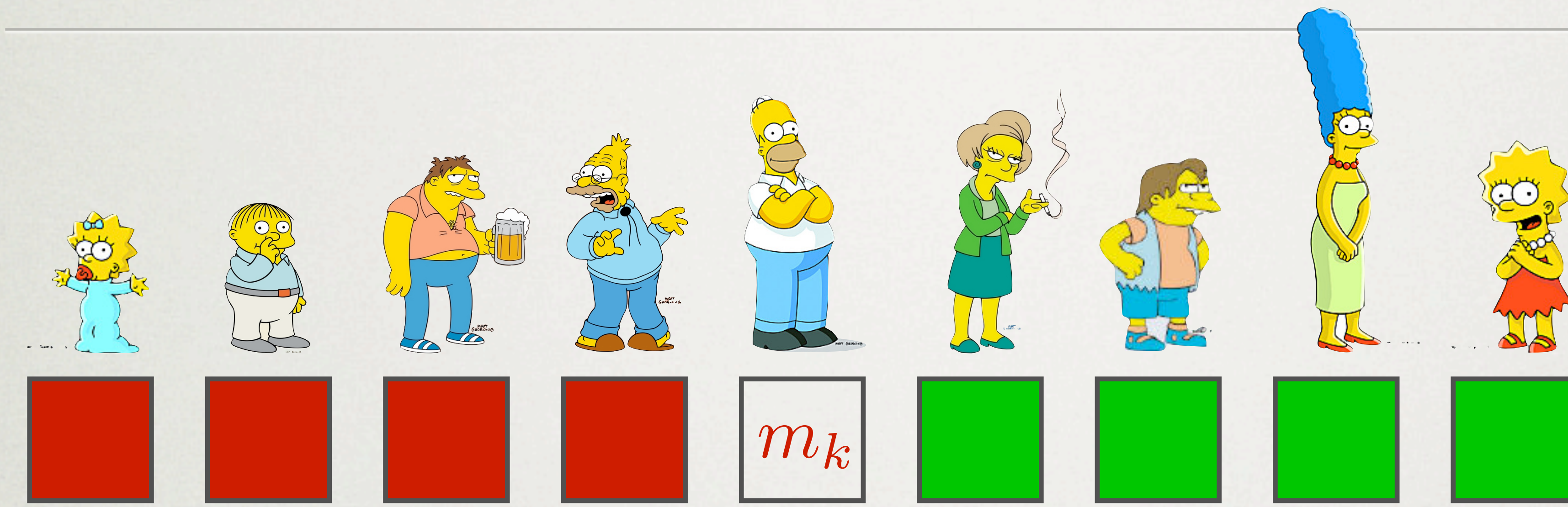
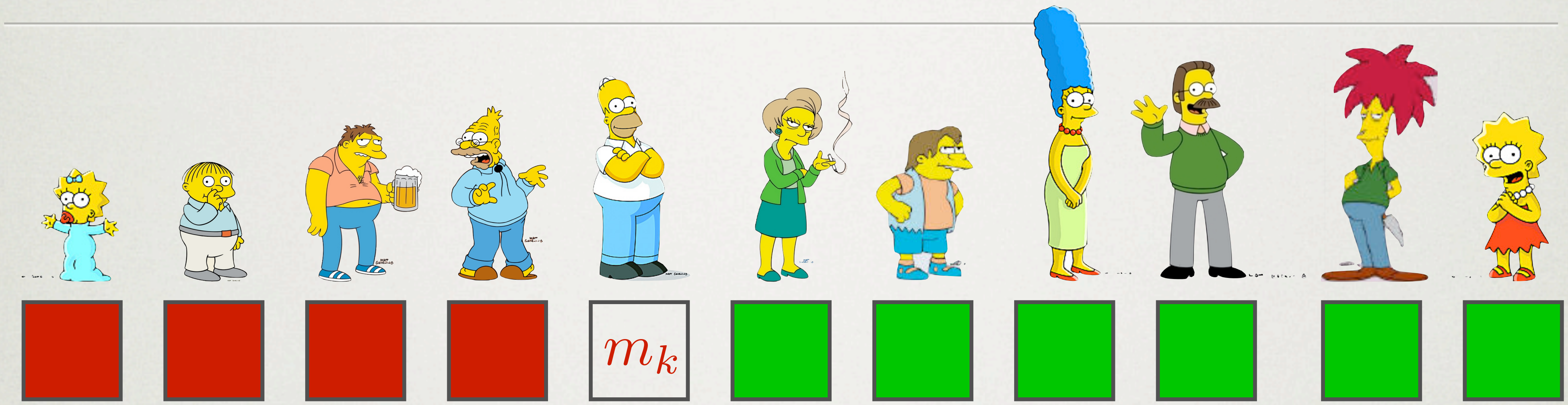


New hires:  with quality above m_k

Hiring Above Median



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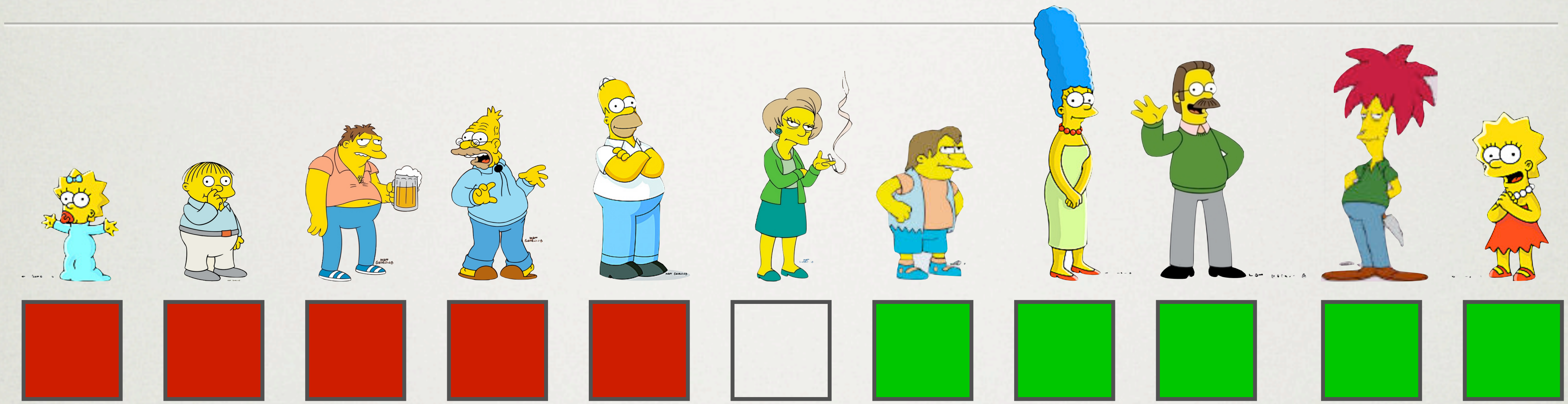
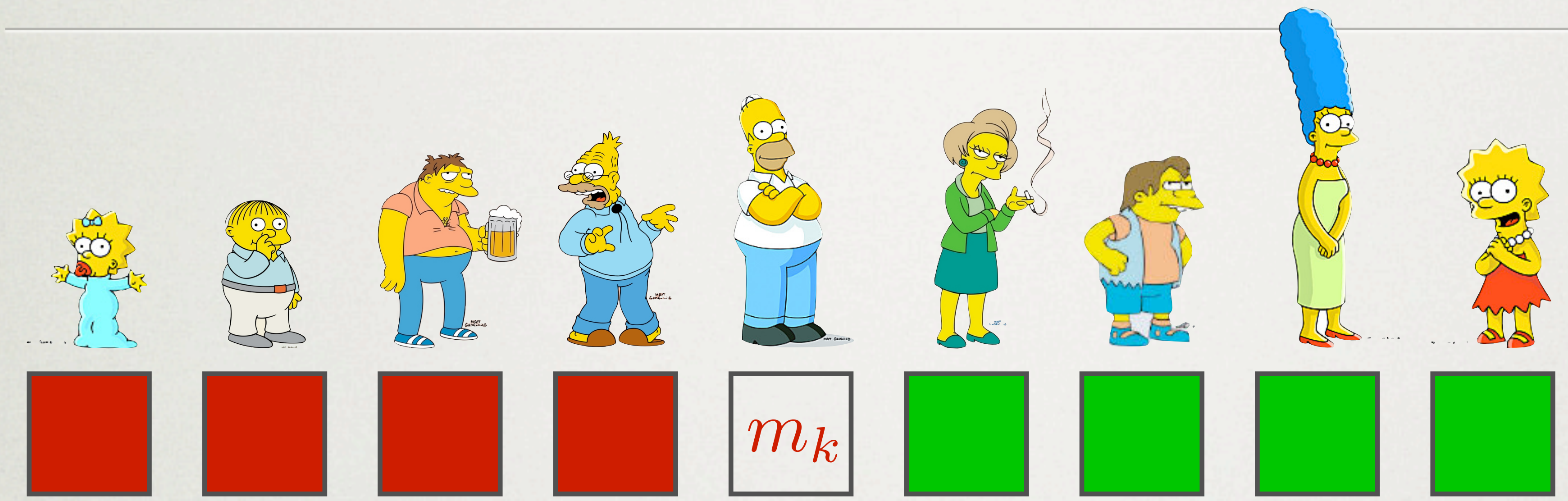


m_{k+1}

Hiring Above Median



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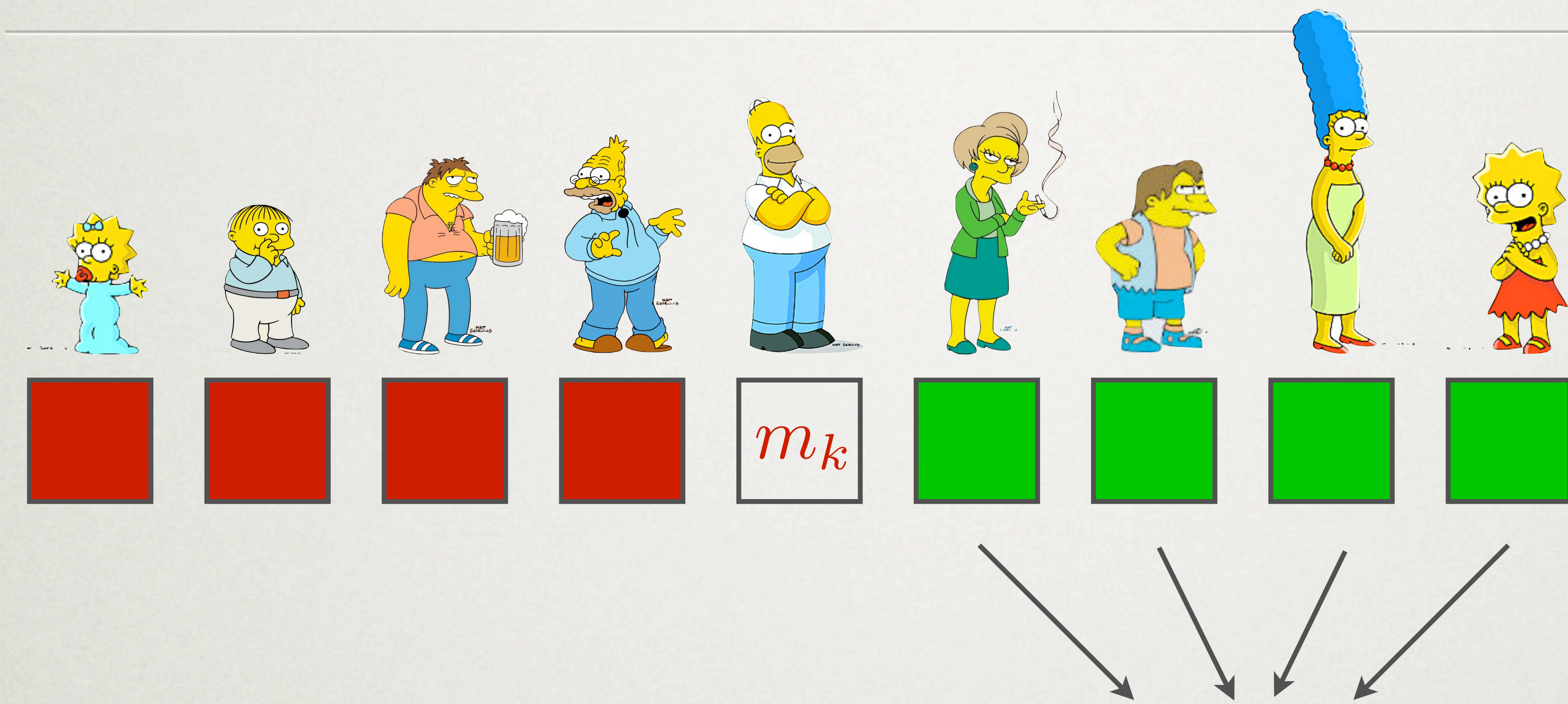
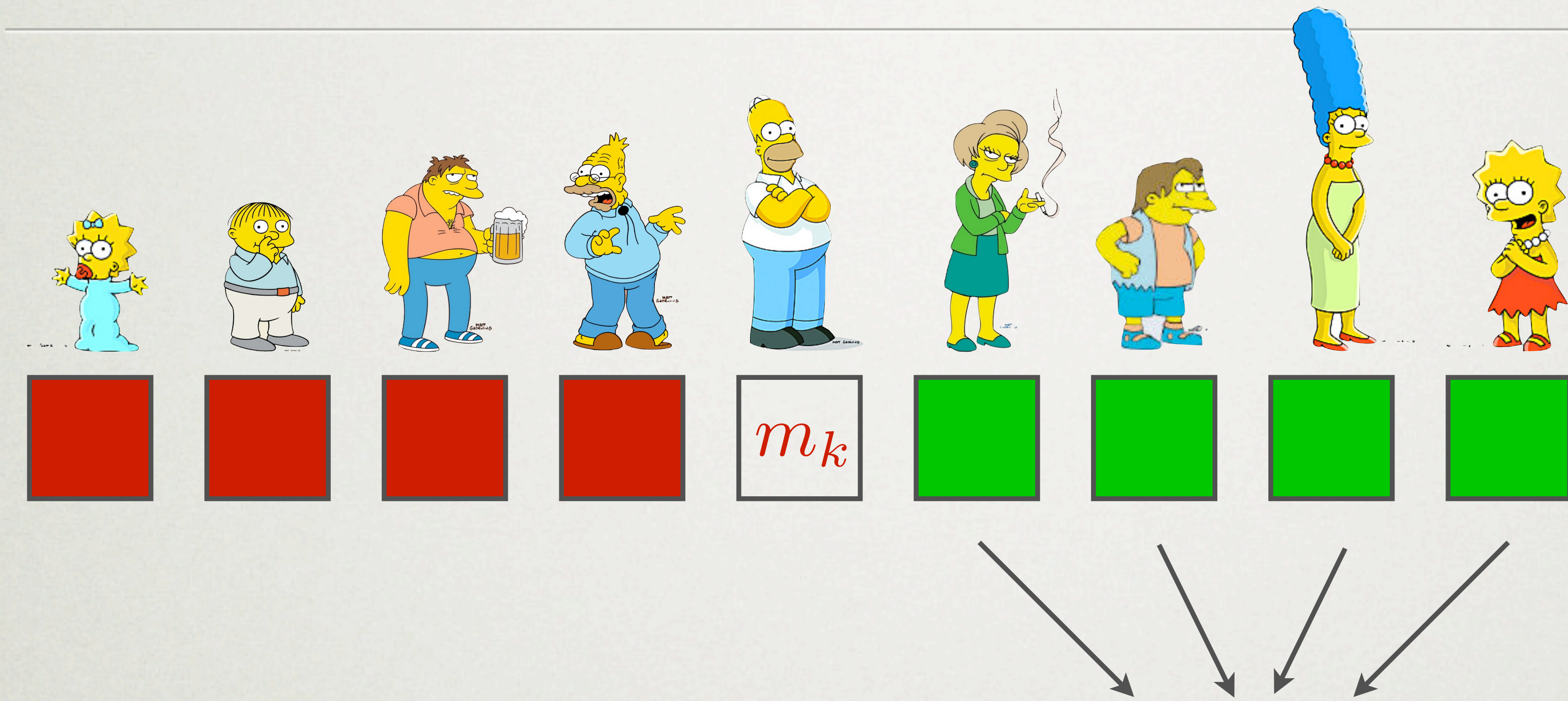


Inductive hypothesis: $Unif(m_k, 1)$

Hiring Above Median



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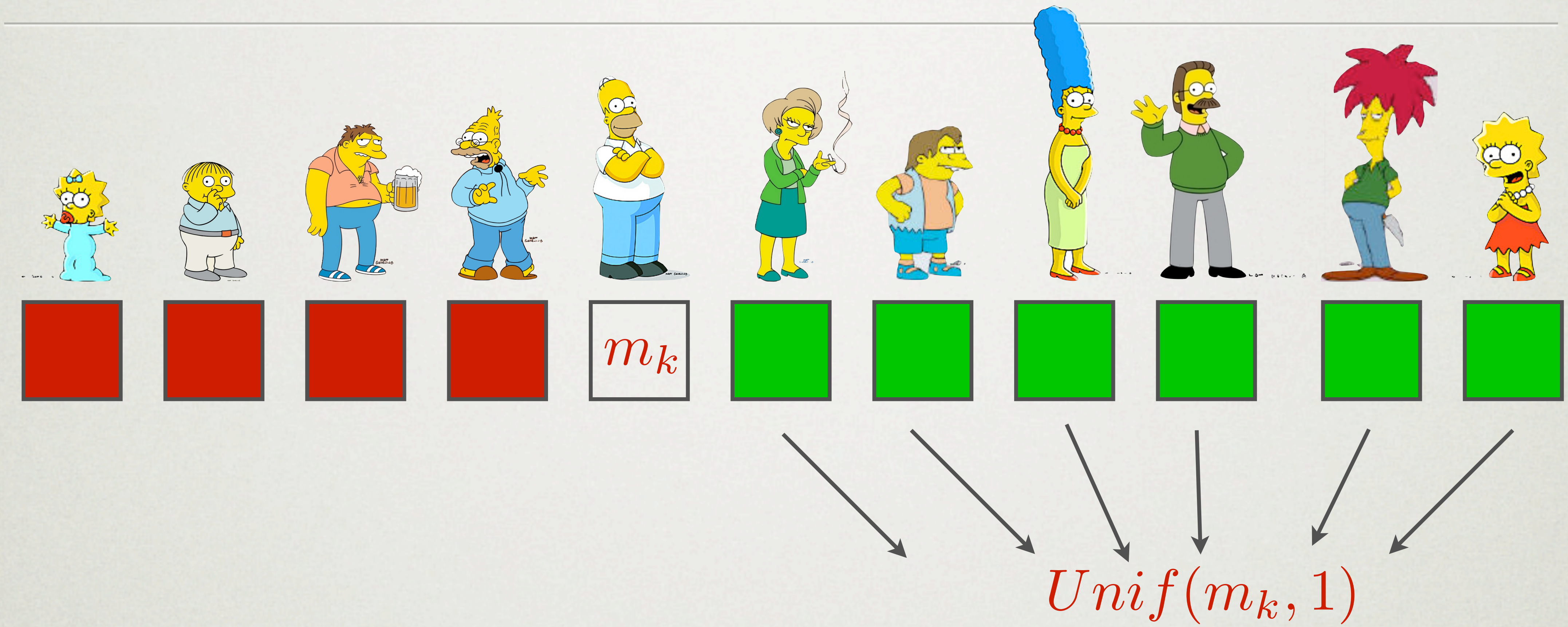
Hiring Above Median



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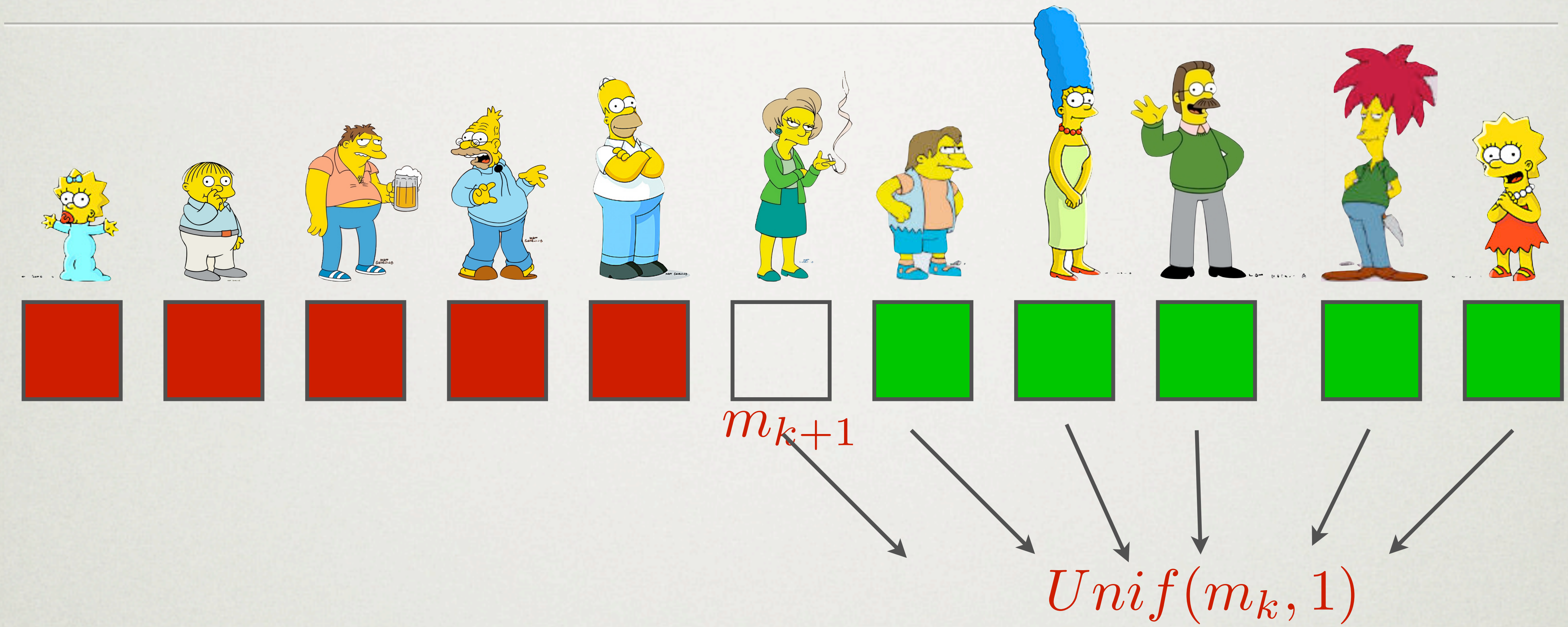

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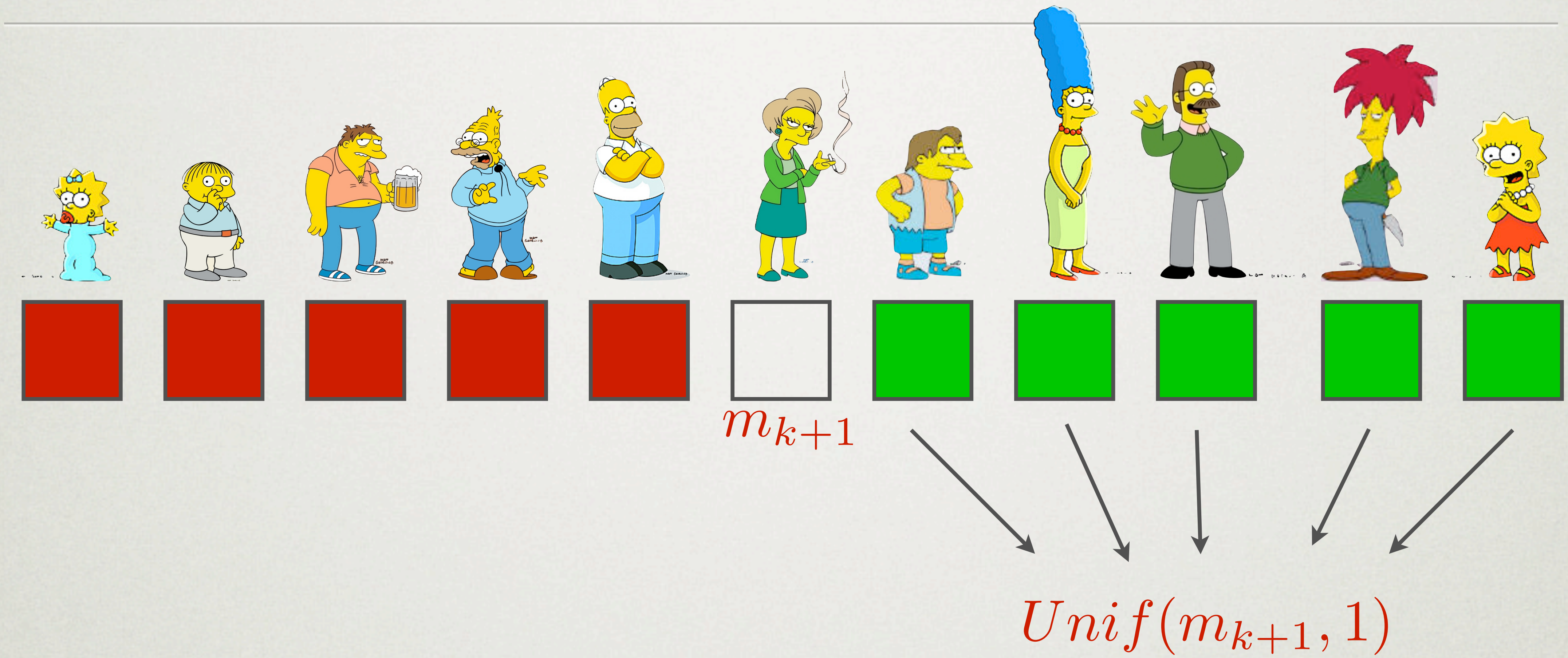

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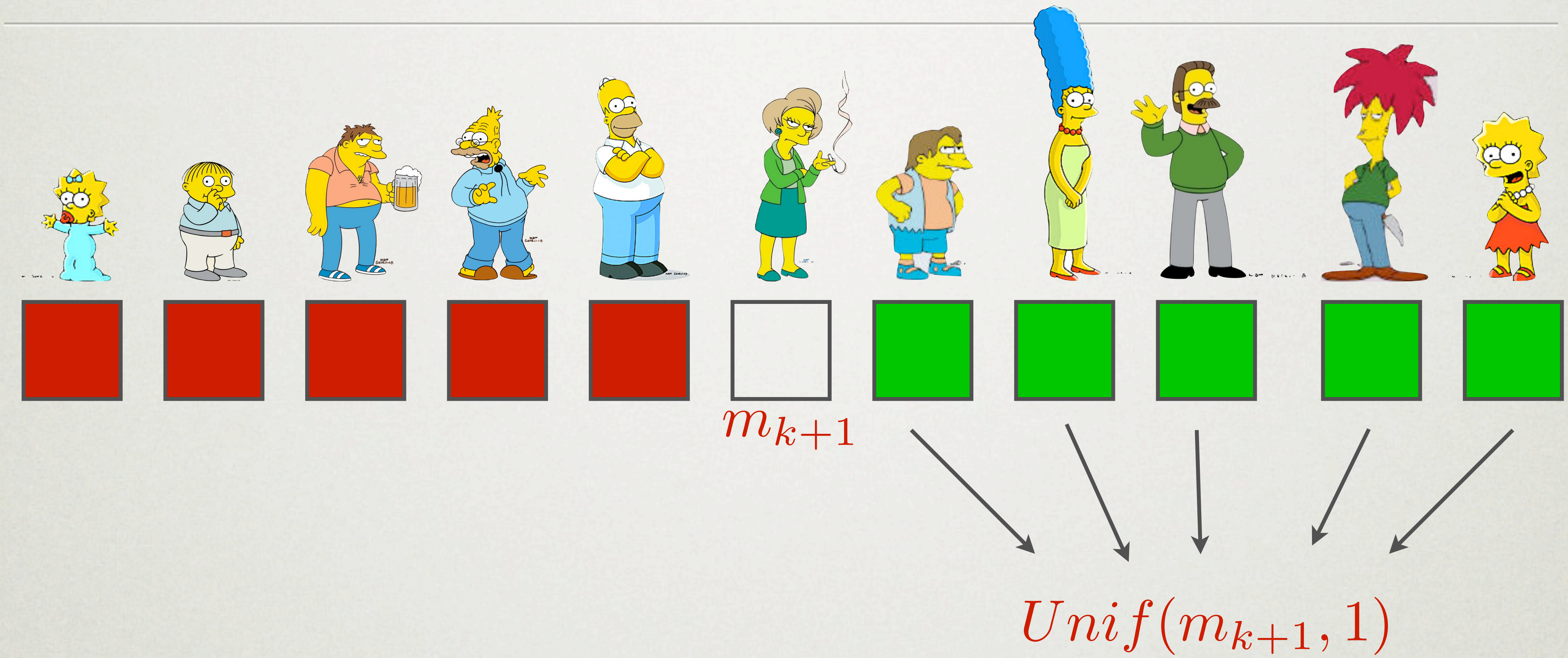
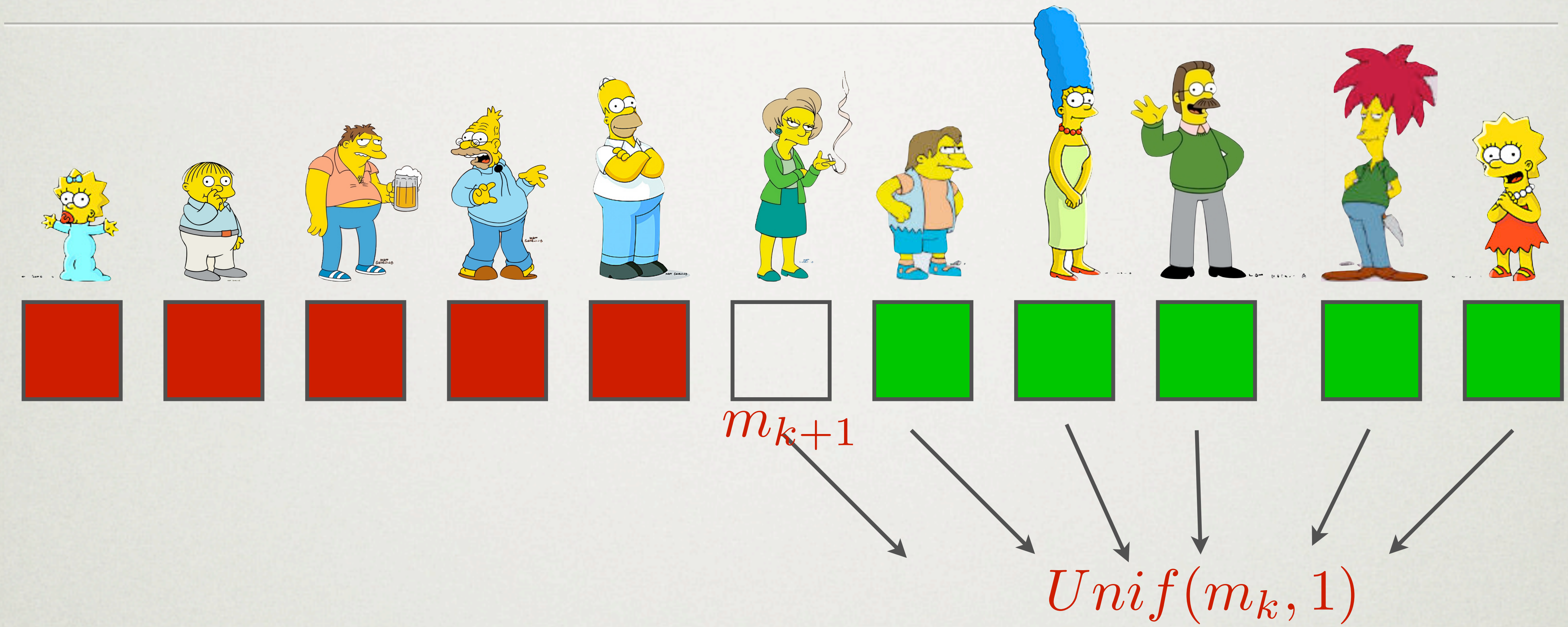


Induction holds.

Hiring Above Median



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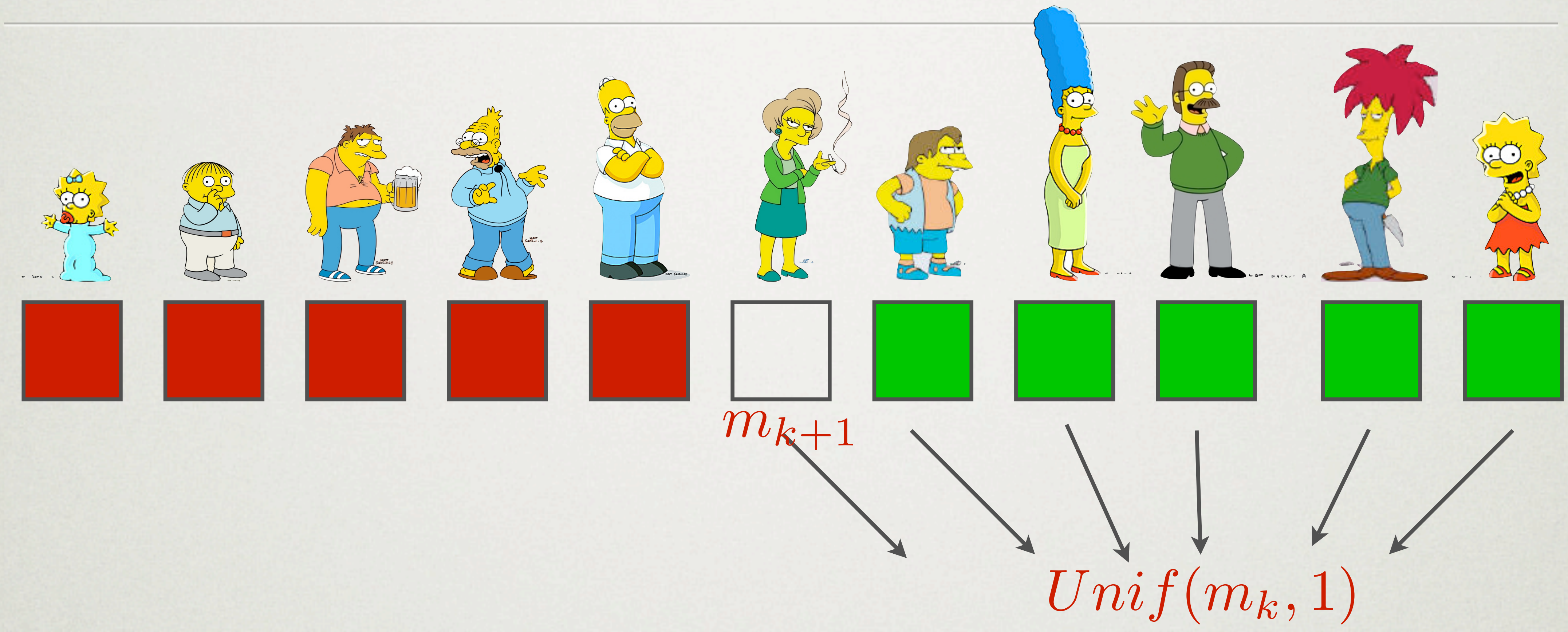
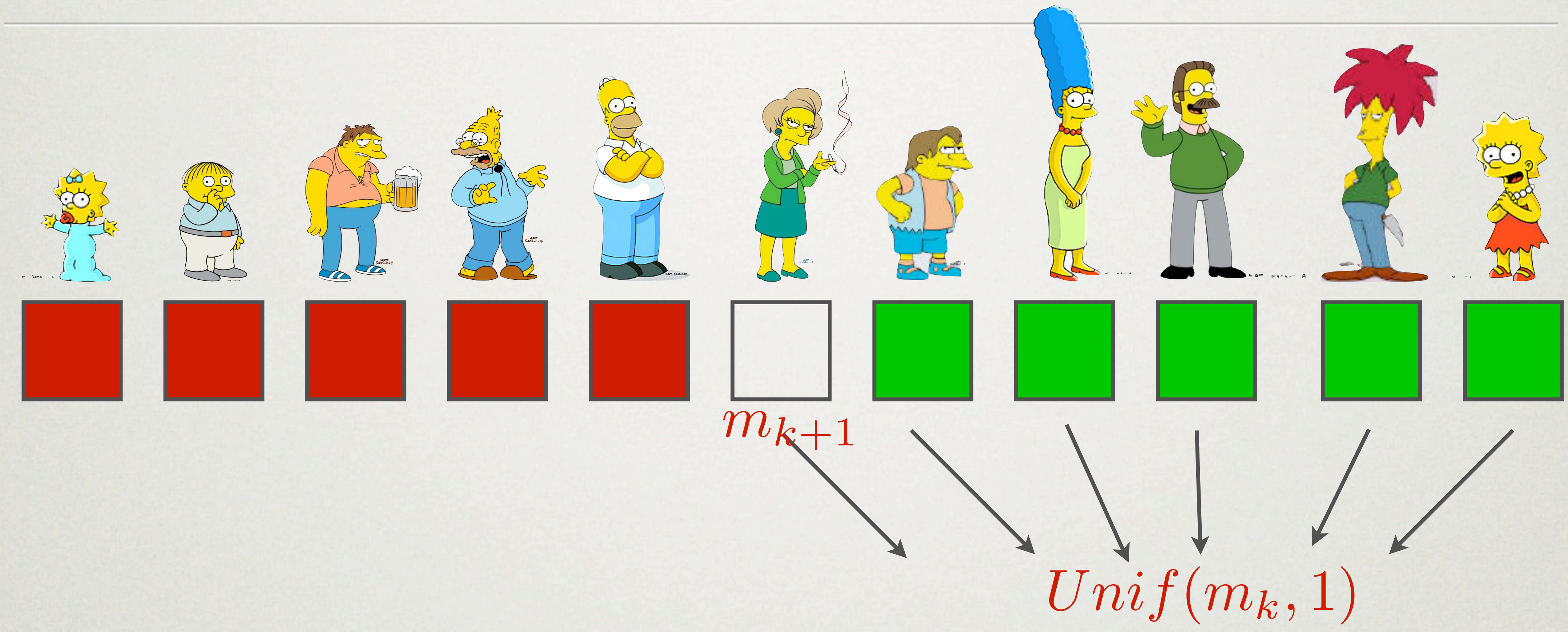


The median:

Hiring Above Median



The median: smallest of $k+1$ uniform r.v., each $Unif(m_k, 1)$

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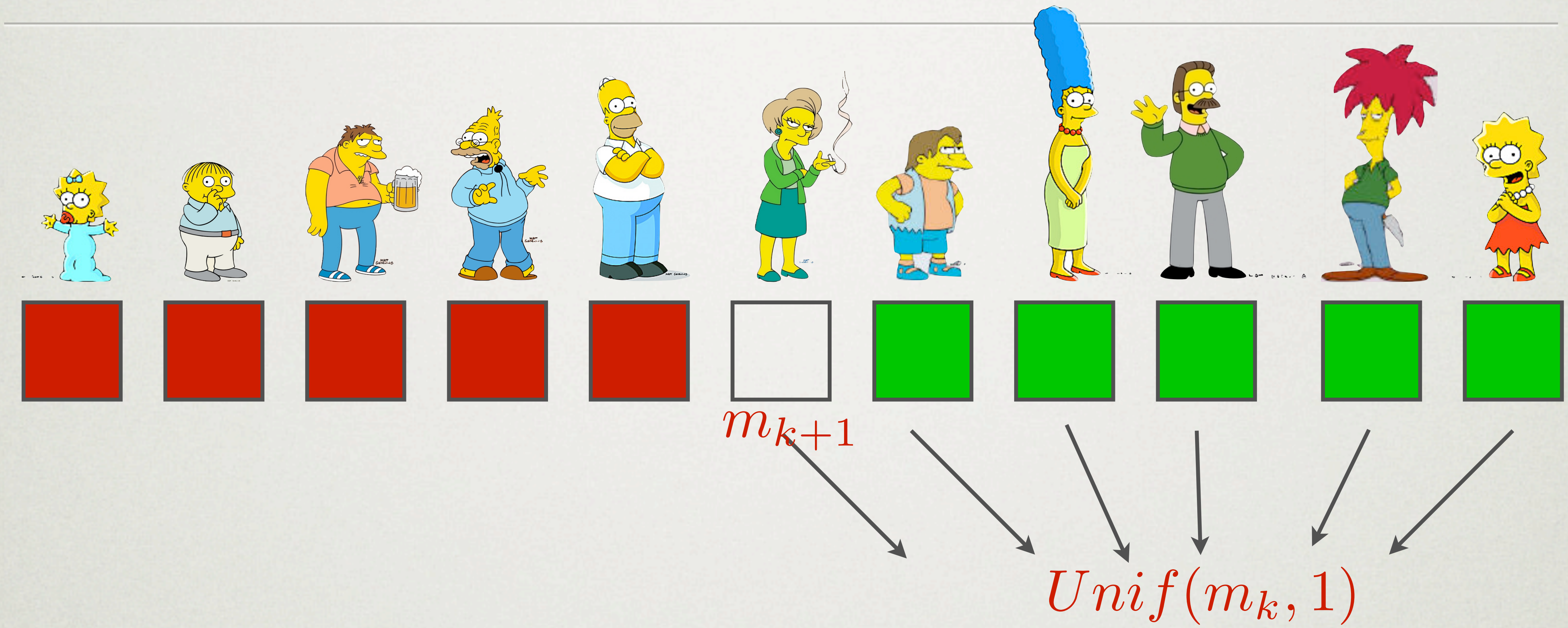

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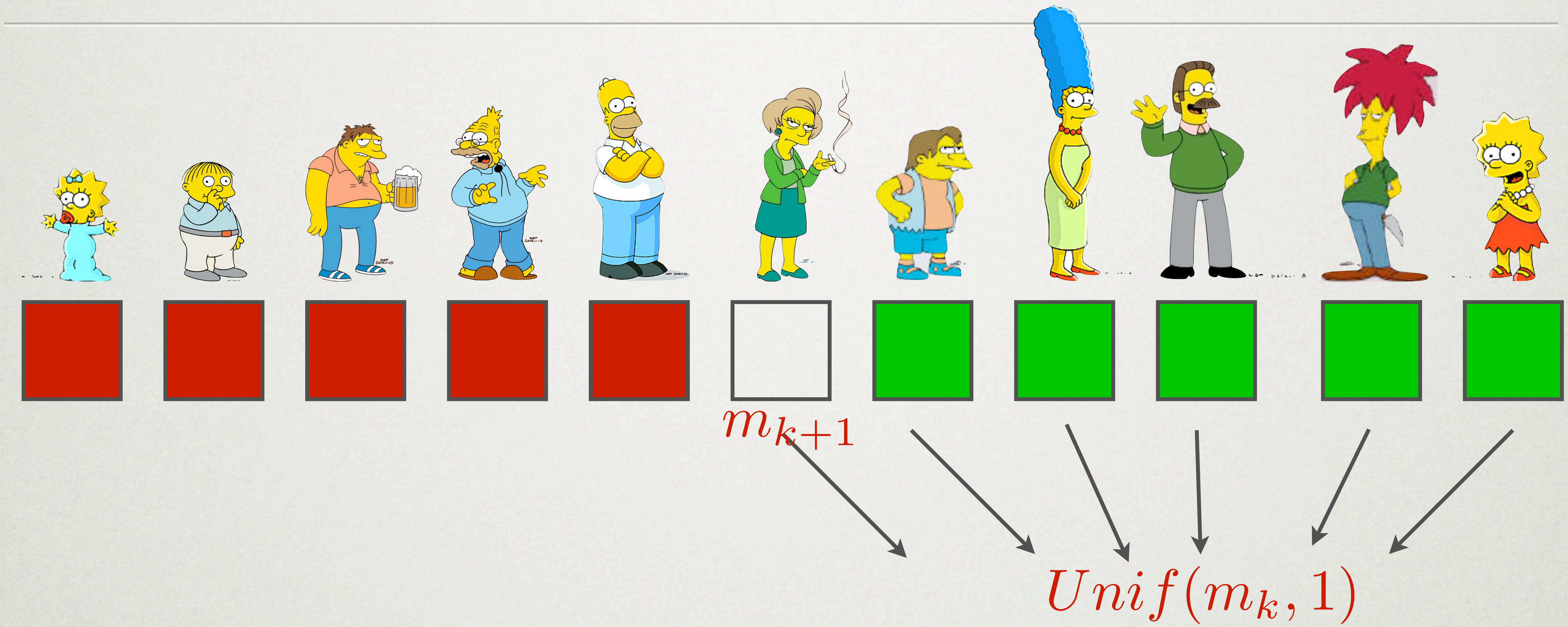

The median: smallest of $k+1$ uniform r.v., each $Unif(0, g'_k)$

$$g'_{k+1} | g'_k \sim g'_k Beta(k+2, 1)$$

Hiring Above Median

Expand (like the means): $g'_k \sim g \prod_{i=1}^{k} Beta(i+1, 1)$

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Caveat: this is the median gap! The mean gap is: $\Theta(\frac{\log n}{n})$

Hiring Above Median

Conclusion:

Average quality after n hires: $1 - \Theta(\frac{\log n}{n})$

Time to hire n employees: $\Theta(n^2)$

Again, very weak concentration results.

Extensions

Interview preprocessing:

- Self selection: quality may increase over time

Errors:

- Noisy estimates on i_q scores.

Firing:

- Periodically fire bottom 10% (Jack Welch Strategy)
- Similar analysis to the median.

Many more...

Thank You