On Threshold Behavior in Query Incentive Networks

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Outline

Some Have Questions Others Answers

Model introduced by Kleinberg and Raghavan [FOCS '05]

- Assume that a user, say *u*, of a social network has a question (e.g. Where to find a good physician?)
- Suppose that some subset of users have an answer
- How would *u* retrieve an answer from those individuals?

An Answer or The Answer Differences

To get an answer, *u* could:

- use a search engine; or
- ask friends.

What's the difference?

- Search engine: many answers *but* may not be reliable
- Friends: trusted answers *but* may not have any

Not enough friends? Reach friends' friends!
$\Rightarrow$ "web of trust".

Ask Your Friends, Please

- Reaching friends' friends through incentives
- Offer payment for answers
  $\hookrightarrow$ utility transfer
- Users act as strategic agents

Natural question: how much should u offer?

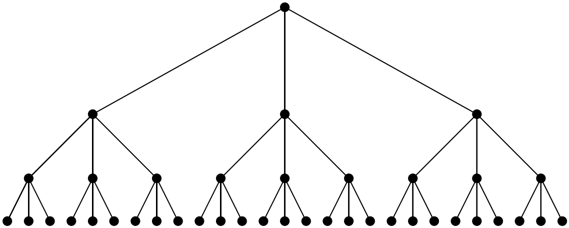
Informal Description
Key Ideas to Model

Key features from Kleinberg and Raghavan's model.

- Nodes and answers:
  - all answers are created equal
  - each person, independently, has an answer with probability $\frac{1}{n}$
- Users aware of only local topology
  $\hookrightarrow$ model with a random graph
- Providing incentives to answer, *not* creating a market

Network, Agents and Incentives

- Underlying network: complete d-ary tree ($d > 1$)
- Root: special node with query (question)
-



Network, Agents and Incentives

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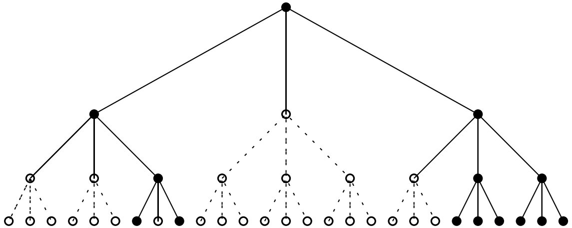
- Root: special node with query (question)
- Realized network: each node has (independently) $0 \leq i \leq d$ children with distribution $\mathcal{C}$ identities of nodes chosen uniformly at random



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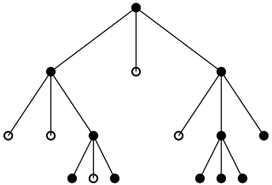

Completing the Model

For the incentives:

- parent node offers reward for answer to children
- if agent has an answer, communicates it to parent
- if there are many answers, choose one uniformly at random
- if providing answer, pay unit cost

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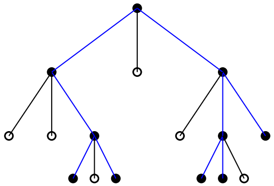
Formally, if a node is offered r and doesn't have an answer
Tradeoff faced by the node: if it offers $f(r)$,

- amount it keeps $r - f(r) - 1$
- probability of finding an answer in subtree increases with $f(r)$

Solution concept: Nash Equilibrium

Schema of Incentives

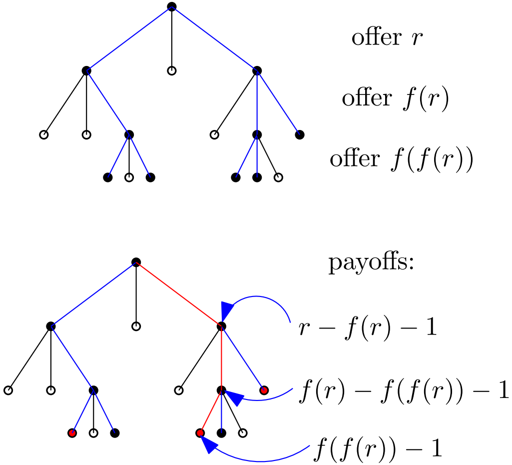


offer r

offer $f(r)$

offer $f(f(r))$

Schema of Incentives



Model as Branching Process Parameters

- $\mathcal{C}$ distribution with support $\{0, ..., d\}$
  let b be its expectation
- Realized network: realization of branching process according to $\mathcal{C}$
- identities of nodes chosen uniformly at random

$$b > 1 \Rightarrow \text{infinite network with constant probability}$$

- Average number of nodes in the first k layers:

$$\frac{1 - b^{k+1}}{1 - b} = \Theta\left(b^k\right)$$

- In $\Theta(\log n)$ layers, one answer with constant probability

Objective

- Given
  - probability of success $1 > \sigma > 0$;
  - the distribution $\mathcal{C}$;
  - the rarity of the answer n; and
  - agents play a Nash Equilibrium given by the function f
- Find minimum offer $R_{\sigma,\mathcal{C}}(n)$ to get answer with probability at least σ
- Study dependency of $R_{\sigma,\mathcal{C}}(n)$ on $\mathcal{C}$ and σ

Kleinberg and Raghavan Main Result

Setting:

- each child present independently at random
  $\hookrightarrow \mathcal{C}$ is a binomial distribution
- expected number of children b
- σ is a constant

Results:

- If $1 < b < 2$, then $R_{\sigma,\mathcal{C}}(n) = n^{\Omega(1)}$
- If $b > 2$, then $R_{\sigma,\mathcal{C}}(n) = O(\log n)$

Phase transition for rewards, but nothing obvious happening from a structural perspective!

Summary of Results

In this paper, we consider the robustness of Kleinberg and Raghavan's original result with respect to

- the distribution $\mathcal{C}$: result is robust; and
- the success probability σ: result is not robust

Robustness with respect to $\mathcal{C}$

Given:

- $\sigma = O(1)$
- $d = O(1)$
- an arbitrary distribution $\mathcal{C}$ with support $\{0, 1, ..., d-1, d\}$

Theorem

For all σ, d and distributions $\mathcal{C}$ as defined above, we have that

- *If $1 < b < 2$, then $R_{\sigma,\mathcal{C}}(n) = n^{\Theta(1)}$*
- *If $b > 2$, then $R_{\sigma,\mathcal{C}}(n) = O(\log n)$*

High Probability Case: Vanishing Threshold

- We want $\sigma = 1 - o(1)$

Given:

- $\sigma_0 = 1 - \frac{1}{n}$
- $d = O(1)$
- an arbitrary distribution $\mathcal{C}$ with support $\{1, ..., d-1, d\}$

Theorem

For all $\sigma > \sigma_0$, d and distributions $\mathcal{C}$ as defined above, we have that

- *If $1 < b < 2$, then $R_{\sigma,\mathcal{C}}(n) = n^{\Theta(1)}$*
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Discussion of Results

Let ℓ be the expected path length to an answer.

For σ constant:

- $\ell = \Theta(\log n)$
- $2 > b > 1$, reward exponential in ℓ
- $b > 2$, reward of same order as ℓ

For $\sigma \geq 1 - \frac{1}{n}$:

- $2 > b > 1$, still exponential in ℓ
- $b > 2$, also exponential in ℓ *but* blowup occurs in the last $O(\log \log n)$ steps

Current Research and Open Problems

Many open directions remain:

- Different network topology
- Aggregate answers

Most important open problem: probabilistic interpretation/proof of results.

Comments?
Questions?

Thank you