Counting Triangles & The Curse of the Last Reducer

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Yahoo! Research

Why Count Triangles?

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Clustering Coefficient:

Given an undirected graph $G = (V, E)$

cc(v) = fraction of v's neighbors who are neighbors themselves

$$= \frac{|\{(u,w) \in E | u \in \Gamma(v) \wedge w \in \Gamma(v)\}|}{\binom{d_v}{2}}$$

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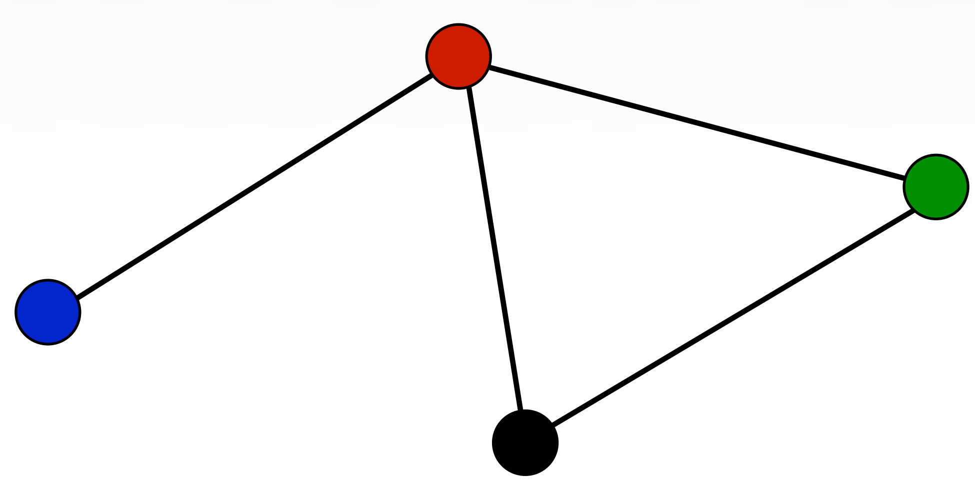

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cc ( ) = 1/3

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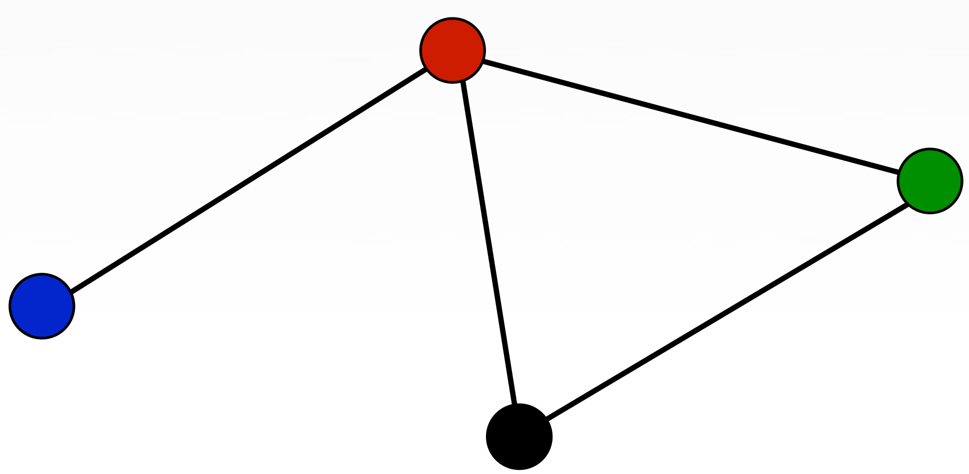

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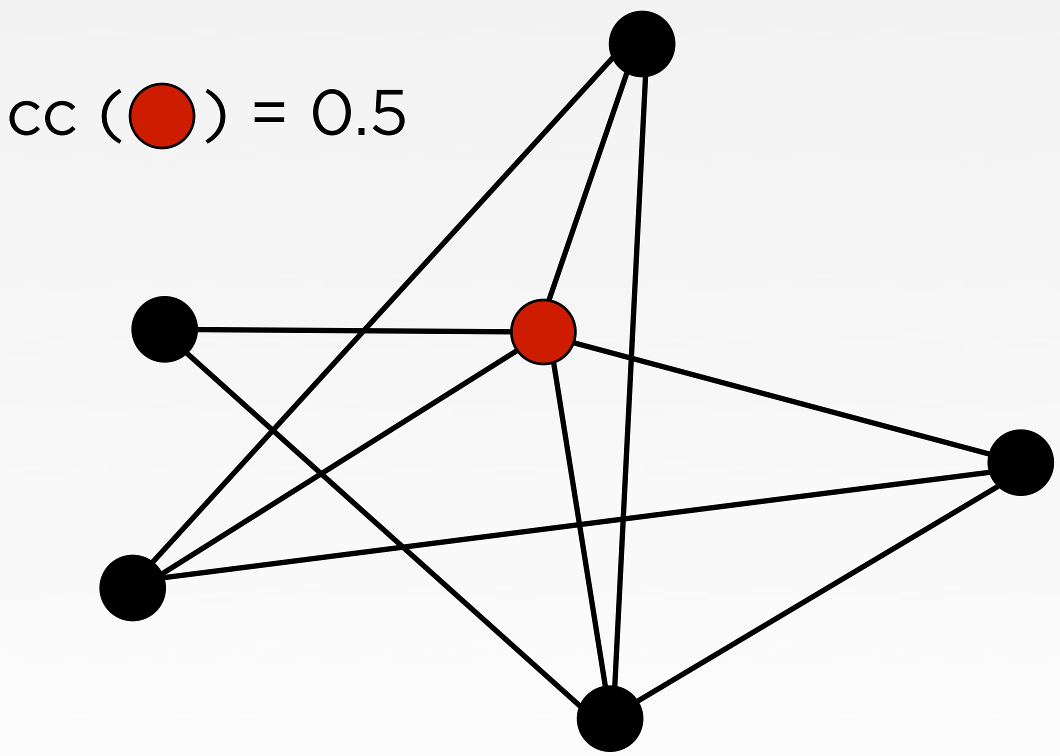
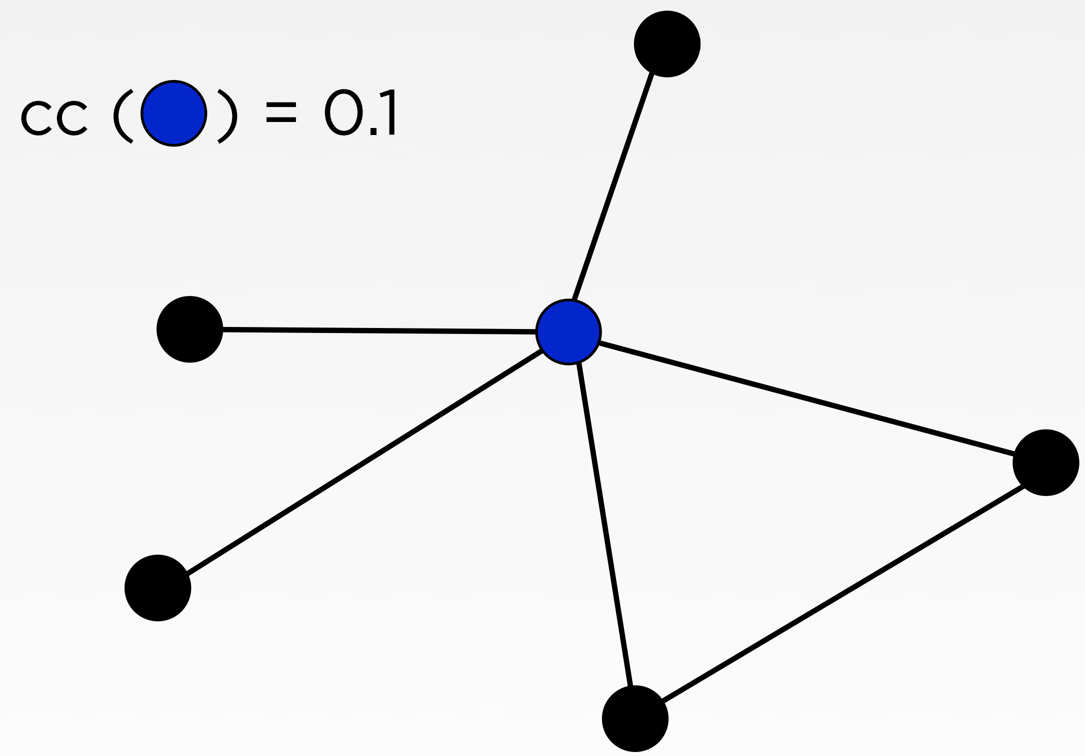
Why Clustering Coefficient?

Captures how tight-knit the network is around a node.



vs.



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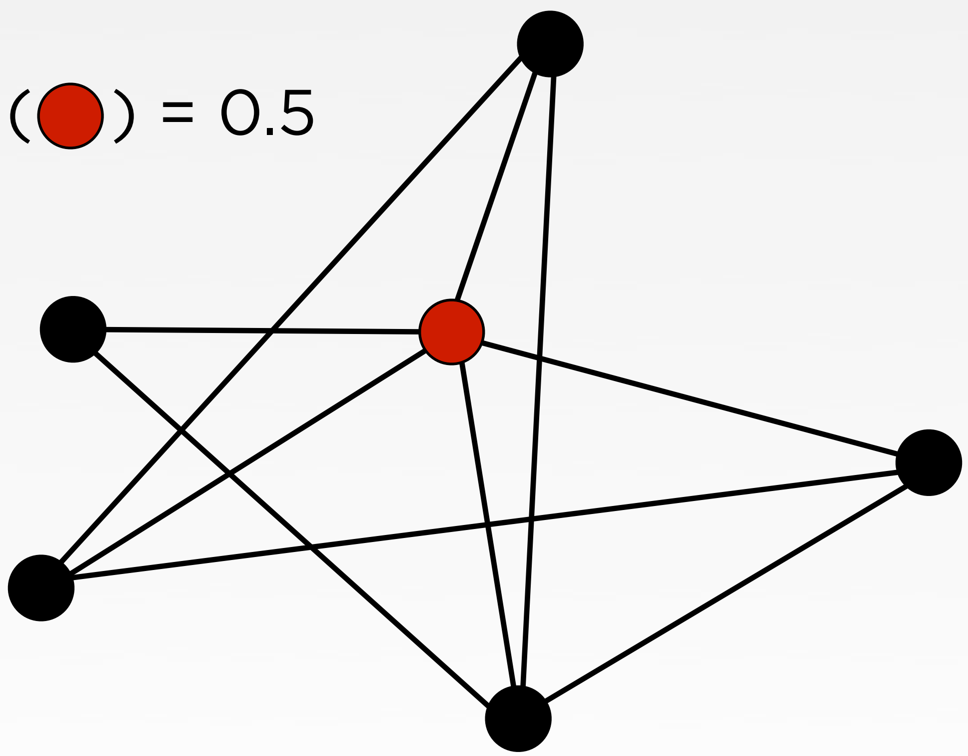
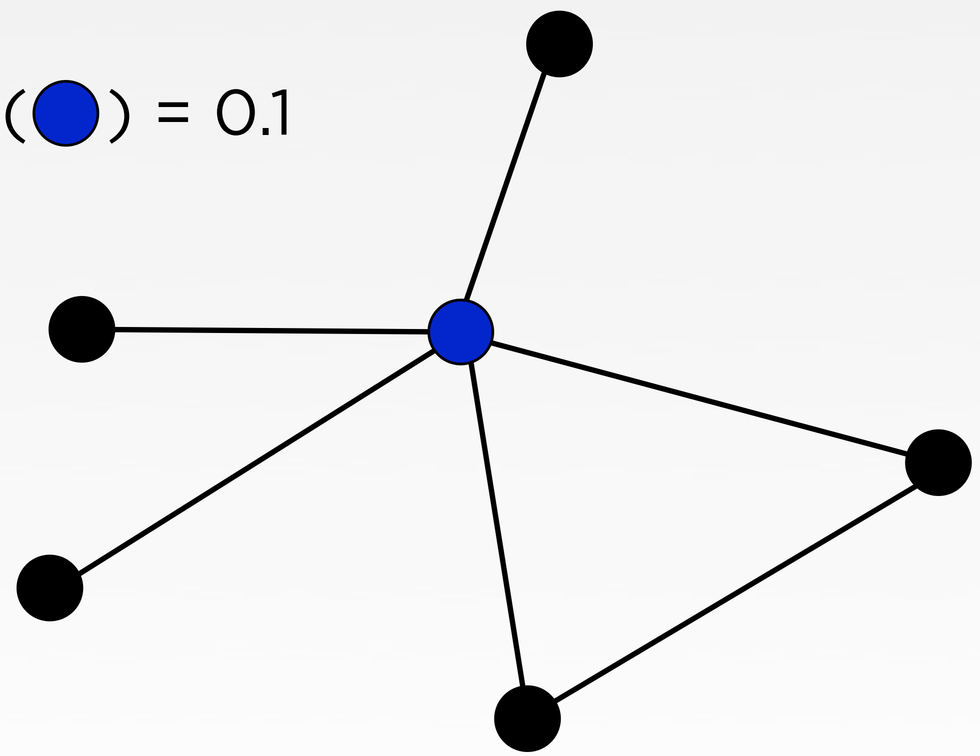
Captures how tight-knit the network is around a node.

cc (●) = 0.5



vs.

cc (●) = 0.1



Network Cohesion:

- Tightly knit communities foster more trust, social norms. [Coleman ’88, Portes ’88]

Structural Holes:

- Individuals benefit form bridging [Burt ’04, ’07]

Why MapReduce?

De facto standard for parallel computation on large data

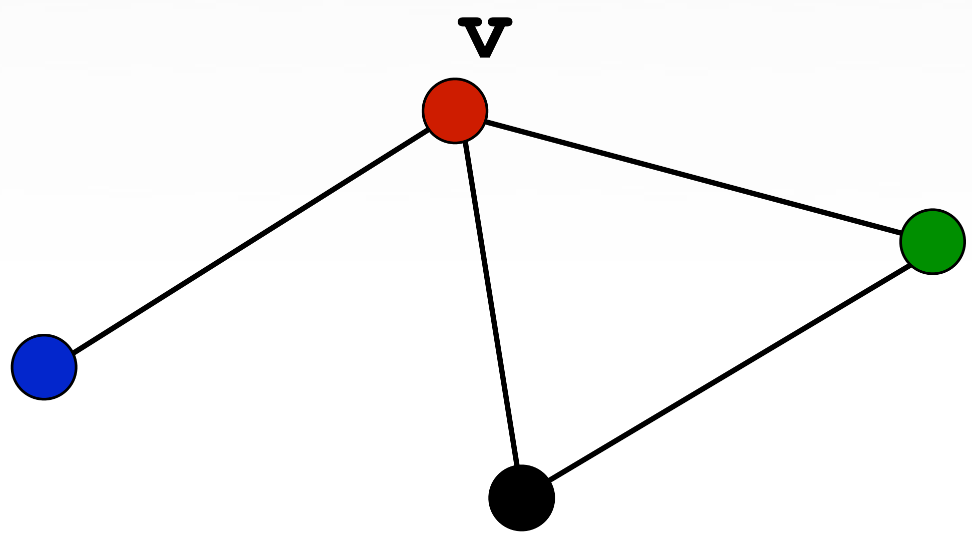
- Widely used at: Yahoo!, Google, Facebook,
- Also at: New York Times, Amazon.com, Match.com, ...
- Commodity hardware
- Reliable infrastructure

- Data continues to outpace available RAM !

How to Count Triangles

Sequential Version:

```
foreach v in V
    foreach u,w in Adjacency(v)
        if (u,w) in E
            Triangles[v]++
```



Triangles[v]=0

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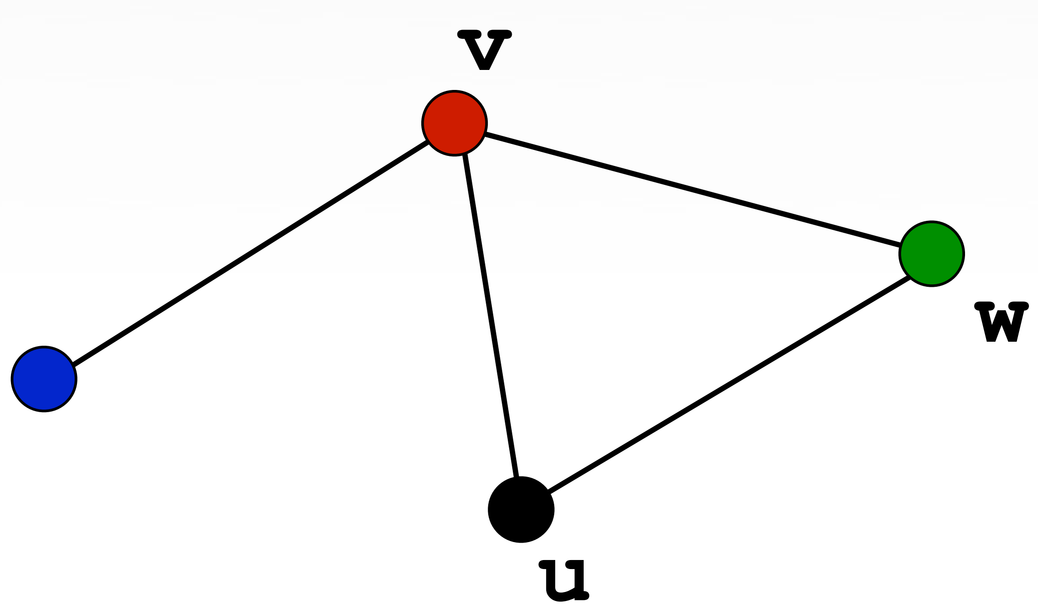

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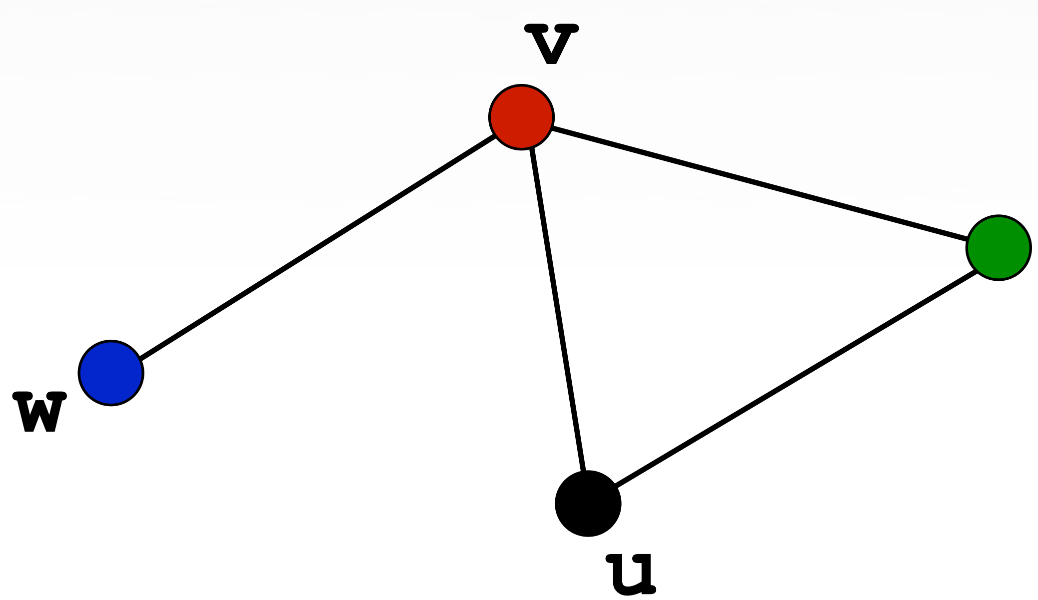

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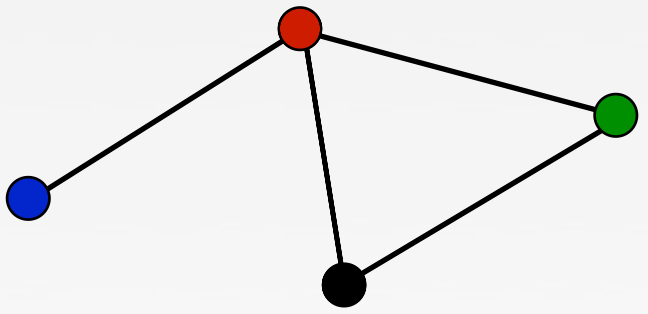
Running time: $\sum_{v \in V} d_v^2$

Even for sparse graphs can be quadratic if one vertex has high degree.

Parallel Version

Parallelize the edge checking phase

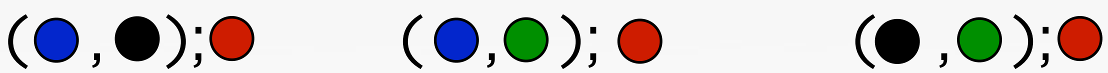


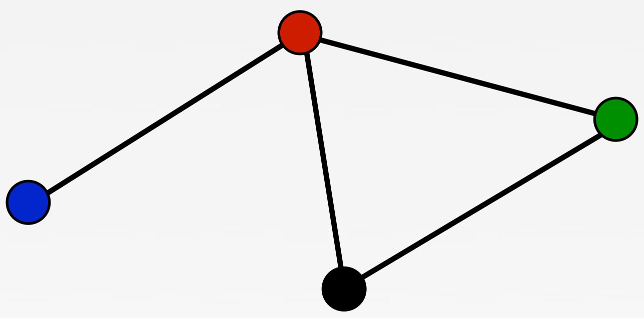
Parallel Version

Parallelize the edge checking phase

- Map 1: For each v send $(v, \Gamma(v))$ to single machine.
- Reduce 1: Input: $\langle v; \Gamma(v) \rangle$

  Output: all 2 paths $\langle (v_1, v_2); u \rangle$ where $v_1, v_2 \in \Gamma(u)$





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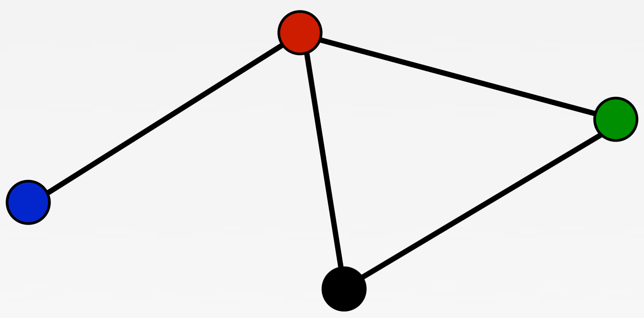
  (●, ●); ●   (●, ●); ●   (●, ●); ●



- Map 2: Send $\langle (v_1, v_2); u \rangle$ and $\langle (v_1, v_2); \$ \rangle$ for $(v_1, v_2) \in E$ to same machine.
- Reduce 2: input: $\langle (v, w); u_1, u_2, \ldots, u_k, \$? \rangle$

  Output: if $\$$ part of the input, then: $u_i = u_i + 1/3$

(●, ●); ●, $\$$ $\longrightarrow$ ● $+1/3$ ● $+1/3$ ● $+1/3$

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Data skew

How much parallelization can we achieve?

- Generate all the paths to check in parallel
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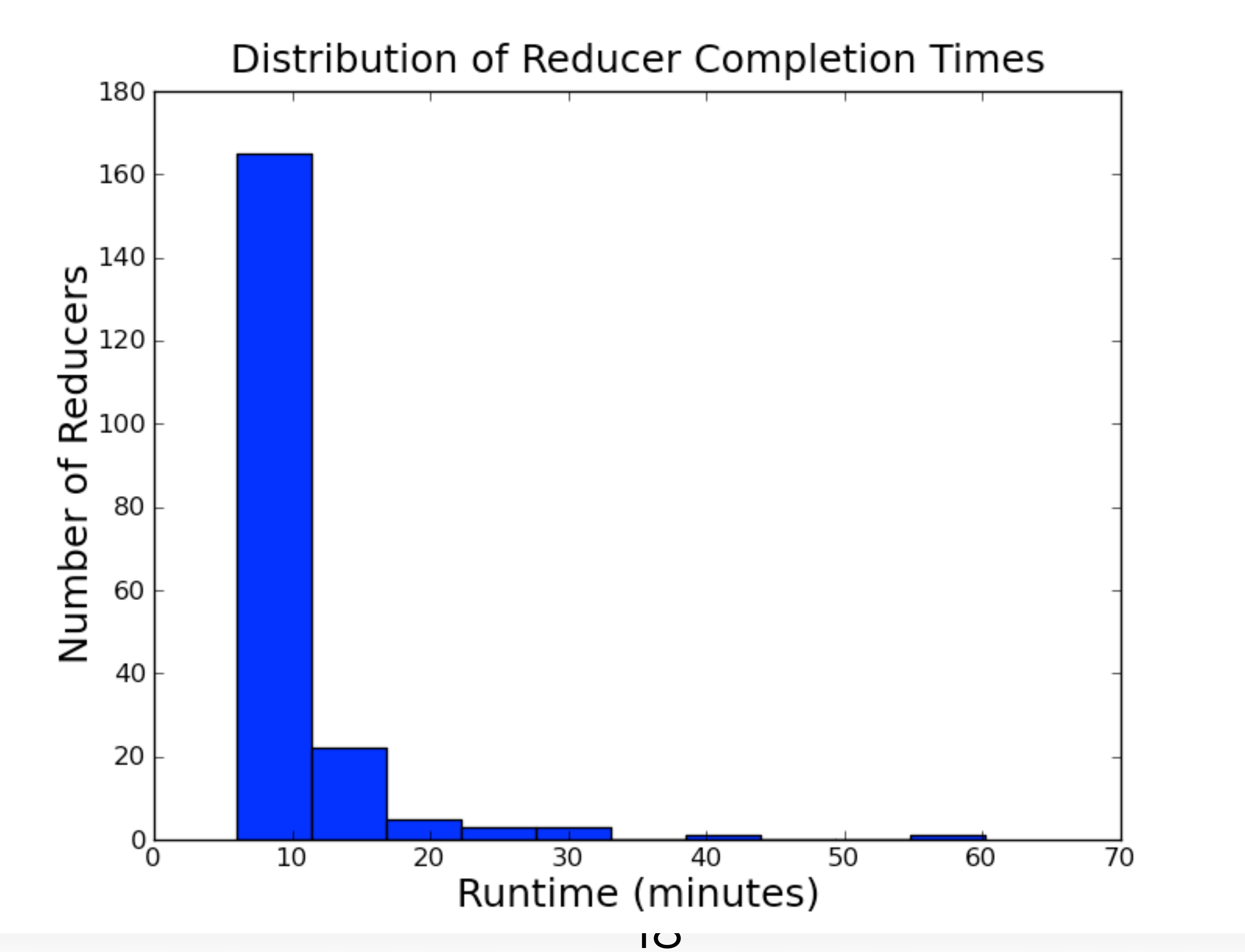
Naive parallelization does not help with data skew

- Some nodes will have very high degree
- Example. 3.2 Million followers, must generate 10 Trillion (10^13) potential edges to check.
- Even if generating 100M edges per second, 100K seconds ~ 27 hours.

"Just 5 more minutes"

Running the naive algorithm on LiveJournal Graph

- 80% of reducers done after 5 min
- 99% done after 35 min



Adapting the Algorithm

Approach 1: Dealing with skew directly

- currently every triangle counted 3 times (once per vertex)
- Running time quadratic in the degree of the vertex
- Idea: Count each once, from the perspective of lowest degree vertex
- Does this heuristic work?

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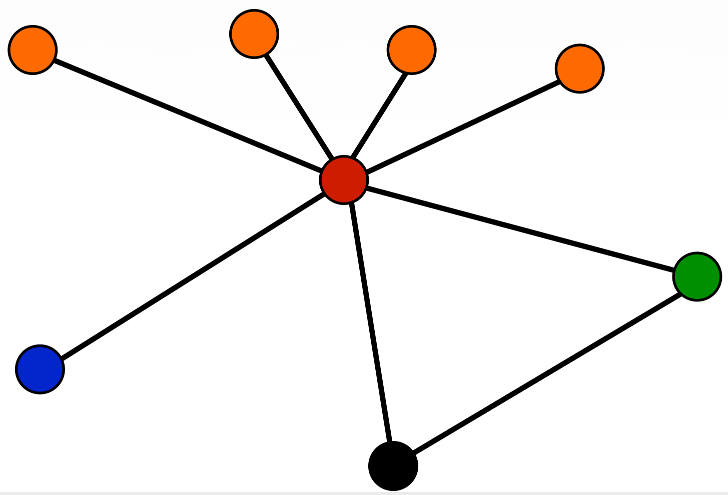
Approach 2: Divide & Conquer

- Equally divide the graph between machines
- But any edge partition will be bound to miss triangles
- Divide into overlapping subgraphs, account for the overlap

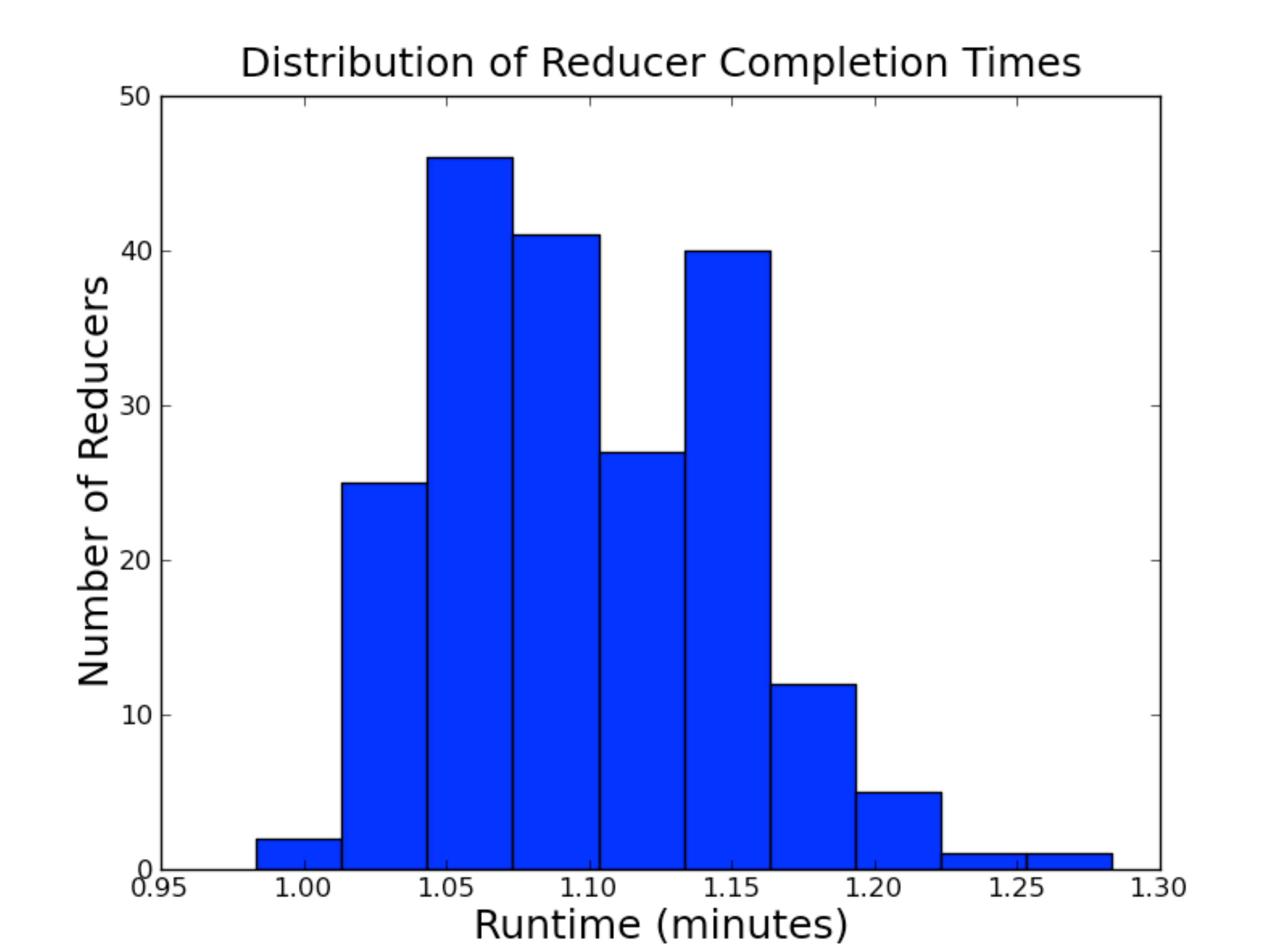
How to Count Triangles Better

Sequential Version [Schank '07]:

```
foreach v in V
   foreach u,w in Adjacency(v)
     if deg(u) > deg(v) && deg(w) > deg(v)
         if (u,w) in E
            Triangles[v]++
```



Does it make a difference?



Dealing with Skew

Why does it help?

- Partition nodes into two groups:
  - Low: $\mathcal{L} = \{v : d_v \leq \sqrt{m}\}$
  - High: $\mathcal{H} = \{v : d_v > \sqrt{m}\}$
- There are at most n low nodes; each produces at most $O(m)$ paths
- There are at most $2\sqrt{m}$ high nodes
  - Each produces paths to other high nodes: $O(m)$ paths per node

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- These two are identical !
- Therefore, no mapper can produce substantially more work than others.
- Total work is $O(m^{3/2})$, which is optimal

Approach 2: Graph Split

Partitioning the nodes:

- Previous algorithm shows one way to achieve better parallelization
- But what if even $O(m)$ is too much. Is it possible to divide input into smaller chunks?

Graph Split Algorithm:

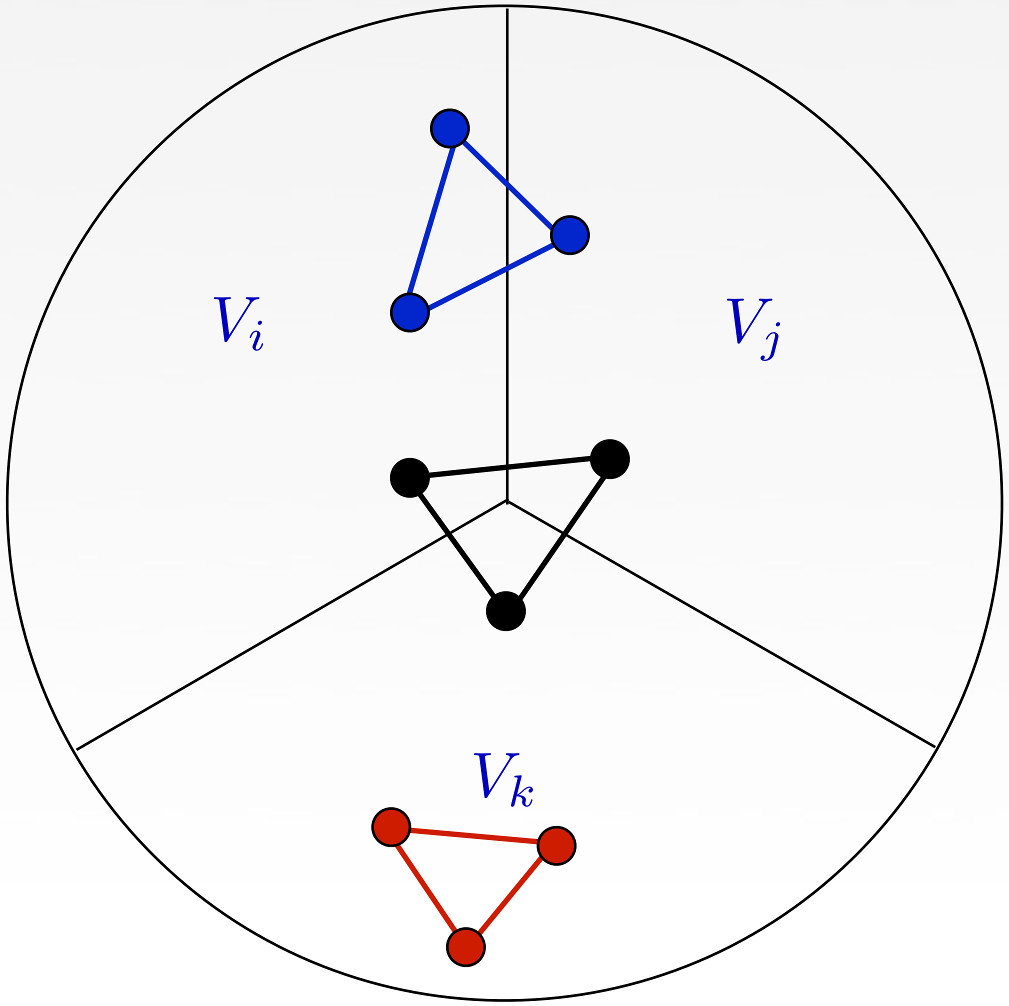
- Partition vertices into p equal sized groups $V_1, V_2, \ldots, V_p$.
- Consider all possible triples (V_i, V_j, V_k) and the induced subgraph:

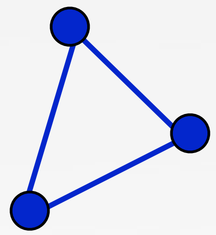
$$G_{ijk} = G\left[V_i \cup V_j \cup V_k\right]$$

- Compute the triangles on each G_{ijk} separately.

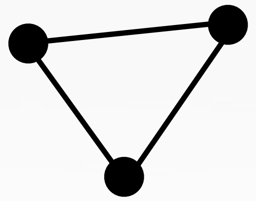
Approach 2: Graph Split

Some Triangles present in multiple subgraphs:

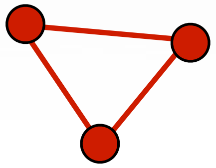




in p-2 subgraphs



in 1 subgraph



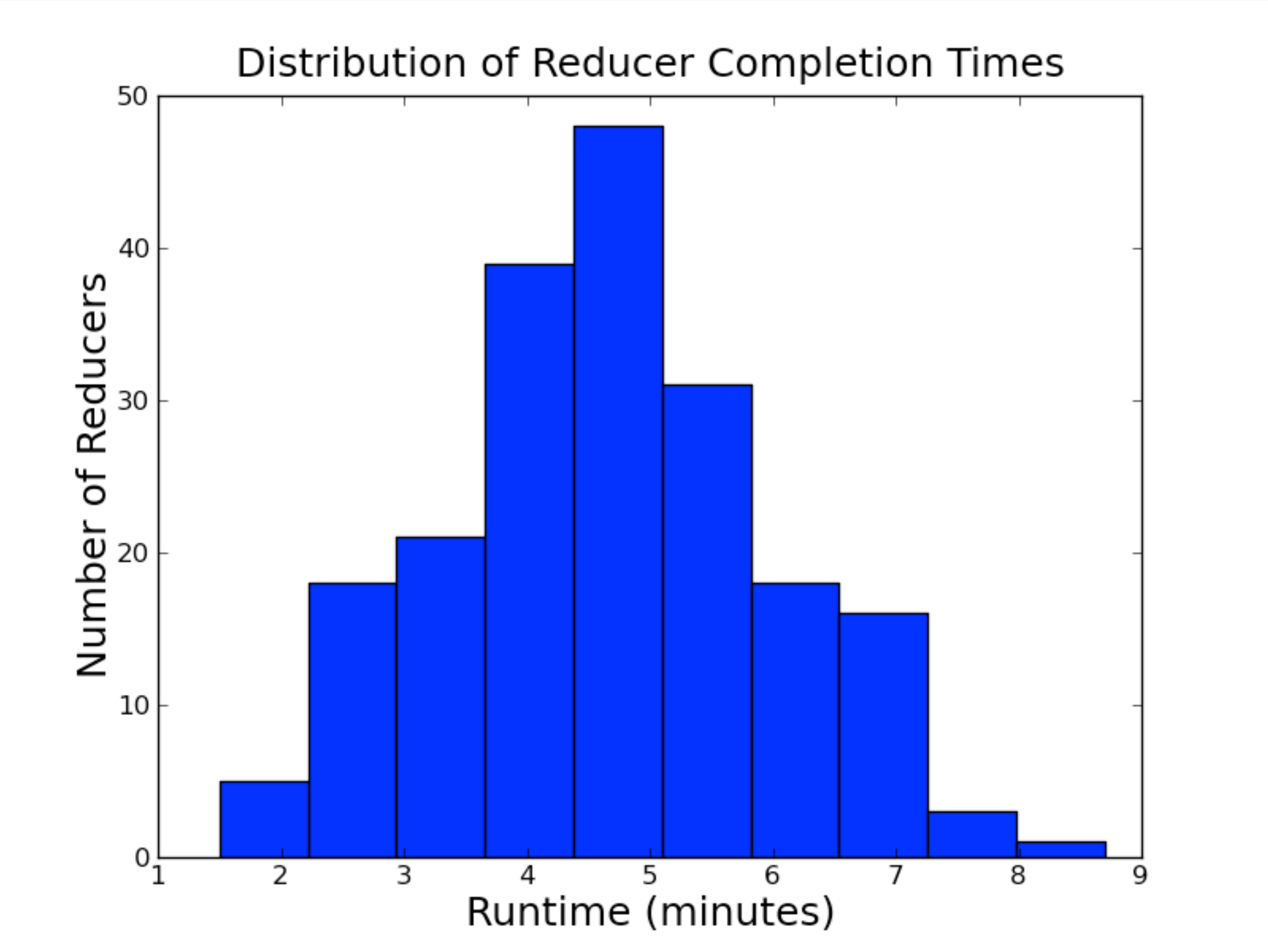
in ~p^2 subgraphs

Can count exactly how many subgraphs each triangle will be in

Approach 2: Graph Split

Analysis:

- Each subgraph has $O(m/p^2)$ edges in expectation.
- Very balanced running times



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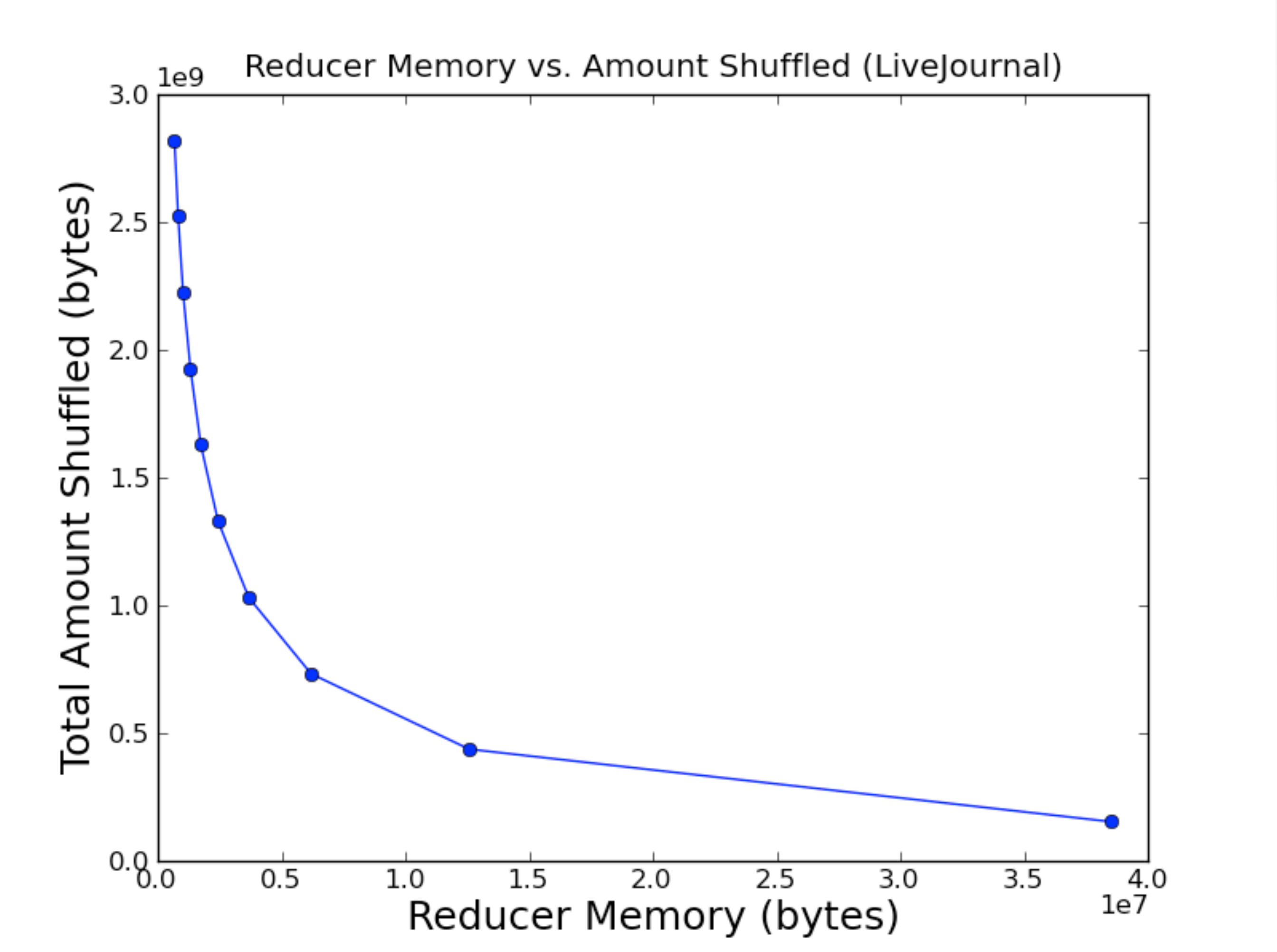

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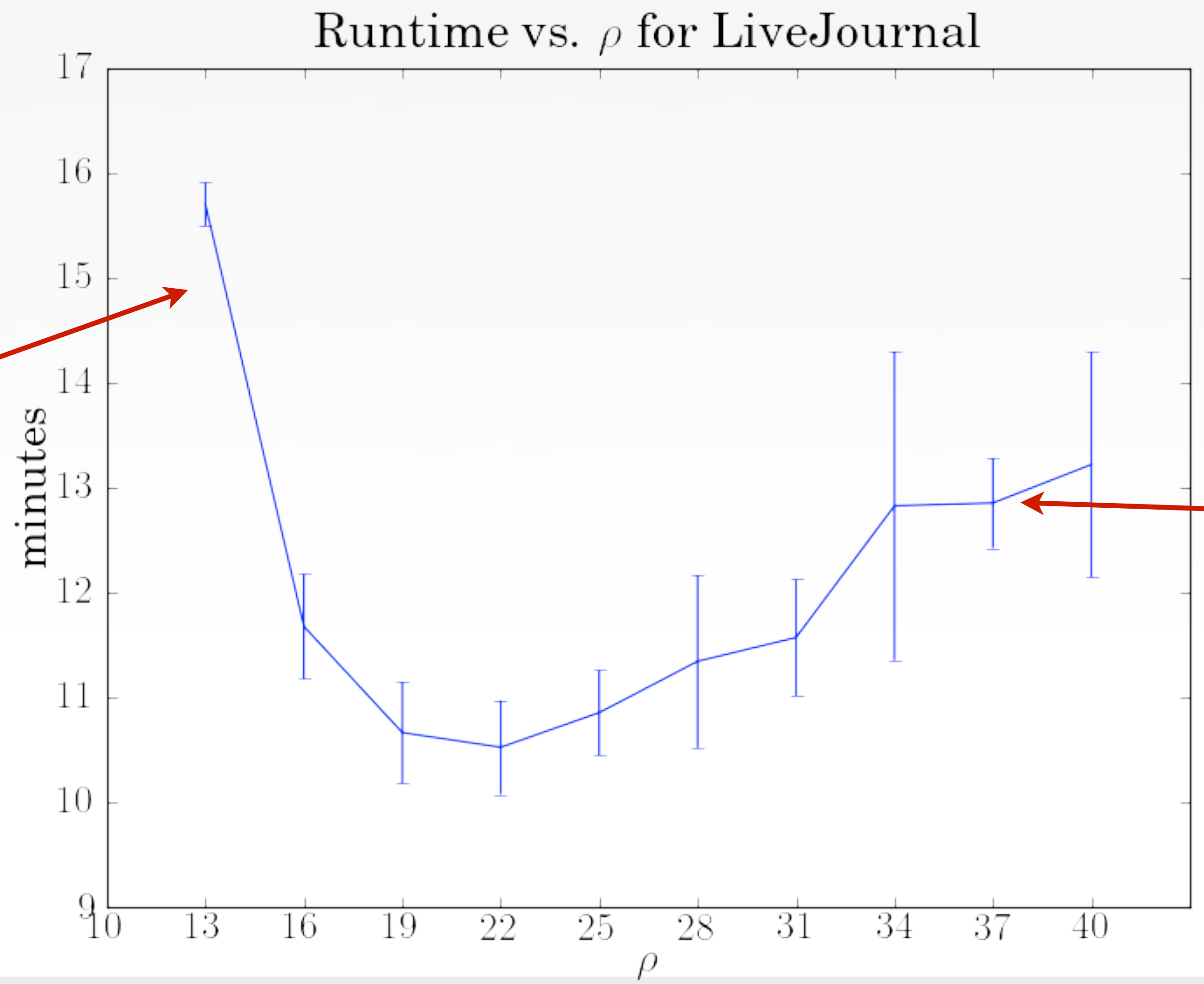

Input too big: paging

Shuffle time increases with duplication

Overall

Naive Parallelization Doesn’t help with Data Skew

Related Work

- Tsourakakis et al. [09]:
  - Count global number of triangles by estimating the trace of the cube of the matrix
  - Don't specifically deal with skew, obtain high probability approximations.
- Becchetti et al. [08]
  - Approximate the number of triangles per node
  - Use multiple passes to obtain a better and better approximation

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Think about data skew.... and avoid the curse

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- The possibilities are endless!

Thank You