CS269I: Exercise Set #2

Due by 11:59 PM on Wednesday, October 12, 2016

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code 9P8K49 to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 3 Exercises

Exercise 8

For exercises 8–13, assume that there is an odd number of voters. For a set of votes (i.e., ranked lists) over a set A of alternatives, we say that alternative a *beats* b if more than half of the voters rank a somewhere above b in their lists. A *Condorcet winner* is an alternative that beats every other alternative.

Show by example that there is not always a Condorcet winner.

Exercise 9

A voting rule satisfies the *Condorcet condition* if it elects a Condorcet winner whenever one exists.

Does the plurality rule satisfy the Condorcet condition?[1] Provide either a proof that it does or a counterexample (i.e., a set of votes where there is a Condorcet winner a and the rule chooses an alternative different from a).

Exercise 10

Does ranked-choice voting satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

[1]Throughout this exercise set, assume that ties are broken in some consistent way, such as lexicographically.

Exercise 11

Does the Borda count satisfy the Condorcet condition? Provide either a proof that it does or a counterexample.

Lecture 4 Exercises

Exercise 12

Recall that the Kemeny voting rule chooses the ranking σ that maximizes the number $\sum_{a,b \in A} k^{\sigma}_{ab}$ of pairwise agreements with the votes, where k^{σ}_{ab} denotes the number of voters who rank a vs. b in the same relative order that σ does.

Does the Kemeny voting rule satisfy the Condorcet condition? Meaning, is the Condorcet winner ranked first in the ranking produced by the Kemeny rule (when one exists)? Provide either a proof that it does or a counterexample.

Exercise 13

Prove that the Kemeny rule can alternatively be described as choosing the ranking σ that minimizes the sum of the bubble-sort distances[2] to voters' lists. That is, given lists $L_1, \ldots, L_n$, the outcome of the Kemeny rule is $\operatorname{argmin}_{\sigma} \sum_{i=1}^{n} d(\sigma, L_i)$, where d denotes bubble-sort distance.

Exercise 14

Recall Knapsack voting. Let B denote the budget.

Knapsack Voting

1. Each voter i approves a set S_i of projects whose total cost is at most B.
2. Projects are ordered according to the number of approving voters. (Ties are broken in some consistent way, such as lexicographically.)
3. Projects are funded in this order until the budget runs out. To make sure that the budget is fully spent, the last project might be only partially funded.

Assume that each voter i has a set S_i^* of projects that she wants funded (with total cost at most B), and that her preferences are for as much money as possible to be spent on projects in S_i^*. That is, if project j is funded at the level b_j, then voter i's "utility" is defined to be $\sum_{j \in S_i^*} b_j$.

Prove that Knapsack voting is strategyproof, meaning that a voter always maximizes her utility by voting for her true set S_i^* (no matter what the other voters do).

Exercise 15

Under the same assumptions as the previous exercise, and assuming that voters vote honestly, prove that the outcome of Knapsack voting is Pareto optimal. That is, every other way of spending B dollars on the projects (and on each project spending at most the project's cost) that increases the utility of some voter also decreases the utility of some other voter.

Bonus Exercise (Extra Credit)

Does the Borda count arise as the maximum likelihood estimator with respect to some probabilistic voting model? Prove your answer.

[2] The *bubble-sort distance* (a.k.a. Kendall tau distance) between two ranked lists is the number of swaps that the bubble sort algorithm would do to place one list in the same order as the other list; this equals the number of pairs of elements that appear in opposite relative orders in the two lists.