CS269I: Exercise Set #4

Due by 11:59 PM on Wednesday, October 26, 2016

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code 9P8K49 to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 7 Exercises

Exercise 23

Braess's paradox shows that adding a new edge to a selfish routing network can increase the commute time of all traffic by a factor of 4/3. Show that if nonlinear cost functions are allowed, then adding a new edge can increase the commute time of all traffic by a factor arbitrarily close to 2. (Your cost functions should still be nonnegative, nondecreasing, and continuous.)

Exercise 24

Compute the price of anarchy in the nonlinear Pigou's example, where there are two parallel edges from the source s to the destination t, one with the constant cost function $c(x) = 1$, and the other with the cost function $c(x) = x^p$ (Figure 1). You should be able to obtain a (somewhat messy) closed-form formula for the price of anarchy as a function of $p \geq 1$. Plot the result, with p on the x-axis, and the corresponding price of anarchy on the y-axis.

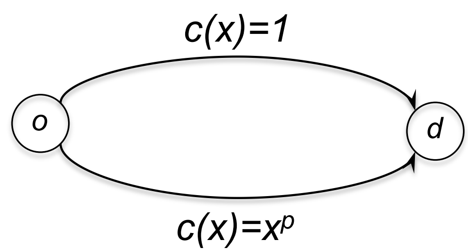

Figure 1: Nonlinear Pigou's example.

Lecture 8 Exercises

Exercise 25

Consider an AS graph that is connected (i.e., every AS can reach the destination d via a sequence of direct physical connections). Consider the following procedure that attempts to compute a stable routing:

1. Initialize $H = \{d\}$ and $P_d = \emptyset$. (Here d is the destination AS.)

   [We maintain the invariants that every AS v in H has a path P_v to d that lies entirely in H, and that $\cup_{v \in H} P_v$ forms a tree directed into d.]

2. For $u \notin H$, call a u-d path P *consistent with* H if it concludes with the v-d path P_v, where v is the first vertex of P that lies in H.

3. While there is a vertex $u \notin H$ such that u's favorite path consistent with H has the form $(u, v) \oplus P_v$ for a vertex $v \in H$:[1]

   (a) Add u to H.

   (b) Set $P_u = (u, v) \oplus P_v$.

4. If H is the entire vertex set, return the tree $\cup_{v \in H} P_v$. Otherwise, FAIL.

Prove that if this procedure does not FAIL, then it terminates with a stable routing.

Exercise 26

Prove that the procedure in the previous exercise can only FAIL if the given AS graph has a dispute wheel.

[Hint: use arguments reminiscent of those used to prove uniqueness in lecture.]

[1] $P \oplus Q$ denotes the concatenation of the paths P and Q. (The ending vertex of P should be the same as the starting vertex of Q.)