CS269I: Exercise Set #7

Due by 11:59 PM on Wednesday, November 16, 2016

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code 9P8K49 to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 13 Exercises

Exercise 27

Consider a single-item auction with at least three bidders. Prove that awarding the item to the highest bidder, at a price equal to the third-highest bid, yields an auction that is *not* truthful.

Exercise 28

Prove that for every false bid $b_i \neq v_i$ by a bidder in a second-price auction, there exist bids $\mathbf{b}_{-i}$ by the other bidders such that i's utility when bidding b_i is strictly less than when bidding v_i.

Exercise 29

Suppose there are k identical copies of an item and $n > k$ bidders. Suppose also that each bidder can receive at most one item. What is the analog of the second-price auction? Prove that your auction is truthful.

Exercise 30

Suppose a subset S of the bidders in a second-price single-item auction decide to collude, meaning that they submit their bids in a coordinated way to maximize the sum of their utilities. Assume that bidders outside of S bid truthfully. State and prove necessary and sufficient conditions on the set S such that the bidders of S can increase their combined utility via non-truthful bidding.

Lecture 14 Exercises

Exercise 31

Suppose you're participating in a first-price single-item auction with two other bidders, and that you know the following: (i) your own valuation is $v_1 \in [0, 1]$; (ii) the other valuations v_2, v_3 are independent and drawn uniformly from the interval $[0, 1]$; (iii) the second bidder uses the bidding strategy $b_2(v_2) = \alpha v_2$ for a constant $\alpha \in (0, 1)$; and (iv) the third bidder uses the bidding strategy $b_3(v_3) = \beta v_3$ for a constant $\beta \in (0, 1)$. You know the values of both α and β (but you don't know the exact value of v_2 or v_3.)

For this exercise, assume that $\alpha = \beta$. What is your best response—that is, the bid that maximizes your expected (quasilinear) utility—as a function of α, β, and v_1?

Exercise 32

Consider the following extension of the sponsored search setting described in lecture. Each bidder i now has a publicly known *quality* β_i, in addition to a private valuation v_i per click. As usual, each slot j has a CTR α_j, and $\alpha_1 \geq \alpha_2 \geq \cdots \geq \alpha_k$. We assume that if bidder i is placed in slot j, then the probability of a click is $\beta_i \alpha_j$. Thus bidder i derives value $v_i \beta_i \alpha_j$ from the jth slot.

Describe a truthful and welfare-maximizing auction for this generalized sponsored search setting. Explicitly discuss both how you assign bidders to slots, and how you define the payments.

Bonus Question (Extra Credit)

Solve the same problem as in Exercise 31, except with $\alpha, \beta \in (0, 1)$ allowed to be different.