CS269I: Exercise Set #8

Due by 11:59 PM on Wednesday, November 30, 2016

Instructions:

(1) You can work individually or in a pair. If you work in a pair, the two of you should submit a single write-up.

(2) Submission instructions: We are using Gradescope for the homework submissions. Go to www.gradescope.com to either login or create a new account. Use the course code 9P8K49 to register for CS269I. Only one person needs to submit the assignment. When submitting, please remember to add your partner's name (if any) in Gradescope.

(3) Please type your solutions if possible. We encourage you to use the LaTeX template provided on the course home page.

(4) Write convincingly but not excessively. You should be able to fit all of your solutions into two pages, if not less.

(5) Except where otherwise noted, you may refer to the course lecture notes and the specific supplementary readings listed on the course Web page *only*.

(6) You can discuss the exercises verbally at a high level with other groups. And of course, you are encouraged to contact the course staff (via Piazza or office hours) for additional help.

(7) If you discuss solution approaches with anyone outside of your group, you must list their names on the front page of your write-up.

(8) No late assignments will be accepted, but we will drop your lowest exercise set score.

Lecture 15 Exercises

Exercise 33

Exhibit an equilibrium of a GSP sponsored search auction that is not social welfare-maximizing.

Exercise 34

Consider extending the sponsored search auction model with click-through rates α_{ij} that can depend arbitrarily on the advertiser i and slot j.[1] Assume that, for each bidder i, higher slots are better: $\alpha_{i1} \geq \alpha_{i2} \geq \cdots \geq \alpha_{ik}$.

Consider the following greedy algorithm:

1. For $j = 1, 2, \ldots, k$:

   (a) Among all bidders not yet assigned to a slot, assign to slot j the bidder with the highest value of $v_i \alpha_{ij}$.

Show by example that (assuming truthful bids) this greedy assignment does not always maximize the social welfare. (In this context, the social welfare of an assignment is $\sum_{i=1}^{k} \alpha_{is(i)} v_i$, where $s(i)$ is the slot to which i is assigned, and where we interpret $\alpha_{is(i)} = 0$ if i does not receive a slot.)

[1] This is related to the problem that the Facebook ad auction faces, with different advertisers bidding on different events and therefore having different CTRs.

Exercise 35

Show that the general VCG mechanism is individually rational, meaning that a truthful bidder is guaranteed nonnegative utility. [Hint: prove that $p_i \le b_i(\omega^*)$, where p_i is the VCG payment by bidder i, ω^* is the outcome chosen by the mechanism, and $b_i(\omega^*)$ is the bid by bidder i for the outcome ω^*.]

Exercise 36

The point of this exercise is to demonstrate that exact optimization is crucial for the nice properties of the VCG mechanism.

Consider the following error-prone algorithm $\mathcal{A}$ for computing the maximum of a set $\{b_1, \ldots, b_n\}$ of numbers: (i) if the second-highest number is exactly one less than the largest number, then output the former; (ii) otherwise, output the largest number.

Consider the following error-prone version of the Vickrey auction:

1. Accept bids $b_1, \ldots, b_n$.
2. Award the item to the bidder i^* corresponding to the output of $\mathcal{A}(b_1, \ldots, b_n)$.
3. Charge the winning bidder $\mathcal{A}(\mathbf{b}_{-i^*})$, where $\mathbf{b}_{-i^*}$ denotes the bids by bidders other than i^*.

Prove that the error-prone Vickrey auction is neither truthful nor individually rational.

Exercise 37

By extending the previous exercise, show by example that substituting a heuristic $\mathcal{A}$ (i.e., a non-exact algorithm) for welfare maximization in the VCG mechanism can result in negative payments (i.e., payments from the mechanism to the bidders).

[Hint: go beyond single-item auctions. For example, consider the setting in Exercise 29.]

Lecture 16 Exercises

Exercise 38

Compute the monopoly price of the following distributions.

(a) The uniform distribution on $[0, a]$ with $a > 0$.

(b) The exponential distribution with rate $\lambda > 0$ (on $[0, \infty)$).

Exercise 39

A valuation distribution meets the *monotone hazard rate (MHR)* condition if its *hazard rate* $\frac{f_i(v_i)}{1-F_i(v_i)}$ is nondecreasing in v_i.[2]

(a) Prove that every distribution meeting the MHR condition is regular.

(b) Which of the distributions in the previous exercise satisfy the MHR condition?

[2] For intuition behind the MHR condition, consider waiting for a light bulb to burn out. Given that the bulb hasn't burned out yet, the probability that it burns out right now is increasing in the amount of time that has elapsed.