

# Comment on “Computing Correlated Equilibria in Multi-Player Games”

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Noah Stein, Pablo Parrilo, and Asu Ozdaglar pointed out an error on page 10, lines 13-14 of our paper “Computing Correlated Equilibria in Multi-Player Games” (Journal of the ACM, 2008). The incorrect phrase is “...which contradicts our assumption that the violating  $y$  is within the current ellipsoid”. This conclusion would be valid for the bounding ellipsoid used for the original dual program (D), but could be incorrect for the larger bounding ellipsoid that is required for the modified dual program (D’) because of its increased bit complexity.

This error is not serious and can be rectified in various ways. Stein, Parillo, and Ozdaglar proposed explicitly bounding the variables of the original dual program (D) by an appropriately large number. The most elegant fix, described next, was suggested by Albert Xin Jiang and Kevin Leyton-Brown (see their arXiv article 1011.0253). Recall that Lemma 3.2 shows that, given a candidate dual solution  $y \geq 0$ , one can compute efficiently a product distribution  $x$  over outcomes such that  $xU^T y = 0$ . Then, by going through the players one at a time and repeatedly using the method of conditional expectations,  $x$  can be converted efficiently into a *pure* strategy profile  $s$  such that  $U_s^T y \geq 0$  (where  $U_s^T$  is the constraint of (D) that corresponds to  $s$ ). This constraint is violated at  $y$ . Proceeding as in the paper but with these “purified” violated constraints yields a modified dual (D’) with bit complexity no larger than that of (D), and the bounding ellipsoid for (D) can now be safely reused for (D’). Solving the new modified primal (P’) yields a correlated equilibrium. This corrected algorithm computes a correlated equilibrium that has polynomial support, an improvement over the less explicit “polynomial combination of products” representation that was produced by the original algorithm.