Game Theory Through The Computational Lens

Some Points of Contact Between Theoretical Computer Science and Economics

Tim Roughgarden (Stanford & LSE)
First Point of Contact

Origins of game theory:
- "Zur Theorie der Gesellschaftsspiele" (1928)
- *Theory of Games and Economic Behavior* (1944, with Morgenstern)

Early contributions to computing:
- ENIAC (UPenn, 1945)
- IAS machine (1945-1951)
Games and Nash Equilibria

Nash’s Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.
Games and Nash Equilibria

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Not an equilibrium

Nash
Example: Chicken

Nash’s Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

\[
\begin{array}{c|cc}
\text{Go straight} & \text{Swerve} \\
\hline
\text{Go straight} & -1, -1 & 1, 0 \\
\text{Swerve} & 0, 1 & 0, 0 \\
\end{array}
\]

Equilibrium #1

Equilibrium #2

Nash
Example: Rock-Paper-Scissors

Nash’s Theorem (1950): every finite noncooperative game has at least one (Nash) equilibrium.

<table>
<thead>
<tr>
<th></th>
<th>Rock</th>
<th>Paper</th>
<th>Scissors</th>
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</thead>
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<tr>
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Rock-Paper-Scissors Championship
Example: Rock-Paper-Scissors

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Nash

Janken robot (University of Tokyo)
Example: Chicken (Reprise)

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Turing and Unsolvable Problems

Origins of computer science:

ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTScheidungsproblem

By A. M. Turing.

[Received 28 May, 1936.—Read 12 November, 1936.]

Importance:
- formal model of computation ("Turing machine")
Turing and Unsolvable Problems

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THE ENTSCHEIDUNGSPROBLEM

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Importance:

- formal model of computation
- existence of unsolvable problems
  - example: the “halting problem”
Outline

1. Introduction: von Neumann, Nash, and Turing
2. **Approximation: A Constructive Compromise**
3. Auction Design: The Rubber Meets the Road
4. Complexity: Critiquing the Nash Equilibrium
5. Conclusions
THE PRICE OF ANARCHY
IS IT OKAY TO BE SELFISH?
**Pigou's Example**

**Example**: one unit of traffic travelling from \( s \) to \( t \)

- \( c(x) = x \) \( \Rightarrow \) delay depends on congestion
- \( c(x) = 1 \) \( \Rightarrow \) no congestion effects

**Question**: what will selfish drivers do?

- assume everyone wants smallest-possible delay
- [Pigou 1920]
Equilibrium in Pigou’s Example

Claim: all traffic will take the top route.

Reason:
- $\varepsilon > 0 \rightarrow$ traffic on bottom is envious
- $\varepsilon = 0 \rightarrow$ equilibrium
  - all traffic incurs one unit of delay
Can We Do Better?

Consider instead: traffic split equally

Improvement:
- half of traffic has delay 1 (same as before)
- half of traffic has delay $\frac{1}{2}$ (much improved!)
- “price of anarchy” [Kousoupias/Papadimitriou 99] = $\frac{4}{3}$
Braess’s Paradox (1968)

Initial Network:

Augmented Network:

Commute time = 1.5

Now what?
Braess’s Paradox (1968)

Initial Network: 

Augmented Network: 

Commute time = 1.5 

Commute time = 2 

Price of anarchy = 4/3 in augmented network (again!)
Strings and Springs

Cohen and Horowtiz (Nature, 1991)
When Is the Price of Anarchy Bounded?

Examples so far:

Question: does the price of anarchy stay small in bigger networks?
Defn: *affine cost function* has the form
\[ c_e(x) = a_e x + b_e \quad [\text{for } a_e, b_e \geq 0]. \]

Theorem: [Roughgarden/Tardos 02] for every network with affine cost functions:

\[ \text{average delay of equilibrium} \leq \frac{4}{3} \times \text{optimal average delay} \]

Metaphor: electrical current in networks of resistors.
Question: why wasn’t this proved in the 20th century by economists, or transportation scientists, or...?
Digression: Map Coloring

Rule: adjacent countries get distinct colors.

need 3 colors (one per country)
Digression: Map Coloring

Rule: adjacent countries get distinct colors.

needs 3 colors (odd cycle)
Digression: Map Coloring

Rule: adjacent countries get distinct colors.

- needs a 4\textsuperscript{th} color (touches entire odd cycle)
- needs 3 colors (odd cycle)
Digression: Map Coloring

Rule: adjacent countries get distinct colors.
NP-Completeness


- solvable in principle via exhaustive search
- no significantly better algorithms exist (if $P \neq NP$)
- compromises required (use heuristics, tackle special/small cases, buy lots of hardware, etc.)

Stephen Cook  Richard Karp  Leonid Levin
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WASHINGTON — The government took a big step on Friday to aid the creation of new high-speed wireless Internet networks that could fuel the development of the next generation of smartphones and tablets, and devices that haven’t even been thought of yet.

The five-member Federal Communications Commission unanimously approved a sweeping, though preliminary, proposal to reclaim public airwaves now used for broadcast television and auction them off for use in wireless broadband networks, with a portion of the proceeds paid to the broadcasters.

The initiative, which the F.C.C. said would be the first in which any government would pay to reclaim public airwaves with the intention of selling them, would help satisfy what many industry experts say is booming demand for wireless Internet capacity.

Mobile broadband traffic will increase more than thirtyfold by 2015, the commission estimates. Without additional airwaves to handle the traffic, officials say, consumers will face more dropped calls, connection delays and slower downloads of data.
FCC Incentive Auction

Broadcast Television Incentive Auction (3/16-3/17):

- Reverse Auction: buy TV broadcast licenses
  - Final tally: ≈$10 billion cost
- Forward Auction: sell wireless broadband licenses
  - Final tally: ≈$20 billion revenue
- Revenue to cover auction costs, fund a new first responder network, reduce the deficit (!)
  - “Middle Class Tax Relief and Job Creation Act”
Bad Designs Cost Billions

New Zealand, 1990:
- simultaneous sealed-bid 2\textsuperscript{nd}-price auctions for 10 interchangeable TV broadcasting licenses
  - creates tricky coordination problem
- projected revenue: 250M; actual = 36M
- often huge difference between top two bids

US, 2016: $10s of billions at stake.
Reverse Auction Format

“Descending Clock Auction”:

[Milgrom/Segal 14] (extending [Moulin/Shenker 01],[Mehta/Roughgarden/Sundararajan 09])

- each round, each broadcaster offered a buyout price (decreases over time)
  - declined \(\Rightarrow\) exits, retains license
  - accepted \(\Rightarrow\) moves to next round
The Stopping Rule

**Intuition:** stop auction when prices are as low as possible, subject to clearing enough spectrum.

**Example goal:** from channels 38-51, clear 10 of them nationwide.

**Issue:** buyouts scattered across channels.

**Solution:** repack remaining TV stations into a smaller subset of channels (e.g., 38-41).
The Repacking Problem
The Repacking Problem
The Repacking Problem
The Need for Algorithms

Cool fact: state-of-the-art algorithms for solving NP-complete problems both necessary and sufficient to solve repacking problem quickly.

[Leighton-Brown et al. 13, 14, 17]

- encode as satisfiability (SAT)
- use presolvers, solver configurations tuned to interference constraints, caching tricks
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Constructive Nash’s Theorem?

Nash’s Theorem (1950): every finite game has at least one Nash equilibrium.

- many interpretations require the determination of an equilibrium
  - by the players, designer, etc.
- fixed-point proof offers no help

Challenge: “more constructive” version.

- cf., “bounded rationality” [Simon]
Classifying the complexity of computing a Nash equilibrium

Idea: is computing a Nash equilibrium NP-complete?

Answer: [Megiddo 88] probably not.
- reason: guaranteed existence

Upshot: Need to refine NP to obtain the right complexity class.

Proposal: [Papadimitriou 94] PPAD.

Papadimitriou
Nash Meets Turing

Theorem: [Daskalakis/Goldberg/Papadimitriou 06, Chen/Deng/Teng 06] Computing a Nash equilibrium is PPAD-complete.

- also intractable in other senses [Etessami/Yannakakis 07, Hart/Mansour 09]
- even approximate Nash equilibria [Rubinstein 15,16], [Roughgarden/Weinstein 16], [Babichenko/Rubinstein 17]

Interpretation: no general constructive version of Nash’s theorem. Compromises required.
Conclusions

- many points of contact between theory CS and game theory/econ over past 15 years
- many 21st-century computer science applications require economic reasoning
  - routing in communication networks
  - auctions for online advertising
  - cryptocurrencies
  - etc.
Conclusions

- many points of contact between theory CS and game theory/econ over past 15 years
- many 21st-century computer science applications require economic reasoning
- theory CS can articulate computational barriers and offers constructive compromises
  - approximation
  - workarounds for NP-complete problems
  - intractability of Nash equilibria