CS364B: Exercise Set #3

Due by the beginning of class on Wednesday, January 29, 2014

Instructions:

(1) Turn in your solutions to all of the following exercises directly to the TA (Okke). Please type your solutions if possible and feel free to use the LaTeX template provided on the course home page. Email your solutions to `cs364b-win1314-submissions@cs.stanford.edu`. If you prefer to hand-write your solutions, you can give it to Okke in person at the start of the lecture.

(2) Your solutions will be graded on a "check/minus/zero" system, with "check" meaning satisfactory and "minus" meaning needs improvement.

(3) Solve these exercises and write up your solutions on your own. You may, however, discuss the exercises verbally at a high level with other students. You may also consult any books, papers, and Internet resources that you wish. And of course, you are encouraged to contact the course staff (via Piazza or office hours) to clarify the questions and the course material.

(4) No late assignments will be accepted.

Lecture 5 Exercises

Exercise 18

Let $v_1, v_2, \ldots, v_m$ be nonnegative numbers. Define a valuation function v on the subsets of $U = \{1, 2, \ldots, m\}$ by

$$v(S) = \max_{T \subseteq S \,:\, |T| \leq k} \sum_{j \in T} v_j.$$

Prove that v satisfies the gross substitutes condition.

Exercise 19

Let v be a valuation function defined on the item set U. For a good $j \in U$, define $v|_j$ as the marginal valuation given that a bidder already has the item j. That is, for $S \subseteq U \setminus \{j\}$, $v|_j(S)$ is defined as $v(S \cup \{j\}) - v(\{j\})$.

Prove that if v satisfies the gross substitutes condition (with item set U), then so does $v|_j$ (with item set $U \setminus \{j\}$).

Exercise 20

Recall the two-bidder two-item example from lecture, with $U = \{A, B\}$, $v_1(U) = 3$, $v_1(S) = 0$ for $S \neq U$, and $v_2(S) = 2$ for every non-empty S. Prove that there is no Walrasian equilibrium.

Lecture 6 Exercises

Exercise 21

Suppose you are given explicitly as input a value $v_i(S)$ for each bidder i and each bundle $S \subseteq U$. You can assume that the $v_i(S)$'s are nonnegative and monotone (i.e., $S \subseteq T$ implies $v_i(S) \leq v_i(T)$) but should make no other assumptions. Explain how to compute an allocation $(S_1, \ldots, S_n)$ that maximizes the welfare $\sum_{i=1}^{n} v_i(S_i)$ in time polynomial in the input size.

[Hint: dynamic programming.]

Exercise 22

In lecture we constructed a subroutine $\mathcal{A}$ that, given black-box access (i.e., value and demand queries) to gross substitutes valuations $v_1, \ldots, v_n$, computes the welfare of an optimal allocation in a polynomial (in n and m) number of steps. Explain how to use $\mathcal{A}$ repeatedly to compute an optimal allocation.

[Hint: you might want to look up "self reducibility." You might also find Exercise 19 relevant.]