Exercise 11

Recall that in the maximum-weight bipartite matching problem, the input is a bipartite graph $G = (V \cup W, E)$ with a nonnegative weight $w_e$ per edge, and the goal is to compute a matching $M$ that maximizes $\sum_{e \in M} w_e$.

In the minimum-cost perfect bipartite matching problem, the input is a bipartite graph $G = (V \cup W, E)$ such that $|V| = |W|$ and $G$ contains a perfect matching, and a nonnegative cost $c_e$ per edge, and the goal is to compute a perfect matching $M$ that minimizes $\sum_{e \in M} c_e$.

Give a linear-time reduction from the former problem to the latter problem.

Exercise 12

Suppose you are given an undirected bipartite graph $G = (V \cup W, E)$ and a positive integer $b_v$ for every vertex $v \in V \cup W$. A $b$-matching is a subset $M \subseteq E$ of edges such that each vertex $v$ is incident to at most $b_v$ edges of $M$. (The standard bipartite matching problem corresponds to the case where $b_v = 1$ for every $v \in V \cup W$.)

Prove that the problem of computing a maximum-cardinality bipartite $b$-matching reduces to the problem of computing a (standard) maximum-cardinality bipartite matching in a bigger graph. Your reduction should run in time polynomial in the size of $G$ and in $\max_{v \in V \cup W} b_v$.

Exercise 13

A graph is $d$-regular if every vertex has $d$ incident edges. Prove that every $d$-regular bipartite graph is the union of $d$ perfect matchings. Does the same statement hold for $d$-regular non-bipartite graphs?

[Hint: Hall’s theorem.]

Exercise 14

Prove that the minimum-cost perfect bipartite matching problem reduces, in linear time, to the minimum-cost flow problem defined in Lecture #6.
Exercise 15

In the edge cover problem, the input is a graph $G = (V, E)$ (not necessarily bipartite) with no isolated vertices, and the goal is to compute a minimum-cardinality subset $F \subseteq E$ of edges such every vertex $v \in V$ is the endpoint of at least one edge in $F$. Prove that this problem reduces to the maximum-cardinality (non-bipartite) matching problem.