Can Nondeterminism Help Complementation?

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Logic in Computer Science
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Determinization and Complementation

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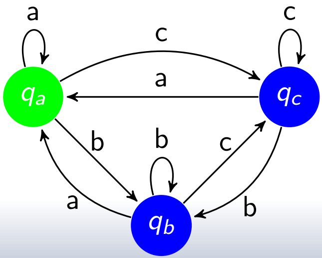
☞ But determinization can do more: execute game strategies.

ω-Automata

A (**nondeterministic**) finite automaton $\mathcal{A}$ is $\langle Q, \Sigma, I, \Delta, \mathcal{F} \rangle$:



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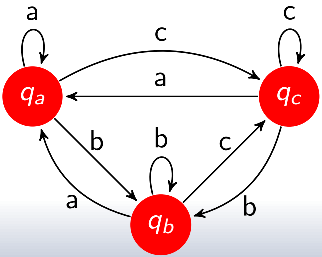

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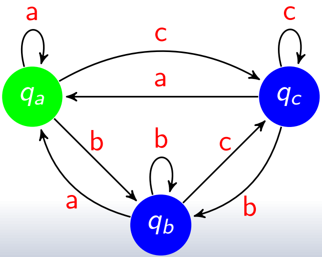

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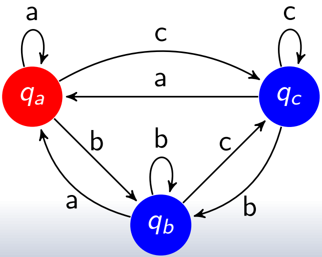

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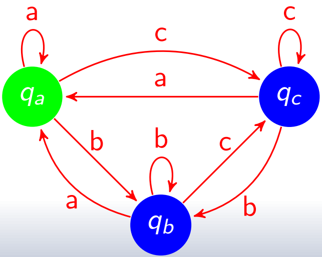

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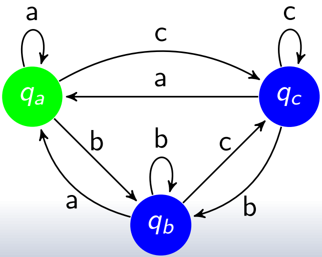

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☞ $\mathcal{F}$: the acceptance condition.

Acceptance Conditions of ω-Automata

$I = [1..k]$, $G, B : I \to 2^Q$

- ☞ Generalized Büchi: $\langle B \rangle_I$: $\forall i \in I, \mathit{Inf}(\varrho) \cap B(i) \neq \emptyset$.

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- Parity: $\langle G, B \rangle_I$ with $B(1) \subset G(1) \subset \cdots \subset B(k) \subset G(k)$.
- ☞ Rabin: $[G, B]_I$: $\exists i \in I,\ \mathit{Inf}(\varrho) \cap G(i) \neq \emptyset \wedge \mathit{Inf}(\varrho) \cap B(i) = \emptyset$.

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Rabin Automata

Express fair termination, that is, all infinite computations are unfair.

Complementation Complexities

Type	Bound		Lower	Upper
Büchi	$2^{\Theta(n \lg n)}$		Mic88	Saf88
Generalized Büchi	$2^{\Theta(n \lg nk)}$	$k = O(2^n)$	Yan06	KV05
Streett	$2^{\Theta(n \lg n + nk \lg k)}$	$k = O(n)$	CZ11	CZ11
	$2^{\Theta(n^2 \lg n)}$	$k = \omega(n)$		
Rabin	$2^{\Theta(nk \lg n)}$	$k = O(2^n)$	CZL09	KV05
Parity	$2^{\Theta(n \lg n)}$	$k = O(n)$	Mic88	CZ11

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- What kind of help in term of state complexity?
- Is determinization much harder than complementation?

☞ Not much in the case of Büchi and absolutely zero in the case of NFA.

Determinization Complexities

Type	Lower Bound	Upper Bound	
Büchi	$2^{\Omega(n \lg n)}$	$2^{O(n \lg n)}$	
Generalized Büchi	$2^{\Omega(n \lg nk)}$	$2^{O(nk \lg nk)}$	$k = O(2^n)$
Streett	$2^{\Omega(n \lg n + nk \lg k)}$	$2^{O(nk \lg nk)}$	$k = O(n)$
	$2^{\Omega(n^2 \lg n)}$	$2^{O(nk \lg nk)}$	$k = \omega(n)$
Rabin	$2^{\Omega(nk \lg n)}$	$2^{O(nk \lg nk)}$	$k = O(2^n)$
Parity	$2^{\Omega(n \lg n)}$	$2^{O(nk \lg nk)}$	$k = O(n)$

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Streett Determinization

- Our Streett complementation uses two tree structures: TOP and ITS.
- TOPs are for counting the number of certain integer matrices.
- Surprisingly, TOPs are very similar to Safra trees for Büchi determinization.
- Does ITS play a role in Safra trees for Streett determinization?

☞ YES!

Streett Determinization

Cost	Safra's Original	Our Improved	
Trees	$2^{O(nk)}$	$2^{O(n \lg n)}$	$k = O(2^n)$
Node Names	$2^{O(nk \lg nk)}$	$2^{O(n \lg n)}$	$k = O(2^n)$
Node Labels	$2^{O(n \lg n)}$	$2^{O(n \lg n)}$	$k = O(2^n)$
Edge Labels	$2^{O(nk \lg nk)}$	$2^{O(n \lg n + nk \lg k)}$	$k = O(n)$
		$2^{O(n^2 \lg n)}$	$k = \omega(n)$
Total	$2^{O(nk \lg nk)}$	$2^{O(n \lg n + nk \lg k)}$	$k = O(n)$
		$2^{O(n^2 \lg n)}$	$k = \omega(n)$

Complementation and Determinization Complexities

Type	Bound	
Büchi	$2^{\Theta(n \lg n)}$	
G Büchi	$2^{\Theta(n \lg nk)}$	$k = O(2^n)$
Streett	$2^{\Theta(n \lg n + nk \lg k)}$	$k = O(n)$
	$2^{\Theta(n^2 \lg n)}$	$k = \omega(n)$
Rabin	$2^{\Theta(nk \lg n)}$	$k = O(2^n)$
Parity	$2^{\Theta(n \lg n)}$	$k = O(n)$
NFA	$\Theta(2^n)$	

Concluding Remarks

Our Investigation

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☞ Are these bounds just coincidence?

Thank you for your attention!

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