Problem 1

Consider the following recursive definition of a function $f : \mathbb{N} \to \mathbb{N}$.

$$f(1) = 1$$
$$f(n) = 2f(n-1)$$

Find a non-recursive definition for f, and prove by induction that this definition is correct.

Problem 2

Prove that $3^n < n!$ whenever n is an integer greater than 6.

Problem 3

Consider the following game for two players. Begin with a pile of n coins for some $n \geq 0$. The first player then takes between one and ten coins out of the pile, then the second player takes between one and ten coins out of the pile. This process repeats until some player has no coins to take; at this point, that player loses the game. Prove that if the pile begins with a multiple of eleven coins in it, the second player can always win.

Problem 4

Prove by induction that for any $m, n \in \mathbb{N}$, we have $m!n! \leq (m+n)!$.