

## Problem 1

Prove or disprove the validity of the following statement in propositional logic:

$$\text{De Morgan's law: } \neg(p \vee q) \rightarrow (\neg p \wedge \neg q)$$

## Problem 2

State whether the following statements in propositional logic are equivalent or not, using truth tables:

$$\neg(p \rightarrow q) \text{ and } p \rightarrow \neg q$$

## Problem 3

Translate the lyrics of Led Zeppelin's "Stairway to Heaven" into FOL using the predicates:

$$\begin{aligned} & \textit{Lady}(x), \text{ which states that } x \text{ is a lady;} \\ & \textit{Glitters}(x), \text{ which states that } x \text{ glitters;} \\ & \textit{IsSureIsGold}(x, y), \text{ which states that } x \text{ is sure that } y \text{ is gold;} \\ & \textit{Buying}(x, y), \text{ which states that } x \text{ buys } y; \\ & \textit{StairwayToHeaven}(x), \text{ which states that } x \text{ is a Stairway to Heaven} \end{aligned}$$

Write a statement in FOL that says "There's a lady who's sure all that glitters is gold, and she's buying a Stairway to Heaven."

## Problem 4

**Constructing DFAs:** Suppose that you are taking your dog out on a walk on a straight line path. Your dog is on a leash that is two units long, so the distance between you and your dog cannot be more than two units. You and your dog start at the same positions. Consider the alphabet  $\Sigma = \{ \mathbf{Y}, \mathbf{D} \}$ . A string in  $\Sigma^*$  can be thought of as a series of events in which either you or your dog moves forward one unit. For example, the string "YYDD" means that you took two steps, and then your dog took two. Let  $L = \{w \in \Sigma^* \mid w \text{ describes a series of steps that ensures that you and your dog are never more than two steps apart}\}$ . Construct a DFA for  $L$ .

## Problem 5

Construct equivalent DFA and NFA for the language  $L$  consisting of the alphabet  $\Sigma = \{0, 1\}$  such that  $L$  consists of all words that are zero or more repetitions of the string "10".

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