This homework is due on April 18, by start of class, 12:50pm.

1. **Problem 1**

   This question is designed to get you used to the notation and mathematical conventions surrounding sets. We strongly suggest working through this problem and double-checking that your answers are correct before starting Problem 2.

   Consider the following sets:

   - $W = \{1, 2, 3, 4\}$
   - $X = \{2, 2, 2, 1, 4, 3\}$
   - $Y = \{1, \{2\}, 1, \{3, 4\}, \{\{3, 4\}\}, \{\{4, 4, 3\}\}\}$
   - $Z = \{1, 3\}$

   Answer the following questions. No explanations are necessary except for the last part.

   (a) Which pairs of the above sets, if any, are equal to each other?

   (b) Is $Z \in W$? Is $Z \subseteq W$?

   (c) Is $Z \in \mathcal{P}(W)$? Is $Z \subseteq \mathcal{P}(W)$? ($\mathcal{P}(W)$ is the power set of W, that is, the set of all subsets of W. For example, the power set of $\{0, 2, 3\}$ is: $\{\emptyset, \{0\}, \{2\}, \{3\}, \{0, 2\}, \{0, 3\}, \{2, 3\}, \{0, 2, 3\}\}$)

   (d) What is $W \cap Y$? What is $W \cup Y$?

   (e) What is $|X|$? What is $|Y|$? Briefly explain each answer.

2. **Problem 2** The following are logical statements about *finite* sets. Identify the false statements (the ones that are not true for all finite sets A, B, and C), and for each explain why it is false. In each explanation, give concrete examples of sets for which the statement does not hold. No credit will be given for correct identifications without explanations.

   (a) $(A \in B \wedge B \in C) \rightarrow A \in C$

(b) $(A \subseteq B \wedge B \subseteq C) \rightarrow A \subseteq C$

(c) $A \in B \rightarrow \mathcal{P}(A) \in \mathcal{P}(B)$

(d) $A \cap \mathcal{P}(A) = A$

3. **Problem 3**

   (a) Prove that if n is a multiple of 3 then n^2 is a multiple of 3

   (b) Prove that if n is not a multiple of 3 then the remainder when dividing n^2 by 3 is always 1. (Hint: use proof by cases)

4. **Problem 4**

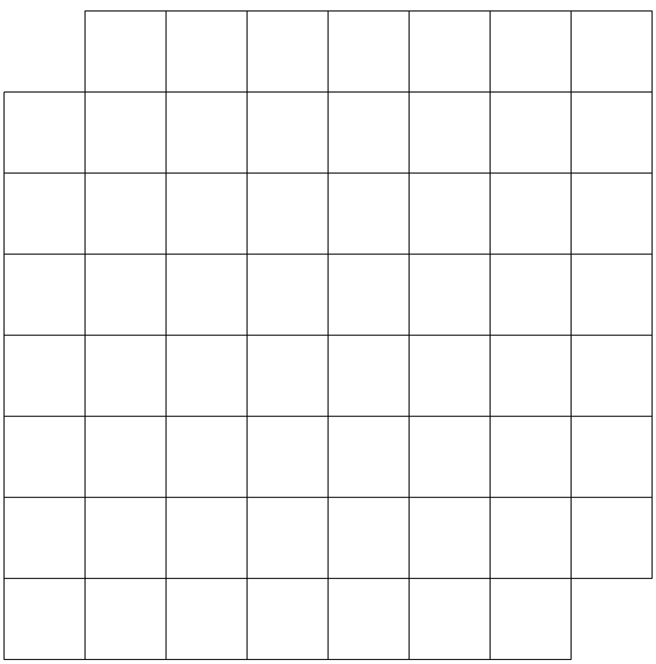
   Prove that $\sqrt{3}$ is irrational using a method similar to the one we used in class.

5. **Problem 5**

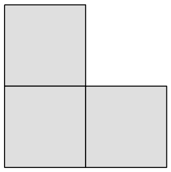
   Explain where this proof method goes wrong when it is used to prove that $\sqrt{4}$ is irrational.

6. **Problem 6**

   Suppose that you have a standard 8×8 chessboard with two opposite corners removed:



   In the course notes (page 62), there's a proof that it's impossible to tile this board using 2×1 dominoes. This question considers what happens if you try to tile the board using right triominoes, L-shaped tiles that look like this:



(a) Prove that it is impossible to tile an 8×8 board missing two opposite corners with right triominoes.

(b) For $n \geq 3$, is it ever possible to tile an $n \times n$ board missing two opposite corners with right triominoes? If so, find a number $n \geq 3$ such that its possible and show how to tile that board with right triominoes. If not, prove that for every $n \geq 3$, it's impossible to tile an $n \times n$ board missing two opposite corners with right triominoes.